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VORTEX MASS IN BCS SYSTEMS: KOPNIN AND BAYM-CHANDLER CONTRIBUTIONS

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The Kopnin mass and the Baym-Chandler mass of the vortex have the same origin. Both represent the mass of the normal component trapped by the vortex. The Kopnin mass of the vortex is formed by quasiparticles localized in the vicinity of the vortex. In the superclean limit it is calculated as linear response, exactly in the same way as the density of the normal component is calculated in homogeneous superfluid. The Baym-Chandler mass is the hydrodynamical (associated) mass trapped by vortex. It is analogous to the normal component formed by inhomogeneities, such as pores and impurities. Both contributions are calculated for the generic model of the continuous vortex core.

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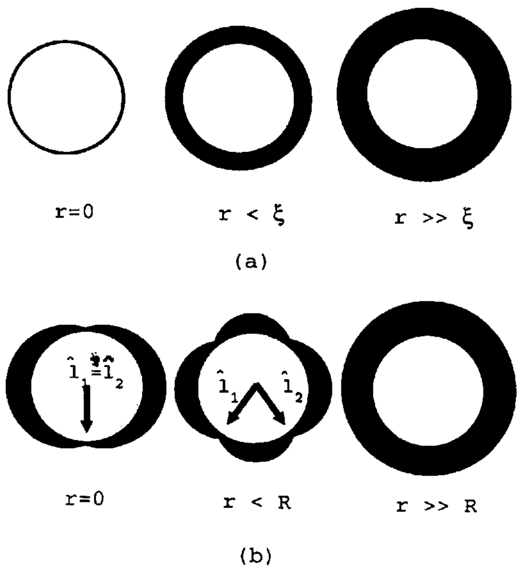
It is well known that in the BCS superfluids and superconductors the most important contribution to the vortex mass originates from the vortex core. The core mass in these systems is proportional to the area of the core $A \sim \xi^2$, where ξ is the coherence length (see [1] for the vortex mass in superconductors and [2] for the vortex mass in superfluid ^{3}He-B). This core mass is essentially larger than the logarithmically divergent contribution, which comes from the compressibility. Inspite of the logarithmic divergence, the latter contains the speed of sound in the denominator and thus is smaller by factor $(a/\xi)^2 \ll 1$, where a is the interatomic distance. The compressibility mass of the vortex dominates in Bose superfluids, where the core size is small, $\xi \sim a$.

According to Kopnin theory the core mass comes from the fermions trapped in the vortex core [1 – 4]. Recently the problem of another vortex mass of the hydrodynamic origin was raised in Ref.[5]. It is the so-called backflow mass discussed by Baym and Chandler [6], which also can be proportional to the core area. Here we compare these two contributions in the superclean regime and at low $T \ll T_c$ using the model of the continuous core.

The continuous-core vortex is one of the best models which helps to solve many problems in the vortex core physics. Instead of consideration of the singular core, one can smoothen the $1/r$-singularity of the superfliuid velocity by introducing the point gap nodes in the core region. As a result the superfluid/superconducting state in the vortex core of any system acquires the properties of the A-phase of superfluid ^{3}He with its continuous vorticity and point gap nodes [7, 8]. Using the continuous-core model one can show for example that the Kopnin (spectral flow) force comes directly from the Adler – Bell – Jackiw chiral anomaly equation [9] and this shows the real origin of this anomalous force. In this model one can easily separate different contributions to the vortex mass. Actually this is not only the model: The spontaneous smoothing of the velocity singularity occurs in the core of both types of vortices observed in ^{3}He-B [8]; in heavy fermionic and high-T_c

superconductors such smoothening can occur due to admixture of different pairing states in the vortex core.

It appears that both the Kopnin mass and the Baym-Chandler mass are related to the normal component. In general the normal component of the superfluid liquid comes from two sources: (i) the local contribution, which comes from the system of quasiparticles, and (ii) the associated mass related to the backflow. The latter is dominating in porous materials, where some part of superfluid component is hydrodynamically trapped by the pores and thus is removed from the overall superfluid motion. The normal component, which gives rise to the vortex mass contains precisely the same two contributions. (i) The local contribution comes from the quasiparticles localized in the vortex core and thus moving with the core. This is the origin of the Kopnin mass according to Ref.[4]. (ii) The associated mass contribution arises because the profile of the local density of the normal component in the vicinity of the vortex core disturbs the superflow around the vortex, when the vortex moves. This creates the backflow and thus some part of the superfluid component is trapped by the moving vortex, resulting in the Baym-Chandler mass of the vortex.



