Pis'ma v ZhETF, vol.67, iss.7, pp.478 - 483 

# ANOMALOUS PROXIMITY EFFECT IN *d*-WAVE SUPERCONDUCTORS

*A.A.Golubov*[+*], *M.Yu.Kupriyanov*[+□]

[+]*Department of Applied Physics, University of Twente, The Netherlands*

[*]*Institute of Solid Physics RAS, 142432 Chernogolovka, Russia*

[□]*Nuclear Physics Institute, Moscow State University*
*119899 Moscow, Russia*

Submitted 12 January 1998
Resubmitted 24 February 1998

Anomalous proximity effect between *d*-wave superconductor and thin disordered normal layer is studied theoretically in the framework of Eilenberger equations. It is shown that disorder of the quasiparticle reflection from this thin layer leads to the formation of the *s*-wave component localised near the boundary. Angular and spatial structure of the pair potential near interface is studied.

PACS: 74.50.+r, 74.72.-h, 74.80.F

There is a continuous experimental evidence that behavior of high temperature superconductors (HTS) can be understood in terms of the *d*-wave pairing scenario, rather than in the conventional *s*-wave picture. On the other hand it is well known that *d*-wave order parameter is strongly reduced by the electron scattering on impurities and therefore can be formed only in clean materials. However, the condition of the clean limit is not fulfilled in the vicinity of the grain boundaries or other HTS interfaces even if these materials are clean in the bulk. There are at least two reasons of that. The first one is that quasiparticle reflection from realistic interfaces is diffusive rather than specular, thus providing the isotropisation in the momentum space and suppression of the order parameter in the *d*-wave channel. The second one is the contamination of the material near interfaces as a result of fabrication process or electromigration in the large scale application devices. Therefore the formation of a thin disordered layer near a HTS interface is highly probable. What kind of superconducting correlation one can expect in such a layer? An answer on this fundamental question is very important both for small and large scale applications of HTS materials.

Peculiarities on the surface of a *d*-wave superconductor were discussed theoretically in the models based on specular quasiparticle reflection from clean interfaces [1 – 6]. Zero- and finite-bias anomalies predicted in these papers were recently observed experimentally in Refs. [7, 8]. In this paper we focus on the problem of the anomalous proximity effect between *d*-wave superconductor and thin disordered layer. We will show that disorder of the quasiparticle reflection from such a layer leads to the formation of the *s*-wave component localized near the boundary, even if the coupling constant in the *s*-wave pairing channel is zero. The magnitude of this component is studied as a function of interface orientation and temperature. We will argue that the *s*-wave state is gapless and the peculiarities in the density of states predicted in Refs.[1 – 6] are smeared out.

There are two approaches to the description of the influence of disorder of quasiparticle reflection at the interfaces on the properties of the interface region. In the first one it assumed that interface consists of the facets with random orientation of their interface normal compare to $a$ axis of the HTS material [5]. According to the second approach, both sides of the ideal interface are coated by Ovchinnikov's thin dirty layer [4, 9, 10]. In the latter case the degree of disorder (or interface roughness) was measured by the ratio of the layer thickness $d$ to the quasiparticle mean free path in the layer $\ell$. Both models were focused only on the study of Andreev surface bound states and do not take into account the effect of surface-induced changes of the symmetry of an order parameter near the boundaries.

To make the last effect more evident we consider the situation in which the crossover from the clean limit to the dirty limit takes place in the thin layer near the boundary. We will assume also that the lattice constant $a < \ell$ and that the thickness of the layer $d \ll \sqrt{\xi_0 \ell}$, where $\xi_0$ is the coherence length of the bulk material.

