Supplemental Material to the article
"Model for description of relaxation of quantum systems with closely spaced energy levels"

In this Supplemental material we present the derivation of the master equation from the main text.

In the main text it has been mentioned that the Hamiltonian of two coupled harmonic oscillators with the same frequencies has the form

$$\hat{H}_S = (\omega + \Omega)\hat{b}^\dagger \hat{b} + (\omega - \Omega)\hat{c}^\dagger \hat{c}, \tag{1}$$

$$\hat{b} = (\hat{a}_1 + \hat{a}_2)/\sqrt{2}, \quad \hat{c} = (\hat{a}_1 - \hat{a}_2)/\sqrt{2}. \tag{2}$$

The Hamiltonian (13) from the main text has the form $\hat{\hat{H}}_{SR} = \lambda \sum_k \hat{S}_k \hat{R}_k$ where k is the index of the reservoir. This Hamiltonian describes the interaction between oscillators and their reservoirs. In the second order in small parameter λ, the change of density matrix over time Δt can be presented as

$$\begin{gathered}\hat{\hat{\rho}}_{S2}(t_0 + \Delta t) = -\sum_{k=1,2} \int\limits_{t_0}^{t_0+\Delta t} dt_1 \int\limits_{t_0}^{t_1} dt_2 \\ \Big(\hat{\hat{\rho}}_{S0}(t_0)\hat{\hat{S}}_k(t_2)\hat{\hat{S}}_k(t_1) TR_k(t_2 - t_1) - \\ -\hat{\hat{S}}_k(t_2)\hat{\hat{\rho}}_{S0}(t_0)\hat{\hat{S}}_k(t_1) TR_k(-(t_2 - t_1)) - \\ -\hat{\hat{S}}_k(t_1)\hat{\hat{\rho}}_{S0}(t_0)\hat{\hat{S}}_k(t_2) TR_k(t_2 - t_1) + \\ +\hat{\hat{S}}_k(t_1)\hat{\hat{S}}_k(t_2)\hat{\hat{\rho}}_{S0}(t_0) TR_k(-(t_2 - t_1))\Big),\end{gathered} \tag{3}$$

where $\hat{\hat{S}}_k(t)$ is an operator of the interaction between the system and kth reservoir in the interaction representation. To calculate $\hat{\hat{S}}_k(t)$, we use Baker–Campbell–Hausdorff formula

$$e^{\hat{A}} \hat{B} e^{-\hat{A}} = \hat{A} + [\hat{A}, \hat{B}]/1! + [\hat{A}, [\hat{A}, \hat{B}]]/2! + ... \tag{4}$$

It is obvious that $[i\hat{H}_S, \hat{b}] = -i(\omega + \Omega)\hat{b}$, $[i\hat{H}_S, \hat{c}] = -i(\omega - \Omega)\hat{c}$, $\hat{\hat{S}}^\dagger = (\hat{\hat{S}})^\dagger$, therefore Eq. (4) gives

$$\begin{gathered}\hat{\hat{S}}_k(t) = \exp(i\hat{H}_S t)(\hat{a}_k + \hat{a}_k^\dagger)\exp(-i\hat{H}_S t) = \\ = \frac{1}{\sqrt{2}}\left(\hat{b}e^{-i(\omega+\Omega)t} + (-1)^{k-1}\hat{c}e^{-i(\omega-\Omega)t}\right) + \\ + \frac{1}{\sqrt{2}}\left(\hat{b}^\dagger e^{i(\omega+\Omega)t} + (-1)^{k-1}\hat{c}^\dagger e^{i(\omega-\Omega)t}\right).\end{gathered} \tag{5}$$

