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EXACT RENORMALIZATION GROUP AND RUNNING NEWTONIAN COUPLING IN HIGHER-DERIVATIVE GRAVITY

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We discuss exact renormalization group (RG) in R^2-gravity using effective average action formalism. The truncated evolution equation for such a theory on de Sitter background leads to the system of non perturbative RG equations for cosmological and gravitational coupling constants. Approximate solution of these RG equations shows the appearence of antiscreening and screening behaviour of Newtonian coupling what depends on higher-derivative coupling constants.

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In the absence of consistent quantum gravity it could be that consideration of effective models for quantum gravity (QG) is the only possibility to take into account gravitational phenomena in high energy physics. One may start from particular model of QG (see [1] for a review) and to formulate effective model which describes theory in some region. In such a way, effective theory for conformal factor to describe QG in far infrared (at large distances) has been formulated [2]. Such theory which is based on higher-derivative scalar gives the way to estimate the behaviour of Newtonian coupling [3].

One may consider Einstein gravity as effective theory and estimate quantum corrections to Newtonian coupling [4] using effective field theory technique. Moreover, as non-renormalizability is not a problem in such approach one can apply exact RG [5], say in a form of effective average action, in order to formulate the non perturbative RG equations for coupling constants in Eistein gravity [6, 7]. In the same way it is very interesting to consider R^2-gravity as effective model. Such model attracts a lot of attention (see [1] for a review and list of references), being multiplicatively renormalizable (but eventually non-unitary in perturbative approach). Note that perturbative RG equations for higher-derivative gravity have been first considered in ref.[8] (see [1] for an introduction). A kind of effective R^2-gravity leads to more or less successful inflationary Universe [9].

In the present letter we formulate the evolution equation and non-perturbative RG equations for coupling constants in R^2-gravity [1]. The action to start with is given by the following (in Euclidean notations)

$$S=\int d^4x\sqrt{g}\left\{\epsilon R^*R^*+\frac{1}{2f^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}-\frac{1}{6\nu^2}R^2-2\kappa^2R+4\kappa^2\Lambda\right\}, \qquad (1)$$

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where $R^*R^* = 1/4\epsilon^{\mu\nu\alpha\beta}\epsilon_{\lambda\rho\gamma\delta}R^{\lambda\rho}_{\mu\nu}R^{\gamma\delta}_{\alpha\beta}$, $C_{\mu\nu\alpha\beta}$ is the Weyl tensor, $\kappa^{-2} = 32\pi\bar{G}$ is the Newton constant, ϵ, f^2, ν^2 are gravitational coupling constants. It is well known that the theory with action (1) is multiplicatively renormalizable and asymptotically free. Note that perturbative running of Newtonian coupling constant in a theory (1) with matter has been disscussed in ref. [10].

Following the approach of ref. [6] we will write the evolution equation for the effective average action $\Gamma_k[g,\bar{g}]$ defined at non zero momentum ultraviolet scale k below some cut-off $\Lambda_{cut\text{-}off}$. The truncated form of such evolution equation has the following form:

$$\partial_t\Gamma_k[g,\bar{g}] = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)}[g,\bar{g}] + R_k^{grav}[\bar{g}]\right)^{-1}\partial_t R_k^{grav}[\bar{g}]\right] -$$

$$-\sum_i c_i \mathrm{Tr}\left[\left(-M_i[g,\bar{g}] + R_{ki}^{gh}[\bar{g}]\right)\partial_t R_{ki}^{gh}[\bar{g}]\right], \tag{2}$$

here $t = \ln k$, R_k are cutoffs in gravitational and ghosts sectors, c_i are the weights for ghosts (we have Fadeev–Popov ghost with $c_{FP} = 1$ and so called third ghost with weight $c_{TG} = 1/2$), $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ where $h_{\mu\nu}$ is the quantum gravitational field, $\Gamma_k^{(2)}$ is the Hessian of $\Gamma_k[g,\bar{g}]$ with respect to $g_{\mu\nu}$ at fixed $\bar{g}_{\mu\nu}$, M_i are ghost operators. Note that the RHS of eq. (2) is very similar to the one-loop effective action.

At the next steep we have to specify the truncated evolution equation for the theory (1). Starting from UV scale $\Lambda_{cut\text{-}off}$, evolving the theory down to smaller scales $k << \Lambda_{cut\text{-}off}$ we may use the truncation of the form

$$\kappa^2 \to Z_{Nk}^{-1}\kappa^2, \quad \frac{1}{f^2} \to Z_{Nk}\frac{1}{f^2}, \quad \frac{1}{\nu^2} \to Z_{Nk}\frac{1}{\nu^2}, \quad \Lambda \to \lambda_k, \tag{3}$$

where k-dependence is denoted by index k. We will be limited here only to lower derivatives terms in the reduction of Γ_k, i.e. higher-derivative coupling constants may be considered as free parameters.