Singular vortex vs soft-core vortex: (*a*) in the singular vortex the gap continuously decreases and becomes zero on the vortex axis (at $r = 0$); (*b*) for some vortices it is energetically favourable to escape the nullification of the order parameter at $r = 0$. Instead, within the smooth core, $r < R$, the point gap nodes appear in the spectrum of fermions [7]. The unit vectors $\hat{\mathbf{l}}_1$ and $\hat{\mathbf{l}}_2$ show the directions to the nodes at different r. Close to the gap nodes the spectrum of fermions is similar to that in ^{3}He-A

Let us consider this on the example of the simplest continuous-core vortex (Figure). It has the following distribution of the unit vector $\hat{\mathbf{l}}(\mathbf{r})$ which shows the direction of the pont gap nodes in the smooth core

$$\hat{\mathbf{l}}(\mathbf{r}) = \hat{\mathbf{z}} \cos \eta(r) + \hat{\mathbf{r}} \sin \eta(r), \tag{1}$$

where z, r, ϕ are cylindrical coordinates. For superfluid ^{3}He-A the $\hat{\mathbf{l}}$-vector in the smooth core changes from $\hat{\mathbf{l}}(0) = -\hat{\mathbf{z}}$ to $\hat{\mathbf{l}}(\infty) = \hat{\mathbf{z}}$, which represents the doubly quantized continuous vortex. For the smoothed singly quantized vortices in ^{3}He-B and superconductors one has two $\hat{\mathbf{l}}$-vectors with $\hat{\mathbf{l}}_1(0) = \hat{\mathbf{l}}_2(0) = -\hat{\mathbf{z}}$ and $\hat{\mathbf{l}}_1(\infty) = -\hat{\mathbf{l}}_2(\infty) = \hat{\mathbf{r}}$ [8]. The region of radius R, where the texture of $\hat{\mathbf{l}}$-vectors is concentrated, is called the smooth (or soft) core of the vortex.

Kopnin mass. Let us remind the phenomenological derivation of the Kopnin mass of the vortex at low T and in the superclean regime [4]. If the vortex moves with velocity $\mathbf{v}_L$ with respect to the superfluid component, the fermionic energy spectrum in the vortex frame is Doppler shifted: $E = E_0(\nu) - \mathbf{k}\cdot\mathbf{v}_L$, where ν stands for the fermionic degrees of freedom in the stationary vortex. The summation over fermionic degrees of freedom leads to the extra linear momentum of the vortex $\propto \mathbf{v}_L$:

$$\mathbf{P} = \sum_\nu \mathbf{k}\theta(-E) = \sum_\nu \mathbf{k}(\mathbf{k}\cdot\mathbf{v}_L)\delta(E_0) = M_{\mathbf{Kopnin}}\mathbf{v}_L. \tag{2}$$

Note that this vortex mass is determined in essentially the same way as the normal component density in the bulk system. The Kopnin vortex mass is nonzero if the density of fermionic states is finite in the vortex core. The density of states (DOS) is determined by the interlevel spacing ω_0 in the core: $N(0) \propto 1/\omega_0$, which gives for the Kopnin vortex mass an estimation: $M_{\mathbf{Kopnin}} \sim k_F^3/\omega_0$ (The exact expession is $M_{\mathbf{Kopnin}} = \int_{-k_F}^{k_F}(dk_z/4\pi)(k_\perp^2/\omega_0(k_z)))$. For the soft-core vortex the interlevel spacing is $\omega_0 \sim \hbar^2/(m\xi R)$ which gives the Kopnin mass $M_{\mathbf{Kopnin}} \sim \rho\xi R$ [10] where ρ is the mass density of the liquid.

For our purposes it is instructive to consider the normal component associated with the vortex as the local quantity, determined at each point in the vortex core. Such consideration is valid for the smooth core with the radius $R \gg \xi$, where the local classical description of the fermionic spectrum can be applied. The main contribution comes from the point gap nodes, where the classical spectrum has the form $E_0 = \sqrt{v_F^2(k-k_F)^2 + \Delta_0^2(\hat{\mathbf{k}}\times\hat{\mathbf{l}})^2}$ and Δ_0 is the gap amplitude. In the presence of the gradient of $\hat{\mathbf{l}}$-field, which acts on the quasiparticles as an effective magnetic field, this gapless spectrum leads to the nonzero local DOS and finally to the following local density of the normal component at $T = 0$ (see Eq.(5.24) in the review [11]):

$$(\rho_n)_{ij}(\mathbf{r}) = \rho_n \hat{l}_i\hat{l}_j \ , \ \rho_n = \frac{k_F^4}{2\pi^2\Delta_0}|(\hat{\mathbf{l}}\cdot\vec{\nabla})\hat{\mathbf{l}}| = \frac{k_F^4 \sin\eta}{2\pi^2\Delta_0}|\partial_r\eta|. \tag{3}$$

The integral of this normal density tensor over the cross section of the soft core gives the same Kopnin mass of the vortex but in the local density representation [4]

$$M_{\mathbf{Kopnin}} = \frac{1}{2}\int d^2r\rho_n(r)\sin^2\eta(r) \sim \rho\xi R \ . \tag{4}$$

He we used that $v_F/\Delta_0 \sim \xi$. Note that area law for the vortex mass is valid only for vortices with $R \sim \xi$, but in general one has the linear law: $M_{\mathbf{Kopnin}} \sim \rho\xi R$ [10].