We substitute (5) into (3) and introduce the variable $\tau = t_2 - t_1$, as a result the first term from Eq. (3) takes the form

$$\int_{t_0}^{t_0+\Delta t} dt_1 \int_{t_0}^{t_1} dt_2 \hat{\hat{\rho}}_{S0}(t_0)\hat{\hat{S}}_1(t_2)\hat{\hat{S}}_1(t_1)TR_1(\tau) = \tag{6}$$

$$= \frac{1}{2}\hat{\hat{\rho}}_{S0}(t_0) \int_{t_0}^{t_0+\Delta t} dt_1 \int_{t_0}^{t_1} dt_2 TR_1(\tau)$$

$$\Big(\hat{b}\hat{b}^\dagger e^{-i(\omega+\Omega)\tau} + \hat{b}^\dagger \hat{b} e^{i(\omega+\Omega)\tau} +$$

$$\hat{b}\hat{c}^\dagger e^{-i(\omega+\Omega)\tau - i2\Omega t_1} + \hat{b}^\dagger \hat{c} e^{i(\omega+\Omega)\tau + i2\Omega t_1} +$$

$$\hat{c}\hat{b}^\dagger e^{-i(\omega-\Omega)\tau + i2\Omega t_1} + \hat{c}^\dagger \hat{b} e^{i(\omega-\Omega)\tau - i2\Omega t_1} +$$

$$\hat{c}\hat{c}^\dagger e^{-i(\omega-\Omega)\tau} + \hat{c}^\dagger \hat{c} e^{i(\omega-\Omega)\tau} +$$

$$\hat{b}\hat{b} e^{-i(\omega+\Omega)(\tau+2t_1)} + \hat{b}\hat{c} e^{-i(\omega+\Omega)\tau - i2\omega t_1} +$$

$$\hat{c}\hat{b} e^{-i(\omega-\Omega)\tau - i2\omega t_1} + \hat{c}\hat{c} e^{-i(\omega-\Omega)(\tau+2t_1)} +$$

$$\hat{b}^\dagger \hat{b}^\dagger e^{i(\omega+\Omega)(\tau+2t_1)} + \hat{b}^\dagger \hat{c}^\dagger e^{i(\omega+\Omega)\tau + i2\omega t_1} +$$

$$\hat{c}^\dagger \hat{b}^\dagger e^{i(\omega-\Omega)\tau + i2\omega t_1} + \hat{c}^\dagger \hat{c}^\dagger e^{i(\omega-\Omega)(\tau+2t_1)}\Big).$$

The terms of Eq. (6) that contain the factor $\exp(\pm(\omega \pm \Omega)\tau)$ and do not depend on t_1, can be transformed according to

$$\int_{t_0}^{t_0+\Delta t} dt_1 \int_{t_0-t_1}^{0} d\tau e^{-i(\omega+\Omega)\tau} TR_1(\tau) \approx \tag{7}$$

$$\approx \int_{t_0}^{t_0+\Delta t} dt_1 \int_{-\infty}^{0} d\tau e^{-i(\omega+\Omega)\tau} TR_1(\tau) = G_{1-}(\omega+\Omega)\Delta t,$$

where $G_{k\pm}(\omega)$ are one-sided Fourier transformations of reservoir correlation function

$$G_{1-}(\omega) = \int_{-\infty}^{0} d\tau e^{-i\omega\tau} TR_1(\tau), \tag{8}$$

$$G_1(\omega) = \int_{-\infty}^{+\infty} d\tau e^{-i\omega\tau} TR_1(\tau) = G_{1-}(\omega) + G_{1+}(\omega). \tag{9}$$

We consider the case $\Omega \ll \omega$ and $\omega^{-1} \ll \Delta t \ll \Omega^{-1}$. Thus, the terms depending on t_1 can be separated into two groups each of which contains either fast or slowly oscillating terms. Fast oscillating terms are proportional to $\exp(\pm i(\omega \pm \Omega)t_1)$ or to $\exp(\pm i\omega t_1)$. Slowly oscillating terms are proportional to $\exp(\pm i\Omega t_1)$. Fast oscillating terms give zero after averaging over time. For instance, for the term from Eq. (3) which is proportional to $\hat{b}\hat{c}$, we have

$$\int_{t_0}^{t_0+\Delta t} dt_1 e^{-i2\omega t_1} \int_{t_0-t_1}^{0} d\tau e^{-i(\omega+\Omega)\tau} TR_1(\tau) \approx \tag{10}$$

$$\approx [t_0 - t_1 = -\infty] \approx G_{1-}(\omega+\Omega) \int_{t_0}^{t_0+\Delta t} dt_1 e^{-i2\omega t_1} = 0,$$