To study the proximity effect at the interface we can use the quasiclassical Eilenberger equations [11], which may be conveniently rewritten in terms of functions $\Phi_+ = (f(r,\theta)+ +f(r,\theta+\pi))/2$ and $\Phi_- = f(r,\theta) - f(r,\theta+\pi)$:

$$4\omega\Phi_+ + v\cos\theta\frac{d\Phi_-}{dx} = 4\Delta g + \frac{2}{\tau}(g\langle\Phi_+\rangle - \Phi_+\langle g\rangle), \tag{1}$$

$$2v\cos\theta\frac{d\Phi_+}{dx} = -(2\omega + \frac{1}{\tau}\langle g\rangle)\Phi_-, \tag{2}$$

$$2v\cos\theta\frac{dg}{dx} = (2\Delta + \frac{1}{\tau}\langle\Phi_+\rangle)\Phi_-, \tag{3}$$

$$\Delta\ln\frac{T}{T_c} + 2\pi T\sum_{\omega>0}\left(\frac{\Delta}{\omega} - \langle\lambda(\theta,\theta')\Phi_+\rangle\right) = 0. \tag{4}$$

Here $\omega = \pi T(2n+1)$ are the Matsubara frequencies, $v$ is the Fermi velocity, $x$ is a coordinate in the direction of the interface normal, $\theta$ is the angle between the interface normal and quasiparticle trajectory, $\tau = \ell/v$ and $\langle ...\rangle = (1/2\pi)\int_0^{2\pi}(...)d\theta$. In the assumption that the coupling constant $\lambda(\theta,\theta')$ in a $d$-wave channel has the form [12] $\lambda_d(\theta,\theta') = 2\lambda\cos(2\theta)\cos(2\theta')$ and $\lambda_s = 0$ the self-consistency equation (4) yields for the pair-potential $\Delta = \sqrt{2}\Delta(x)\cos(2(\theta-\alpha))$. We also assume that the Fermi surface has the cylindrical form.

Equations (1) – (4) must be supplemented by the appropriate boundary conditions. Far from the interface functions $\Phi_+$ must coincide with the bulk solution

$$\Phi_+ = \frac{\sqrt{2}\Delta_\infty\cos(2(\theta-\alpha))}{\sqrt{\omega^2 + 2\Delta_\infty^2\cos^2(2(\theta-\alpha))}}. \tag{5}$$

Two boundary conditions for the function $\Phi_+$ and it's derivative $d\Phi_+/dx$ are required at the interface between the clean and disordered regions of a $d$-wave superconductor ($x=0$). These conditions can be derived by integration of the Eilenberger equations (1), (2) in a small region near the interface. In accordance with Ref.[13], the first condition is the continuity of $\Phi_-$ at the interface and can be presented in the form

$$\frac{\ell\cos\theta}{\langle g(-0)\rangle}\frac{d\Phi_+(-0)}{dx} = \frac{v\cos\theta}{2\omega}\frac{d\Phi_+(+0)}{dx}. \tag{6}$$

This condition manifests the current conservation across the interface.

The second boundary condition depends on backscattering properties of the interface. To account for such a backscattering we introduce a strongly disordered thin layer located near the interface at $-\delta \leq x \leq 0$, which is characterized by the mean free path $\ell_\delta$, where $a \ll \ell_\delta, \delta \ll d, \ell$. Assuming that all the interfaces are transparent and integrating equation (1), (2) in the interval $-\delta \leq x \leq 0$ in the limit $\delta \to 0$ we arrive at the second boundary condition

$$D\ell \frac{d\Phi_+(-0)}{dx} = \Phi_+(+0) - \Phi_+(-0), \tag{7}$$

where $D = 2\delta/\ell_\delta$. For $D = 0$ equation (7) is a direct consequence of the continuity of the Eilenberger functions along quasiclassical trajectories which is valid for transparent $SN$-boundary [13]. With increase of $D$ the probability of quasiparticles penetration into $N$-layer ($\Phi_+(-0) \approx D^{-1}\Phi_+(+0)$) decrease as $D^{-1}$. It means that the most of them are diffusively reflected back to the bulk $d$-wave region at the length scale smaller than $\ell$.

