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THEOTY OF AN AVERAGE PULSE PROPAGATION IN HIGH-BIT-RATE OPTICAL TRANSMISSION SYSTEMS WITH STRONG DISPERSION MANAGEMENT

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Theory of a signal transmission in the high-bit-rate optical communication systems with large variations of dispersion (strong dispersion management) is presented. It is found that an averaged propagation of a chirped breathing optical pulse along the line is described by the nonlinear Schroedinger equation with additional parabolic potential. The shape of the averaged pulse is an intermediate state between the sech-type soliton and a Gaussian pulse. Fast decaying Gaussian wings of such pulses allow more dense information packing in comparison with using sech-type fundamental solitons.

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Ultra-fast optical signal transmission is an example of a successful practical utilization of the fundamental results of the modern soliton theory. Impressive results have been achieved recently in the long-distance, high-bit-rate optical data transmission by using optical soliton (a pulse resulting from a balance between fiber nonlinearity and dispersion) as an information carrier. The stable, error-free, multy-channel (10 Gbit/s per channel) soliton transmission has been demonstrated over the transoceanic distances [1]. Theory of the signal transmission in optical fiber lines is based on the nonlinear Schroedinger equation (NLSE) that has been integrated by Zakharov and Shabat in 1971 [2]. The NLSE is one of the fundamental nonlinear models integrable by means of the powerful method of the inverse scattering transform. Properties of the sech-profile soliton solution of the NLSE determine features of the optical communication lines exploiting soliton concept.

One of the main factors limiting transmission capacity achievable by the modern optical soliton-based communication systems [3] is the interaction between two neighbouring solitons. Overlap of the exponential tails of the closely spaced pulses leads to the interaction of the solitons and the information loss. To provide for a stable transmission, a separation between two neighbouring fundamental solitons should be not less than five soliton widths. This is a principal limitation for a transmission based on the soliton with sech shape described by the NLSE.

One possible way to increase transmission capacity is to use as an information carrier a solitary wave with the wings decaying faster than exponential tails of the NLSE soliton. This would result in a substantial suppression of the soliton interaction and, consequently, in a possibility of a more dense information packing. This Letter presents a theory of the nonlinear communication systems that allow a stable transmission of a soliton with fast decaying tails. Specifically, we describe a propagation of a soliton with Gaussian wings in optical transmission systems with dispersion compensation.

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Dispersion compensation technique has been put recently into the focus of intensive research as a promising approach to increase transmission capacity of optical communication systems both in linear and soliton regimes (see e.g. [4–15] and references therein). In the linear regime, compensation of dispersion prevents dispersive broadening of the pulse. An additional advantage is that the impact of the four-wave mixing on a signal transmission is suppressed due to the reduction of the efficiency of the phase matching. In the soliton regime, numerical simulations and experiments demonstrate extremely stable propagation of a soliton in the fiber links with dispersion compensation. Large variation of the dispersion leads to the breathing-like oscillations of a pulse width on the amplification distance, a "slow" dynamics on the larger scales is determined by the fiber nonlinearity and residual dispersion [7]. Numerical simulations reveal the following features of the breathing soliton:

– the form of the asymptotic pulse is closer to a Gaussian shape rather than to a sech-profile;

– a forming pulse is chirped (pulse phase has nontrivial time-dependence);

– energy of the stable breathing pulse is well above that of the NLSE soliton of the corresponding average dispersion.

These observations make clear a difference between soliton-like pulse in a system with dispersion compensation and the soliton of the NLSE. This indicates that an average model describing evolution of the breathing pulse should differ from the NLSE.

In this Letter, a basic equation describing slow dynamics of the chirped pulse in the transmission systems with strong dispersion management is derived in the leading order. It is found that the average propagation of the chirped pulse is described by the NLSE with additional quadratic potential. It is demonstrated that a stationary pulse is an intermediate state between the NLSE sech-type soliton and a Gaussian pulse. The theory of the soliton with Gaussian wings propagating in the transmission systems with dispersion compensation developed in this Letter explains numerical and experimental observations mentioned above.