At the same time, slowly oscillating terms are transformed as follows

$$\begin{aligned}
&\int\limits_{t_0}^{t_0+\Delta t} dt_1 e^{i2\Omega t_1} \int\limits_{t_0-t_1}^{0} d\tau e^{i(\omega+\Omega)\tau} TR_1(\tau) \approx \\
&\approx \int\limits_{t_0}^{t_0+\Delta t} dt_1 e^{i2\Omega t_1} \int\limits_{-\infty}^{0} d\tau e^{i(\omega+\Omega)\tau} TR_1(\tau) = \\
&= G_{1-}(-(\omega+\Omega)) e^{i2\Omega t_0} \frac{\exp(i2\Omega\Delta t)-1}{i2\Omega} \approx \qquad (11)\\
&\approx G_{1-}(-(\omega+\Omega)) e^{i2\Omega t_0} \Delta t. \qquad (12)
\end{aligned}$$

The same calculations can be done for the second, the third and the forth terms of Eq. (3). As a result, the right hand side of Eq. (6) becomes proportional to the Δt. Finally, after replacing $\Delta/\Delta t$ with $\partial/\partial t$, we obtain Eq. (16) from the main text.

It should be mentioned that one can move back to the Schrödinger picture via the formula

$$\begin{aligned}
\dot{\hat{\rho}} &= d\left(\exp\left(-i\hat{H}_S t\right) \tilde{\hat{\rho}} \exp\left(i\hat{H}_S t\right)\right)/dt = \\
&= -i\left[\hat{H}_S, \hat{\rho}\right] + \exp\left(-i\hat{H}_S t\right) \dot{\tilde{\hat{\rho}}} \exp\left(i\hat{H}_S t\right) \qquad (13)
\end{aligned}$$

and obtain

$$\begin{aligned}
&\frac{1}{\lambda^2}\frac{\partial \hat{\rho}_S}{\partial t} = -i\frac{1}{\lambda^2}[\hat{H}_S, \hat{\rho}_S] + \\
&\frac{G_1(-(\omega+\Omega))}{4} L[\hat{b}, \hat{b}^\dagger] + \frac{G_1(\omega+\Omega)}{4} L[\hat{b}^\dagger, \hat{b}] + \\
&\frac{G_1(-(\omega-\Omega))}{4} L[\hat{c}, \hat{c}^\dagger] + \frac{G_1(\omega-\Omega)}{4} L[\hat{c}^\dagger, \hat{c}] + \\
&\frac{G_{1-}(-(\omega-\Omega)) + G_{1+}(-(\omega+\Omega))}{4} L[\hat{b}, \hat{c}^\dagger] + \\
&\frac{G_{1-}(\omega+\Omega) + G_{1+}(\omega-\Omega)}{4} L[\hat{c}^\dagger, \hat{b}] + \\
&\frac{G_{1-}(\omega-\Omega) + G_{1+}(\omega+\Omega)}{4} L[\hat{b}^\dagger, \hat{c}] + \\
&\frac{G_{1-}(-(\omega+\Omega)) + G_{1+}(-(\omega-\Omega))}{4} L[\hat{c}, \hat{b}^\dagger] - \\
&- LS_I + LS_C + \left((1) \to (2),\ \hat{c} \to -\hat{c},\ \hat{c}^\dagger \to -\hat{c}^\dagger\right), \qquad (14)
\end{aligned}$$

where $L[\hat{A}, \hat{B}] = 2\hat{A}\hat{\rho}_S\hat{B} - \hat{\rho}_S\hat{B}\hat{A} - \hat{B}\hat{A}\hat{\rho}_S$, and