In the following we will consider the case of strong disorder $\ell \ll d$. Then it follows from [13, 14] that for the totally reflecting free interface ($x = -d$) the boundary condition is

$$\frac{d}{dx}\Phi_+ = 0. \tag{8}$$

In the limit $\ell \ll \xi_0$ it follows from (1) – (4) that in the disordered layer $\Delta = 0$. Then equations (1) – (4) in the region $-d \leq x \ll -\ell$ are reduced to the dirty limit form [15], which formally coincides with the one valued for normal metal with $T_{cn} = 0$. Since in this regime the scale of variation of $\langle\Phi_+\rangle$ and $\langle g\rangle$ is of the order of a dirty limit coherence length $\sqrt{\xi_0\ell}$, the functions $\langle\Phi_+\rangle$ and $\langle g\rangle$ in the disordered layer are $x$-independent when $d \ll \sqrt{\xi_0\ell}$. Then the Eilenberger equations in the region $-d \leq x \leq 0$ are essentially simplified and have the solution

$$\Phi_+ = \langle\Phi_+\rangle + A\frac{\cosh(k(x+d))}{\cosh(kd)}, \quad \Phi_- = -2A\frac{\sinh(k(x+d))}{\cosh(kd)}, \quad k = \frac{1}{\ell\,|\cos\theta|}, \tag{9}$$

$$g = \langle g\rangle - \frac{\langle\Phi_+\rangle}{\langle g\rangle}A\frac{\cosh(k(x+d))}{\cosh(kd)}, \qquad \langle\Phi_+\rangle^2 + \langle g\rangle^2 = 1. \tag{10}$$

Making use of the boundary conditions (6), (7) at $x = 0$ and (9), (10), one can further reduce the problem to the solution of the Eilenberger equations (1) – (4) in the clean $d$-wave superconductor ($x \geq 0$)

$$\kappa^2\frac{d^2\Phi_+}{dx^2} - \Phi_+ = -\sqrt{2}\frac{\Delta(x)}{\omega}\cos(2(\theta-\alpha))g, \qquad \kappa = \frac{v\,|\cos\theta|}{2\omega}, \tag{11}$$

$$\frac{dg}{dx} = -\sqrt{2}\frac{\Delta(x)}{\omega}\cos(2(\theta-\alpha))\frac{d\Phi_+}{dx} \tag{12}$$

with the condition (5) in the bulk ($x \gg \xi_0$) and boundary condition

$$\left\{\kappa\,\langle g(0)\rangle + D\frac{v}{\omega}\right\}\frac{d}{dx}\Phi_+(0) = (\Phi_+(0) - \langle\Phi_+(0)\rangle), \qquad \langle g(0)\rangle = \sqrt{1-\langle\Phi_+(0)\rangle^2} \tag{13}$$

at $x = 0$.

In the following we will limit ourselves to the situation when the disordered layer produces the most strong effect, namely when $\ell \ll d$ and $D = 0$. In this case the boundary condition (13) has the closed form. The isotropic Usadel function $\langle \Phi_+(0) \rangle$ has to be determined selfconsistently as a result of iteration procedure.

In the limit $\kappa \ll 1$ the pair potential $\Delta(x)$ is a smooth function of $x$ at distances of the order of $\kappa$. Then the boundary value problem (11)-(13) is essentially simplified and has the asymptotic solution

$$\Phi_+ = \Psi(x) + \eta \sqrt{\frac{G(x)}{G(0)}} \exp\left\{ - \int_0^x \frac{dy}{\kappa G(y)} \right\},$$

$$\eta = G(0) \frac{\langle \Phi_+(0) \rangle - \Psi(0) + \langle g(0) \rangle \, \kappa \Psi'(0)}{G(0) + \langle g(0) \rangle \left[ 1 - \kappa (\Psi(0)/\Delta)' \right]}, \tag{14}$$

$$G(x) = \frac{\omega}{\sqrt{\omega^2 + 2\Delta^2(x) \cos^2(2(\theta - \alpha))}}, \quad \Psi(x) = \frac{\sqrt{2} \cos(2(\theta - \alpha)) \Delta(x)}{\sqrt{\omega^2 + 2\Delta^2(x) \cos^2(2(\theta - \alpha))}}. \tag{15}$$