Optical pulse propagation down the cascaded transmission system with dispersion compensation is governed by

$$iA_z + d(z)A_{tt} + |A|^2A = iZ_{NL}(-\gamma + [\exp(\gamma Z_a) - 1]\sum_{k=1}^{N}\delta(z - z_k))A = i\,G(z)A. \quad (1)$$

We use here notation of [7]: $Z_{NL} = 1/\sigma P_0$ is the nonlinear length, $Z_{dis} = t_0^2/|\beta_2|$ - the dispersion length corresponding to the transmission fiber (standard mono-mode fiber (SMF)); t_0 and P_0 are an incident pulse width and peak power, β_2 is the group velocity dispersion for SMF; σ is the coefficient of the nonlinearity, γ describes fiber losses. Retarded time is normalized to the initial pulse width $t = T/t_0$, an envelope of the electric field $E = E(T, Z)$ is normalized to the initial pulse power $|E|^2 = P_0|A|^2$, and the coordinate along the fiber z to the nonlinear length $z = Z/Z_{NL}$, Z_a is amplification period, $z_k = kz_a$ are the amplifiers locations. Chromatic dispersion $d(z) = \tilde{d}(z) + \langle d\rangle$ presents a sum of a rapidly varying part ($\tilde{d} \sim Z_{NL}/Z_{dis} \gg 1$) and a constant residual dispersion ($\langle d\rangle \sim Z_{NL}/Z_{RD} \sim 1$), here $Z_{RD} = t_0^2/|\beta_{2RD}|$ is a dispersion length corresponding to the residual dispersion of each section. The simplest optical-pulse equalizing system consists from a transmission fiber and equalizer fiber with the opposite dispersion – dispersion compensating fiber. Incorporation of a fiber with normal dispersion reduces (or in the ideal case eliminates) total dispersion of the fiber span between two amplifiers. Term strong dispersion management means that a variation of the dispersion on the amplification period is large. Consequently, not only pulse power, but also pulse width experiences substantial variations during the amplification period. Formally this can be formulated as a condition $R(z) = \int_0^z d(z')dz' \geq 1$.

As was mentioned, an optical pulse propagating in a system with large variation of dispersion experiences periodic oscillations of the amplitude and width. The breathing rapid oscillations of the pulse are accompanied by slow average changes of the pulse characteristics due to nonlinearity and residual dispersion [7]. In the limit $Z_a, Z_{dis} \ll Z_{NL}, Z_{RD}$, one may treat the nonlinearity and residual dispersion as perturbations. Therefore, let us first recall the well-known exact solution of the linear problem. Neglecting nonlinear term in Eq.(1) fast oscillations of the linear pulse amplitude and width for a Gaussian input signal $A(0,t) = N\exp(-t^2)$ are given by

$$A(z,t) = \frac{N}{\sqrt{\tau(z)}}\exp(-t^2/\tau^2(z) - iCt^2/\tau^2(z) + i\Phi(z))\exp(\int_0^z G(z')dz'), \qquad (2)$$

here $\tau^2(z) = 1 + 16R^2(z)$, $dR(z)/dz = d(z)$, $C = 4R(z)$ and $\Phi = -0.5\arctan[4R(z)]$. This solution shows that the pulse is highly chirped in contrast to the soliton solution of the NLSE. We demonstrate later that this chirping leads to the effective parabolic potential in the equation describing slow dynamics. Nonlinear effects come into play on a large scale compared to Z_a, namely on the distances proportional to Z_{NL}. Nonlinear length Z_{NL} can be comparable with Z_{RD}. Therefore, in the description of the average evolution of the pulse, it is necessary to take into account both the residual dispersion and nonlinearity. Thus, there are two scales in the pulse dynamics [7]: fast processes correspond to the large oscillations of the amplitude and the width of the pulse; and slow dynamics giving the average changes due to nonlinear effects and residual dispersion. Fast oscillations are only slightly modified by nonlinearity and residual dispersion. Note that the slow average dynamics is responsible for the stability of signal transmission. Our goal now is to average Eq.(1) keeping general structure of the rapid oscillations given by (2).