$$\begin{aligned}
LS_I = &\frac{G_{1+}(-(\omega+\Omega)) - G_{1-}(-(\omega+\Omega))}{4}\left[\hat{b}^\dagger\hat{b}, \hat{\rho}_S\right] + \\
&+ \frac{G_{1+}(\omega+\Omega) - G_{1-}(\omega+\Omega)}{4}\left[\hat{b}\hat{b}^\dagger, \hat{\rho}_S\right] + \\
&+ \frac{G_{1+}(-(\omega-\Omega)) - G_{1-}(-(\omega-\Omega))}{4}\left[\hat{c}^\dagger\hat{c}, \hat{\rho}_S\right] + \\
&+ \frac{G_{1+}(\omega-\Omega) - G_{1-}(\omega-\Omega)}{4}\left[\hat{c}\hat{c}^\dagger, \hat{\rho}_S\right], \qquad (15)
\end{aligned}$$

$$LS_C = \frac{G_{1-}(-(\omega-\Omega)) - G_{1+}(-(\omega+\Omega))}{4}\left[\hat{c}^\dagger\hat{b}, \hat{\rho}_S\right] +$$
$$+\frac{G_{1-}(\omega+\Omega) - G_{1+}(\omega-\Omega)}{4}\left[\hat{b}\hat{c}^\dagger, \hat{\rho}_S\right] +$$
$$+\frac{G_{1-}(\omega-\Omega) - G_{1+}(\omega+\Omega)}{4}\left[\hat{c}\hat{b}^\dagger, \hat{\rho}_S\right] +$$
$$+\frac{G_{1-}(-(\omega+\Omega)) - G_{1+}(-(\omega-\Omega))}{4}\left[\hat{b}^\dagger\hat{c}, \hat{\rho}_S\right]. \tag{16}$$

It can be seen that in the Schrödinger picture the relaxation superoperators do not depend on time. The term (15) can be included in the Hamiltonian part of Eq. (14). This will slightly change the eigenfrequencies of the system initially described by the Hamiltonian $\hat{H}_S$. Further, we neglect this change of eigenfrequencies.

Now, we derive the equations for the average occupation operators of the system eigenmodes. The dynamics of operator $\hat{a}$ which does not depend on time explicitly obeys the equation $d\langle\hat{a}\rangle/dt = Tr\left(\dot{\hat{\rho}}_S\hat{a}\right)$. We define the function Λ

$$\Lambda\left(\hat{X},\hat{Y},\hat{a}\right) = Tr\left(L[\hat{X},\hat{Y}]\hat{a}\right) = 2Tr(\hat{\rho}_S\hat{Y}\hat{a}\hat{X}) -$$
$$- Tr(\hat{\rho}_S\hat{Y}\hat{X}\hat{a}) - Tr(\hat{\rho}_S\hat{a}\hat{Y}\hat{X}) = 2\langle\hat{Y}\hat{a}\hat{X}\rangle - \langle\hat{Y}\hat{X}\hat{a}\rangle$$
$$- \langle\hat{a}\hat{Y}\hat{X}\rangle = \langle\hat{Y}[\hat{a},\hat{X}]\rangle + \langle[\hat{Y},\hat{a}]\hat{X}\rangle. \tag{17}$$

Using Equations (17) and (14) one can derive the following relations for $\langle\hat{b}^\dagger\hat{b}\rangle$