Here prime denotes the derivative with respect to coordinate $x$. From (14), (15) it follows that at $x = 0$ the anomalous Green's function $\Phi_+(0)$ equals to the sum of the three terms with different angular symmetry. Two of them are simply isotropic part and the term with the $d$-wave symmetry. The last one is proportional to the product $\Delta'(0) \, |\cos\theta| \cos(2(\theta - \alpha))$ and can be considered as a source for formation of nonzero component in the $s$-wave channel. Thus the gradient of the pair potential at the interface results in spatial variation of function $\Phi_+(x)$ with the characteristic length which depends on the direction in the momentum space. This difference in $\kappa(\theta)$ leads to the larger deformation the $d$-wave angular dependence of $\Phi_+(x)$ the closer $x$ to the interface. As a result, nonzero angle-averaged value $\langle \Phi_+(x) \rangle$ appears, which is localized near the interface.

In a general case of arbitrary $\kappa$ values the problem was solved numerically. The isotropic function $\langle \Phi_+(0) \rangle$ and the spatially dependent pair potential $\Delta(x)$ were calculated by iteration procedure from the boundary condition (13) and the selfconsistency equation (4). The results of numerical calculations shown on Figs.1 – 3 confirm simple considerations presented above.

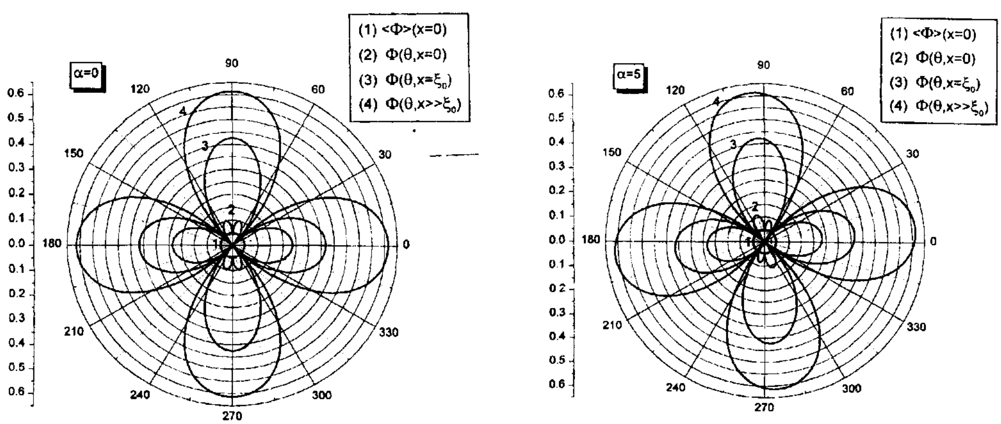


Fig.1. Angular dependencies of $\Phi_+(x)$ at different distances from the interface at $T = 0.7T_c$