Choosing $\bar{g}_{\mu\nu} = g_{\mu\nu}$ (then ghost term disappears) and projecting the evolution equation on the space with low derivatives terms one gets the left-hand side of the evolution eq. (2) as following:

$$\partial_t\Gamma_k[g,g] = 2\kappa^2\int d^4x\sqrt{g}\left[-R(g)\partial_t Z_{Nk} + 2\partial_t\left(Z_{Nk}\lambda_k\right)\right]. \tag{4}$$

The initial conditions for Z_{Nk}, λ_k are choosen as in ref. [6].

The right-hand side of evolution equations may be found after very tedious calculations (choosing de Sitter background $R_{\mu\nu} = 1/4 g_{\mu\nu}R$, calculating the path integral, making expansion on R). We drop the details of these calculations. The final system of non-perturbative renormalization group (RG) equations for Newtonian and cosmological constants is obtained as following:

$$\partial_t g_k = \left[2 + \eta_N(k)\right] g_k, \tag{5}$$

where g_k is the dimensionless renormalized Newtonian constant,

$$g_k = k^2 G_k = k^2 Z_{Nk}^{-1}\bar{G}.$$

The anomalous dimension $\eta_N(k)$ is given by

$$\eta_N(k) = g_k B_1(\alpha_{2k}, \beta_{2k}, \gamma_{2k}, \delta_{2k}) + \eta_N(k) g_k B_2(\alpha_{2k}, \beta_{2k}, \gamma_{2k}, \delta_{2k}), \tag{6}$$

where

$$\begin{aligned} B_1(\alpha_{2k}, \beta_{2k}, \gamma_{2k}, \delta_{2k}) = \frac{1}{12\pi} \{10\Phi_1^1(\alpha_{2k}) + 10\Phi_1^1(\beta_{2k}) - \\ -10\Phi_1^1(0) + 2\Phi_1^1(\gamma_{2k}) + 2\Phi_1^1(\delta_{2k}) - (60\alpha_1 + 5)\,\Phi_2^2(\alpha_{2k}) - (60\beta_1 + 5)\,\Phi_2^2(\beta_{2k}) + \\ +(\frac{24}{K-3} - 6)\Phi_2^2(0) - 12\gamma_1 \Phi_2^2(\gamma_{2k}) - 12\delta_1 \Phi_2^2(\delta_{2k}) \Big\}, \\ B_2(\alpha_{2k}, \beta_{2k}, \gamma_{2k}, \delta_{2k}) = -\frac{1}{12\pi} \{5\tilde{\Phi}_1^1(\alpha_{2k}) + 5\tilde{\Phi}_1^1(\beta_{2k}) + 7\tilde{\Phi}_1^1(0) + \\ +\tilde{\Phi}_1^1(\gamma_{2k}) + \tilde{\Phi}_1^1(\delta_{2k}) - 30(\alpha_1 + \frac{1}{12})\tilde{\Phi}_2^2(\alpha_{2k}) - 30(\beta_1 + \frac{1}{12})\tilde{\Phi}_2^2(\beta_{2k}) - \\ -3\tilde{\Phi}_2^2(0) - 6\gamma_1 \tilde{\Phi}_2^2(\gamma_{2k}) - 6\delta_1 \tilde{\Phi}_2^2(\delta_{2k})\}. \end{aligned} \tag{7}$$

Here

$$\begin{aligned} \alpha_1, \beta_1 = \frac{1}{12} + \frac{f^2 + \nu^2}{6\nu^2} \pm \frac{1}{2} \left(\frac{f^2 + \nu^2}{3\nu^2} - \frac{K+6}{6K} \right) \left[1 + \frac{4\lambda_k}{\kappa^2 f^2 K} \right]^{-1/2}, \\ \gamma_1, \delta_1 = \frac{1}{2(K-3)} \pm \frac{1}{2(K-3)} \left(1 - \frac{8\lambda_k}{\kappa^2 \nu^2 K} \right)^{-1/2}; \end{aligned} \tag{8}$$

$$\begin{aligned} \alpha_{2k}, \beta_{2k} = \frac{\kappa^2 f^2}{k^2} \left\{ 1 \pm \left[1 + \frac{4\lambda_k}{\kappa^2 f^2 K} \right]^{1/2} \right\}, \\ \gamma_{2k}, \delta_{2k} = \frac{\kappa^2 \nu^2}{k^2} \left\{ 1 \pm \left[1 - \frac{8\lambda_k}{\kappa^2 \nu^2 (K-3)} \right]^{1/2} \right\}. \end{aligned} \tag{9}$$