$$\Lambda\Big(\hat{b},\hat{b}^\dagger,\hat{b}^\dagger\hat{b}\Big) = \langle\hat{b}^\dagger[\hat{b}^\dagger\hat{b},\hat{b}]\rangle + \langle[\hat{b}^\dagger,\hat{b}^\dagger\hat{b}]\hat{b}\rangle = -2\langle\hat{b}^\dagger\hat{b}\rangle,$$
$$\Lambda\Big(\hat{b}^\dagger,\hat{b},\hat{b}^\dagger\hat{b}\Big) = \langle\hat{b}[\hat{b}^\dagger\hat{b},\hat{b}^\dagger]\rangle + \langle[\hat{b},\hat{b}^\dagger\hat{b}]\hat{b}^\dagger\rangle = 2(1+\langle\hat{b}^\dagger\hat{b}\rangle),$$
$$\Lambda\Big(\hat{c},\hat{c}^\dagger,\hat{b}^\dagger\hat{b}\Big) = 0, \qquad \Lambda\Big(\hat{c}^\dagger,\hat{c},\hat{b}^\dagger\hat{b}\Big) = 0,$$
$$\Lambda\Big(\hat{b},\hat{c}^\dagger,\hat{b}^\dagger\hat{b}\Big) = \langle\hat{c}^\dagger[\hat{b}^\dagger\hat{b},\hat{b}]\rangle + \langle[\hat{c}^\dagger,\hat{b}^\dagger\hat{b}]\hat{b}\rangle = -\langle\hat{c}^\dagger\hat{b}\rangle,$$
$$\Lambda\Big(\hat{c}^\dagger,\hat{b},\hat{b}^\dagger\hat{b}\Big) = \langle\hat{b}[\hat{b}^\dagger\hat{b},\hat{c}^\dagger]\rangle + \langle[\hat{b},\hat{b}^\dagger\hat{b}]\hat{c}^\dagger\rangle = \langle\hat{c}^\dagger\hat{b}\rangle,$$
$$\Lambda\Big(\hat{b}^\dagger,\hat{c},\hat{b}^\dagger\hat{b}\Big) = \langle\hat{c}[\hat{b}^\dagger\hat{b},\hat{b}^\dagger]\rangle + \langle[\hat{c},\hat{b}^\dagger\hat{b}^\dagger]\hat{b}\rangle = \langle\hat{b}^\dagger\hat{c}\rangle,$$
$$\Lambda\Big(\hat{c},\hat{b}^\dagger,\hat{b}^\dagger\hat{b}\Big) = \langle\hat{b}^\dagger[\hat{b}^\dagger\hat{b},\hat{c}]\rangle + \langle[\hat{b}^\dagger,\hat{b}^\dagger\hat{b}]\hat{c}\rangle = -\langle\hat{b}^\dagger\hat{c}\rangle,$$
$$Tr\left([\hat{c}^\dagger\hat{b},\hat{\rho}_S]\hat{b}^\dagger\hat{b}\right) = -\langle\hat{c}^\dagger\hat{b}\rangle,$$
$$Tr\left([\hat{b}^\dagger\hat{c},\hat{\rho}_S]\hat{b}^\dagger\hat{b}\right) = \langle\hat{b}^\dagger\hat{c}\rangle. \tag{18}$$

Analogous relations are valid for $\langle\hat{c}^\dagger\hat{c}\rangle$. Combining similar terms we arrive at the equations for average values of occupation operators of symmetric and antisymmetric modes:

$$\frac{\partial\langle\hat{b}^\dagger\hat{b}\rangle}{\partial t} = \lambda^2\Bigg(\frac{G_1(\omega+\Omega)+G_2(\omega+\Omega)}{2} + 2A\langle\hat{b}^\dagger\hat{b}\rangle +$$
$$+ (B+C)\langle\hat{b}^\dagger\hat{c}\rangle + (\tilde{B}-\tilde{C})\langle\hat{c}^\dagger\hat{b}\rangle\Bigg). \tag{19}$$

$$\frac{\partial\langle\hat{c}^\dagger\hat{c}\rangle}{\partial t} = \lambda^2\Bigg(\frac{G_1(\omega-\Omega)+G_2(\omega-\Omega)}{2} + 2\tilde{A}\langle\hat{c}^\dagger\hat{c}\rangle +$$
$$+ (B-C)\langle\hat{b}^\dagger\hat{c}\rangle + (\tilde{B}+\tilde{C})\langle\hat{c}^\dagger\hat{b}\rangle\Bigg). \tag{20}$$