Fig.1 shows the angular dependence of $\Phi_+(x)$ far from the boundary ($x \gg \xi_0$) and at $x = \xi_0$, 0 for two different orientations $\alpha$ of $a$ axis with respect to the interface normal. In both cases far from the interface the angular distribution is typical for $d$-wave superconductor. At $x = \xi_0$ the positive lobe is suppressed stronger than the negative one, since $\kappa(\theta)$ in this direction is smaller compared to $\kappa(\theta)$ in the direction of the negative lobe. Hence at $x \approx \xi_0$ negative lobes of $\Phi_+(x)$ practically reach the local value $\Psi(x)$, while positive ones still not. This difference leads to the negative sign of the $s$-component $\langle\Phi_+(x)\rangle$ (see Fig.2). In the vicinity of the interface ($x \leq 0.3\xi_0$) the situation is just the opposite. In accordance with (14) due to angular dependence of $\kappa(\theta) \propto |\cos\theta|$ the negative lobes are suppressed stronger than the positive ones, the function $\langle\Phi_+(x)\rangle$ changes sign to positive and reaches its maximum at $x = 0$. It is important to note that exactly at $\theta = \pi/2$ it follows from (14) that $\Phi_+(0,\pi/2) = \Psi(0,\pi/2)$, while $\lim_{\theta\to\pm\pi/2}\Phi_+(0,\theta) = \Psi(0,\pi/2) + \eta$. This discontinuity is the manifestation of the simple fact that quasiparticles which propagate exactly parallel to the interface have an information about the disordered region only via the local value of $\Delta(0)$, while for all other directions the direct interaction between both regions takes place. However, this discontinuity at $\theta = \pi/2$ does not contribute to the result of the angular averaging of $\Phi_+$ in (13).

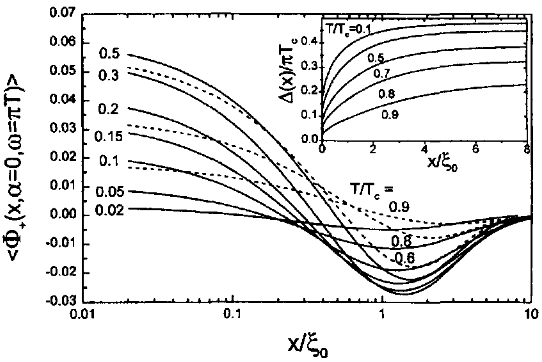


Fig.2. Spatial dependencies of the surface-induced $s$-wave component $\langle\Phi_+(x)\rangle$ at various temperatures. Insert: behavior of the pair potential $\Delta(x)$ near the interface

Fig.3 shows the spatial variations of $\Delta(x)$ for different values of the angle $\alpha$. As is seen from (14), (15), the function $\Phi_+(0,\theta)$ near interface has a contribution proportional to $|\cos\theta|\cos(2(\theta-\alpha))$. This immediately leads to the result that the amplitude of the $s$-component induced into the disordered layer scales with misorientation angle $\alpha$ as $\langle\Phi_+(0,\alpha=0)\rangle\cos 2\alpha$. At $\alpha = \pi/4$ the superconducting correlations are not induced into the disordered layer, i.e. $\langle\Phi_+(0)\rangle = 0$. Further increase of $\alpha$ leads to the sigh change of the $s$-component. As is seen from Fig.3, these qualitative considerations are in a good agreement with the results of exact numerical calculations. In particular, for $d_{xy}$ case ($\alpha = \pi/4$) we have $\langle\Phi_+(0)\rangle = 0$. At the same time, it is important to note that the pair potential $\Delta(0,\alpha=\pi/4)$ is nonzero, in contrast to the case of a specular reflecting boundary. The reason is that in the considered case of diffusive scattering from the interface there is no symmetry requirement for function $\Phi_+(0,\alpha=\pi/4)$ to vanish.

In the whole temperature range the amplitude of the of the $s$-wave component $\langle\Phi_+\rangle$ induced into the disordered layer (see Fig.2) is an order of magnitude smaller compared to the amplitude of the order parameter in the bulk superconductor. That means that $\langle g(0)\rangle$ is practically temperature independent and is close to unity. Thus, taking into account that $\langle g(0)\rangle$ is independent on Matsubara frequencies and that $\xi\Phi'_+(0) \approx \Phi_+(0)$ from the