Note that $K = 3f^2/(f^2 + 2\nu^2)$ what corresponds to the choice of so-called gauge-fixing independent effective action (for a review see [1, 11]). By this choice, we solve the gauge-dependence problem (for a related discussion in case of Einstein gravity, see [7]). The functions $\Phi_n^p(w)$ and $\tilde{\Phi}_n^p$ are given by the integrals

$$\begin{aligned} \Phi_n^p(w) = \frac{1}{\Gamma(n)} \int_0^\infty dz z^{n-1} \frac{R^{(0)}(z) - z R^{(0)'}(z)}{[z + R^{(0)}(z) + w]^p}, \\ \tilde{\Phi}_n^p(w) = \frac{1}{\Gamma(n)} \int_0^\infty dz z^{n-1} \frac{R^{(0)}(z)}{[z + R^{(0)}(z) + w]^p}. \end{aligned} \tag{10}$$

Solving (6)

$$\eta_N(k) = \frac{g_k B_1(\bar{\lambda}_k, \kappa_k)}{1 - g_k B_2(\bar{\lambda}_k, \kappa_k)}, \tag{11}$$

where $\kappa_k^2 = \kappa^2/k^2$, $\bar{\lambda}_k = \lambda_k/k^2$, we see that the anomalous dimension η_N is a non pertubative quantity. The evolution equation for the cosmological constant is obtained as following

$$\partial_t(\bar{\lambda}_k) = -\left[2 - \eta_N(k)\right]\bar{\lambda}_k + \frac{g_k}{4\pi}\left\{10\Phi_2^1(\alpha_{2k}) + 10\Phi_2^1(\beta_{2k}) - 10\Phi_2^1(0) + \right.$$
$$+2\Phi_2^1(\gamma_{2k}) + 2\Phi_2^1(\delta_{2k}) - \eta_N(k)\left[5\tilde{\Phi}_2^1(\alpha_{2k}) + 5\tilde{\Phi}_2^1(\beta_{2k}) + \right.$$
$$\left.\left. +7\tilde{\Phi}_2^1(0) + \tilde{\Phi}_2^1(\gamma_{2k}) + \tilde{\Phi}_2^1(\delta_{2k})\right]\right\}. \tag{12}$$

Eqs. (5) and (12) with (11) determine the value of the running Newtonian constant and cosmological constant at the scale $k << \Lambda_{cut\text{-}off}$. Above evolution eqs. include non-perturbative effects which go beyond a simple one-loop calculation.

Next we estimate the qualitative behaviour of the running Newtonian constant as above system of RG equations is too complicated and cannot be solved analytically. To this end we assume that the cosmological constant is much smaller than the IR cut-off scale, $\lambda_k << k^2$, so we can put $\lambda_k = 0$ that simplify Eqs. (8) and (9). After that, we make an expansion in powers of $(\bar{G}k^2)^{-1}$ keeping only the first term (i.e. we evaluate the functions $\Phi_n^p(0)$ and $\tilde{\Phi}_n^p(0)$) and finally obtain (with $g_k \sim k^2\bar{G}$)

$$G_k = G_o\left[1 - w\bar{G}k^2 + ...\right], \tag{13}$$

where

$$w = -\frac{1}{2}B_1(0,0) = \frac{1}{24\pi}\left[\left(50 + 22\frac{f^2}{\nu^2}\right) - \frac{7\pi^2}{3}\right].$$

In case of Einstein gravity, similar solution has been obtained in refs. [6, 7]. In getting (13) we use the same cut-off function as in [6].

We see that sign of w depends on higher-derivative coupling constants:

$$w > 0, \text{ if } \quad 50 - \frac{7\pi^2}{3} + \frac{22f^2}{\nu^2} > 0. \tag{14}$$

The coupling constant ν^2 may be choosen to be negative (see [1]). So, for example for $f^2 = 1$, $\nu^2 = \pm 1$ we get $w > 0$ and Newtonian coupling decreases as k^2 increases; i.e. we find that gravitational coupling is antiscreening. On the contrary, for $f^2 = 1$, $\nu^2 = -1/2$ we get $w < 0$ and screening behaviour for Newtonian coupling. It means that in such phase gravitational charge (mass) is screened by quantum fluctuations, or, in other words Newtonian coupling is smaller at smaller distances. The sign of quantum correction to Newtonian potential will be different also.

Note that above quantum correction to Newtonian coupling constant has been calculated in ref.[10] using one-loop approach and perturbative RG equations. It is clear that result of such calculation is different from the one presented above as we use nonperturbative RG method. Moreover, as it has been noted at the beginning the theory under discussion is multiplicatively renormalizable in perturbative approach, but most likely it is not unitary in such approach. Hence, the perturbative results may not be trusted in many situations. On the contrary, within nonperturbative approach the theory is considered as an effective theory, so the problems with non-unitarity are not important. The possibility to get some nonperturbative results in models of QG in four dimensions looks very attractive and may help in the construction of new QG models.

Thus, we found that Newtonian coupling may show screening or antiscreening behaviour in R^2-gravity what depends on higher-derivative couplings. That shows explicitly that R^2 quantum gravity may lead to different physical consequences than Einstein gravity even at low energies.

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