Here

$$A = -(\gamma_1(\omega+\Omega) + \gamma_2(\omega+\Omega))/4, \tag{21}$$
$$\tilde{A} = -(\gamma_1(\omega-\Omega) + \gamma_2(\omega-\Omega))/4, \tag{22}$$
$$B + C = -(\gamma_{1-+}(\omega-\Omega) - \gamma_{2-+}(\omega-\Omega))/2, \tag{23}$$
$$\tilde{B} + \tilde{C} = -(\gamma_{1-+}(\omega+\Omega) - \gamma_{2-+}(\omega+\Omega))/2, \tag{24}$$
$$B - C = -(\gamma_{1+-}(\omega+\Omega) - \gamma_{2+-}(\omega+\Omega))/2, \tag{25}$$
$$\tilde{B} - \tilde{C} = -(\gamma_{1+-}(\omega-\Omega) - \gamma_{2+-}(\omega-\Omega))/2, \tag{26}$$

and $\gamma_j(\omega) = (G_j(-\omega) - G_j(\omega))$, $\gamma_{j-+}(\omega) = -(G_{j-}(\omega) - G_{j+}(-\omega))$, $\gamma_{j+-}(\omega) = -(G_{j+}(\omega) - G_{j-}(-\omega))$.

Since $(TR(\tau))^* = TR(-\tau)$, one-sided Fourier transformations obey the condition $(G_-(\omega))^* = G_+(\omega)$. Therefore, $G(\omega) \in \mathbb{R}$, and $\mathrm{Re}(G_\pm(\omega)) = G(\omega)/2$. As a result, $A, \tilde{A} \in \mathbb{R}$, $B = (\tilde{B})^*$, $C = -(\tilde{C})^*$, $B+C = (\tilde{B}-\tilde{C})^*$, $\tilde{B}+\tilde{C} = (B-C)^*$.

Further, we have

$$\begin{aligned}
&\Lambda\Big(\hat{b}, \hat{b}^\dagger, \hat{b}^\dagger\hat{c}\Big) = \langle \hat{b}^\dagger[\hat{b}^\dagger\hat{c}, \hat{b}]\rangle + \langle[\hat{b}^\dagger, \hat{b}^\dagger\hat{c}]\hat{b}\rangle = -\langle\hat{b}^\dagger\hat{c}\rangle,\\
&\Lambda\Big(\hat{b}^\dagger, \hat{b}, \hat{b}^\dagger\hat{c}\Big) = \langle \hat{b}[\hat{b}^\dagger\hat{c}, \hat{b}^\dagger]\rangle + \langle[\hat{b}, \hat{b}^\dagger\hat{c}]\hat{b}^\dagger\rangle = \langle\hat{b}^\dagger\hat{c}\rangle,\\
&\Lambda\Big(\hat{c}, \hat{c}^\dagger, \hat{b}^\dagger\hat{c}\Big) = \langle \hat{c}^\dagger[\hat{b}^\dagger\hat{c}, \hat{c}]\rangle + \langle[\hat{c}^\dagger, \hat{b}^\dagger\hat{c}]\hat{c}\rangle = -\langle\hat{b}^\dagger\hat{c}\rangle,\\
&\Lambda\Big(\hat{c}^\dagger, \hat{c}, \hat{b}^\dagger\hat{c}\Big) = \langle \hat{c}[\hat{b}^\dagger\hat{c}, \hat{c}^\dagger]\rangle + \langle[\hat{c}, \hat{b}^\dagger\hat{c}]\hat{c}^\dagger\rangle = \langle\hat{b}^\dagger\hat{c}\rangle,\\
&\Lambda\Big(\hat{b}, \hat{c}^\dagger, \hat{b}^\dagger\hat{c}\Big) = \langle \hat{c}^\dagger[\hat{b}^\dagger\hat{c}, \hat{b}]\rangle + \langle[\hat{c}^\dagger, \hat{b}^\dagger\hat{c}]\hat{b}\rangle = -\langle\hat{b}^\dagger\hat{b}\rangle - \langle\hat{c}^\dagger\hat{c}\rangle,\\
&\Lambda\Big(\hat{c}^\dagger, \hat{b}, \hat{b}^\dagger\hat{c}\Big) = \langle \hat{b}[\hat{b}^\dagger\hat{c}, \hat{c}^\dagger]\rangle + \langle[\hat{b}, \hat{b}^\dagger\hat{c}]\hat{c}^\dagger\rangle = 2 + \langle\hat{b}^\dagger\hat{b}\rangle + \langle\hat{c}^\dagger\hat{c}\rangle,\\
&\Lambda\Big(\hat{b}^\dagger, \hat{c}, \hat{b}^\dagger\hat{c}\Big) = \Lambda\Big(\hat{c}, \hat{b}^\dagger, \hat{b}^\dagger\hat{c}\Big) = Tr\left([\hat{b}^\dagger\hat{c}, \hat{\rho}_S]\hat{b}^\dagger\hat{c}\right) = 0,\\
&Tr\left([\hat{c}^\dagger\hat{b}, \hat{\rho}_S]\hat{b}^\dagger\hat{c}\right) = -\langle\hat{c}^\dagger\hat{c}\rangle + \langle\hat{b}^\dagger\hat{b}\rangle.
\end{aligned}\tag{27}$$