The associated (or induced) mass appears when, say, an external body moves in the superfluid. This mass depends on the geometry of the body. For the moving cylinder of radius R it is the mass of the liquid displaced by the cylinder,

$$M_{\mathbf{associated}} = \pi R^2\rho, \tag{5}$$

which is to be added to the actual mass of the cylinder to obtain the total inertial mass of the body. In superfluids this part of superfluid component moves with external body and thus can be associated with the normal component. The similar mass is responsible for the normal component in porous materials and in aerogel, where some part of superfluid is hydrodynamically trapped by the pores. It is removed from the overall superfluid motion and thus becomes the part of the normal component.

In the case when the vortex is trapped by the wire of radius $R \gg \xi$, such that the vortex core is represented by the wire, the Eq.(5) gives the vortex mass due to the backflow around the moving core. This is the simplest realization of the backflow mass of the vortex discussed by Baym and Chandler [6]. For such vortex with the wire-core the Baym-Chandler mass is the dominating mass of the vortex. The Kopnin mass which can result from the normal excitations trapped near the surface of the wire is essentially less.

Let us now consider the Baym-Chandler mass for the free vortex at $T = 0$ using again the continuous-core model. In the wire-core vortex this mass arises due to the backflow caused by the inhomogeneity of ρ_s: $\rho_s(r > R) = \rho$ and $\rho_s(r < R) = 0$. Similar but less severe inhomogeneity of $\rho_s = \rho - \rho_n$ occurs in the continuous-core vortex due to the nonzero local normal density in Eq.(3). Due to the profile of the local superfluid density the external flow is disturbed near the core according to continuity equation

$$\nabla \cdot (\rho_s \mathbf{v}_s) = 0. \tag{6}$$

If the smooth core is large, $R \gg \xi$, the deviation of the superfluid component in the smooth core from its asymptotic value outside the core is small: $\delta\rho_s = \rho - \rho_s \sim (\xi/R)\rho \ll \rho$ and can be considered as perturbation. Thus if the asymptotic value of the velocity of the superfluid component with respect of the core is $\mathbf{v}_{s0} = -\mathbf{v}_L$, the disturbance $\delta\mathbf{v}_s = \nabla\Phi$ of the superflow in the smooth core is given by:

$$\rho\nabla^2\Phi = v^i_{s0}\nabla^j(\rho_n)_{ij}. \tag{7}$$

The kinetic energy of the backflow gives the Baym-Chandler mass of the vortex

$$M_{\rm BC} = \frac{\rho}{v^2_{s0}} \int d^2r \, (\nabla\Phi)^2 . \tag{8}$$

In the simple approximation, when the normal component in Eq.(3) is considered as isotropic, one obtains

$$M_{\rm BC} = \frac{1}{2\rho} \int d^2r \rho^2_n(r) \sim \rho\xi^2. \tag{9}$$

The Baym-Chandler mass does not depend on the core radius R, since the large area R^2 of integration in Eq.(9) is compensated by small value of the normal component in the rarified core, $\rho_n \sim \rho(\xi/R)$. That is why if the smooth core is large, $R \gg \xi$, this mass is parametrically smaller than the Kopnin mass in Eq.(4).

In conclusion, both contributions to the mass of the vortex result from the mass of the normal component trapped by the vortex. The difference between Kopnin mass and Baym-Chandler backflow mass is only in the origin of the normal component trapped by the vortex. The relative importance of two masses depends on the vortex core structure: (1) For the free continuous vortex with the large core size $R \gg \xi$, the Kopnin mass dominates: $M_{\rm Kopnin} \sim \rho R\xi \gg M_{\rm BC} \sim \rho\xi^2$. (2) For the vortex trapped by the wire of radius $R \gg \xi$, the Baym-Chandler mass is proportional to the core area, $M_{\rm BC} \sim \rho R^2$, and is parametrically larger than the Kopnin mass. (3) For the free vortex core with the core radius $R \sim \xi$ the situation is not clear since the continuous core approximation does not work any more. But extrapolation of the result in Eq.(9) to $R \sim \xi$ suggests that the Baym-Chandler mass can be comparable with Kopnin mass.

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