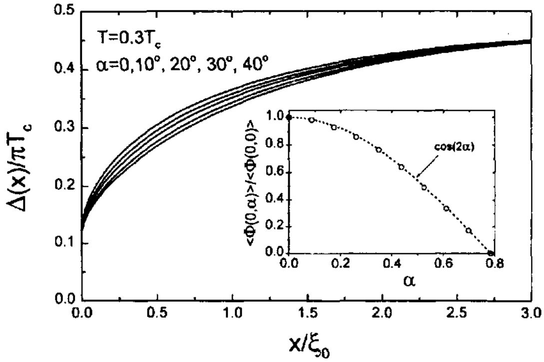


Fig.3. Behavior of the pair potential near the interface for different misorientation angles $\alpha$. Insert: dependence of $\langle\Phi_+(0)\rangle$ on $\alpha$

boundary condition (13) we immediately have that at small temperatures $\langle\Phi_+(0)\rangle \propto \omega$, if $\omega \leq \Delta$ and falls down $\langle\Phi_+(0)\rangle \propto \omega^{-2}$ as soon as $\omega$ exceeds the value of $\Delta$. Since the density of states $N(\varepsilon) = \mathrm{Re}\,\langle g(0, \varepsilon = i\omega)\rangle$, where $\langle g\rangle = \sqrt{1 - \langle\Phi_+\rangle^2}$ the property $\langle\Phi_+\rangle \propto$ $\propto \omega$ at small $\omega$ results in the gapless density of states in the disordered layer. The density of states has only small peculiarities at energies $\varepsilon \approx \Delta$. The behavior of the density of states will be discussed in a more detail elsewhere.

In conclusion, we have studied theoretically the proximity effect between $d$-wave superconductor and thin disordered normal layer. It is shown that disorder of the quasiparticle reflection from this layer leads to the formation of the $s$-wave component localized near the interface. This model might be relevant for description of rough HTS interfaces.

We would like to thank H.Rogalla, G.Gerritsma and Yu.Nazarov for helpful discussions. This work is supported in part by INTAS Grant 93-790ext and by Program for Russian-Dutch Research Cooperation (NWO).

---

1. Y.Tanaka and S.Kashiwaya, Phys. Rev. Lett. **74**, 3451 (1995).
2. Y.Tanaka and S.Kashiwaya, J. Phys. Chem. Solids. **56**, 1761 (1995); Phys. Rev. B **53**, 11957 (1996).
3. S.Kashiwaya, Y.Tanaka, M.Koyanagi, and K.Kajimura, Phys. Rev. B **53**, 2667 (1996).
4. Yu.S.Barash, A.A.Svidzinsky, and H.Burkhardt, Phys.Rev. B **55**, 15282 (1997).
5. M.Fogelström, D.Rainer, and J.A.Sauls, Phys. Rev. Lett. **79**, 281 (1997).
6. D.Rainer, H.Burkhardt, M.Fogelström, and J.A.Sauls, preprint cond-mat/9712234.
7. L.Alff, H.Takashima, S.Kashiwaya et al., Phys. Rev. B **55**, R14757 (1997).
8. M.Covington, M.Aprili, L.H.Greene et al, Phys. Rev. Lett. **79**, 277 (1997).
9. F.J.Culetto, G.Kieselmann, and D.Rainer, in: *Proceedings of the 17th Intern. Conf. on Low Temperature Physics*, Eds. U.Eckern, A.Schmid, W.Weber, and H.Wühl, North Holland, Amsterdam, 1984, p. 1027.
10. Yu.N.Ovchinnikov, Zh. Eksp. Teor. Fiz. **56**, 1590 (1969) [Sov. Phys. JETP **29**, 853 (1969)].
11. G.Eilenberger, Z.Physik **214**, 195 (1968).
12. E.Schachinger, J.P.Carbotte, and F.Marsiglio, Phys. Rev. B **56**, 2738 (1997).
13. A.V.Zaitsev, Zh. Eksp. Teor. Fiz. **86**, 1742 (1984) [Sov. Phys. JETP **59**, 1015 (1984)].
14. M.Yu.Kupriyanov and V.F.Lukichev, Zh. Eksp. Teor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP (1988)].
15. K.D.Usadel, Phys. Rev. Lett. **25**, 507 (1970).