Combining similar terms, we arrive at

$$\begin{aligned}
&\frac{\partial\langle\hat{b}^\dagger\hat{c}\rangle}{\partial t} = i2\Omega\langle\hat{b}^\dagger\hat{c}\rangle + \lambda^2\Bigg(\frac{G_{1-}(\omega+\Omega) + G_{1+}(\omega-\Omega)}{2} -\\
&-\frac{G_{2-}(\omega+\Omega) + G_{2+}(\omega-\Omega)}{2} + (\tilde{B}+\tilde{C})\langle\hat{b}^\dagger\hat{b}\rangle +\\
&+(\tilde{B}-\tilde{C})\langle\hat{c}^\dagger\hat{c}\rangle + (A+\tilde{A})\langle\hat{b}^\dagger\hat{c}\rangle\Bigg).
\end{aligned}\tag{28}$$

$$\begin{aligned}
&\frac{\partial\langle\hat{c}^\dagger\hat{b}\rangle}{\partial t} = -i2\Omega\langle\hat{c}^\dagger\hat{b}\rangle + \lambda^2\Bigg(\frac{G_{1-}(\omega-\Omega) + G_{1+}(\omega+\Omega)}{2} -\\
&-\frac{G_{2-}(\omega-\Omega) + G_{2+}(\omega+\Omega)}{2} + (B-C)\langle\hat{b}^\dagger\hat{b}\rangle +\\
&+(B+C)\langle\hat{c}^\dagger\hat{c}\rangle + (A+\tilde{A})\langle\hat{c}^\dagger\hat{b}\rangle\Bigg).
\end{aligned}\tag{29}$$

For completeness, we derive equations for the average values of the operators $\hat{b}^\dagger\hat{b}$, $\hat{c}^\dagger\hat{c}$, $\hat{b}^\dagger\hat{c}$, $\hat{c}^\dagger\hat{b}$ in the global and local approaches. In the global approach, in Eq. (16) from main text we should replace all terms containing $e^{\pm i2\Omega t}$ with zeros. Then $B, \tilde{B}, C, \tilde{C}$ as well as constant terms in Eqs. (28) and(29) become zero. Finally, the equations for the averages of occupation operators and for the correlations in the global approach take the form

$$\frac{1}{\lambda^2}\frac{\partial\langle\hat{b}^\dagger\hat{b}\rangle}{\partial t} = \frac{G_1(\omega+\Omega) + G_2(\omega+\Omega)}{2} + 2A\langle\hat{b}^\dagger\hat{b}\rangle. \tag{30}$$

$$\frac{1}{\lambda^2}\frac{\partial\langle\hat{c}^\dagger\hat{c}\rangle}{\partial t} = \frac{G_1(\omega-\Omega)+G_2(\omega-\Omega)}{2} + 2\tilde{A}\langle\hat{c}^\dagger\hat{c}\rangle. \tag{31}$$

$$\frac{\partial\langle\hat{b}^\dagger\hat{c}\rangle}{\partial t} = i2\Omega\langle\hat{b}^\dagger\hat{c}\rangle + \lambda^2(A+\tilde{A})\langle\hat{b}^\dagger\hat{c}\rangle. \tag{32}$$

$$\frac{\partial\langle\hat{c}^\dagger\hat{b}\rangle}{\partial t} = -i2\Omega\langle\hat{c}^\dagger\hat{b}\rangle + \lambda^2(A+\tilde{A})\langle\hat{c}^\dagger\hat{b}\rangle. \tag{33}$$

The main difference of these equations from the ones obtained in the partial-secular approach, (see Eqs. (19), (20), (28) and (29)) is that these equations are independent from each other.

In the local approach, the following master equation is considered

$$\begin{aligned}\frac{\partial\hat{\rho}_S}{\partial t} &= -i[\hat{H}_S,\hat{\rho}_S]+\\ &+ \big(G_1(-\omega)L[\hat{a}_1,\hat{a}_1^\dagger] + G_1(\omega)L[\hat{a}_1^\dagger,\hat{a}_1]+\\ &+ G_2(-\omega)L[\hat{a}_2,\hat{a}_2^\dagger] + G_2(\omega)L[\hat{a}_2^\dagger,\hat{a}_2]\big)\lambda^2/2.\end{aligned} \tag{34}$$

This master equation reflects the assumption that relaxation operators of coupled oscillators do not differ from the relaxation operators of uncoupled oscillators. This is equivalent to the zeroth order perturbation theory in small parameter Ω/ω. From Equation (34), one can obtain the following equations for average of the operators $\hat{b}^\dagger\hat{b}$, $\hat{c}^\dagger\hat{c}$, $\hat{b}^\dagger\hat{c}$, $\hat{c}^\dagger\hat{b}$

$$\frac{1}{\lambda^2}\frac{\partial\langle\hat{b}^\dagger\hat{b}\rangle}{\partial t} = K_+ + 2P\langle\hat{b}^\dagger\hat{b}\rangle + M\langle\hat{b}^\dagger\hat{c}\rangle + M\langle\hat{c}^\dagger\hat{b}\rangle, \tag{35}$$

$$\frac{1}{\lambda^2}\frac{\partial\langle\hat{c}^\dagger\hat{c}\rangle}{\partial t} = K_+ + 2P\langle\hat{c}^\dagger\hat{c}\rangle + M\langle\hat{b}^\dagger\hat{c}\rangle + M\langle\hat{c}^\dagger\hat{b}\rangle, \tag{36}$$

$$\frac{1}{\lambda^2}\frac{\partial\langle\hat{b}^\dagger\hat{c}\rangle}{\partial t} = \frac{i2\Omega}{\lambda^2}\langle\hat{b}^\dagger\hat{c}\rangle + K_- + M(\langle\hat{b}^\dagger\hat{b}\rangle + \langle\hat{c}^\dagger\hat{c}\rangle) + 2P\langle\hat{b}^\dagger\hat{c}\rangle, \tag{37}$$

$$\frac{1}{\lambda^2}\frac{\partial\langle\hat{c}^\dagger\hat{b}\rangle}{\partial t} = -\frac{i2\Omega}{\lambda^2}\langle\hat{c}^\dagger\hat{b}\rangle + K_- + M(\langle\hat{b}^\dagger\hat{b}\rangle + \langle\hat{c}^\dagger\hat{c}\rangle) + 2P\langle\hat{c}^\dagger\hat{b}\rangle, \tag{38}$$

where $K_\pm = (G_1(\omega) \pm G_2(\omega))/2$, $P = -(\gamma_1(\omega)+\gamma_2(\omega))/4$, and $M = -(\gamma_1(\omega)-\gamma_2(\omega))/4$.