Large variation of the dispersion on the amplification period is the main obstacle to the direct application of the powerful Lie-transform [16] method to obtain averaged (slow) dynamics in Eq.(1). The main technical idea of the approach suggested here is to use first a transformation that accounts for a fast pulse dynamics and to apply averaging procedure to the transformed equation. As we demonstrate, this allows one to derive an averaged model that is the NLSE with additional parabolic potential responsible for a formation of the Gaussian wings of the soliton. This procedure is a modification of the averaging procedure used in [7]. Important difference is that in the approach developed here, a pulse chirp (phase dependence) is accounted by an exact transform, and, therefore, a local (nonintegral) average equation is obtained. Since nonlinearity and residual dispersion acts as small perturbations to the linear dynamics, to start with, we assume that a pulse dynamics will be close to a structure given by Eq.(2).

Let us make the following transformation that is similar to the so-called "lense" transformation first suggested by Talanov in the theory of self-focusing [17]

$$A(t,z) = \frac{Q(\xi,z)}{\sqrt{\tau(z)}} \exp[i\frac{\nu(z)}{\tau(z)}t^2] \exp(\int_0^z G(z')dz'), \tag{3}$$

here $\xi = t/\tau$ and

$$d\tau/dz = 4d(z)\nu \ . \tag{4}$$

Eq.(1) is transformed to

$$iQ_z + \frac{d(z)}{\tau^2}Q_{\xi\xi} + \frac{c(z)}{\tau}|Q|^2Q - \tau\nu_z\xi^2Q = 0. \tag{5}$$

We still have freedom to choose the equation for ν_z. Let us fix the latter as

$$\nu_z = a^2[\frac{d(z)}{\tau^3} - \frac{c(z)}{\tau^2}]; \quad c(z) = \exp(2\int_0^z G(z')dz'). \tag{6}$$

Note that Eqs. (4), (6) have been derived in [7] using variational approach, but here these equations describe exact transformation of the Eq.(1). Neglecting nonlinearity one can find exact linear solution of these equations with initial conditions $\tau(0) = 1$ and $\nu(0) = \nu_0$.

$$\tau^2 = \frac{a^2 + 4[(a^2+4\nu_0^2)R(z)+\nu_0]^2}{a^2+4\nu_0^2}; \quad \nu = \frac{(a^2+4\nu_0^2)R(z)+\nu_0}{\tau}; \quad \frac{dR}{dz} = d(z). \tag{7}$$

When nonlinear effects and residual dispersion are negligible $(R(z_a) = R(0) = 0)$ the pulse recovers its original form. A combined action of the residual dispersion and nonlinearity modifies the periodic solutions (7) and they cannot be expressed in the explicit form. Numerical periodic solutions of Eqs.(4), (6) has been presented in [14]. It should be pointed out that nonlinearity and residual dispersion presents small perturbation to the linear solution and (7) can be used as a first approximation of the solution in the general case. As it will be shown below, a structure of the equation describing averaged dynamics does not depend on the specifics of the dispersion compensation scheme. However, oscillatory behavior of the pulse given by $\tau(z)$ and $\nu(z)$ are determined, evidently, by the dispersion map. Substitution of (6) into (5) yields

$$iQ_z + \frac{d(z)}{\tau^2}[Q_{\xi\xi} - a^2\xi^2Q] + \frac{c(z)}{\tau}(|Q|^2Q - a^2\xi^2Q) = 0. \tag{8}$$

The most of presently used in practice dispersion maps $d(z)$ are built from pieces of fibers with different dispersion (negative or positive). Note that for such dispersion maps a condition $d^2 > 0$ is satisfied. Therefore, in what follows we consider only dispersion compensation schemes with $d^2 > 0$. This allows to rewrite the coefficient before last term in Eq.(8) as

$$\frac{c(z)}{\tau(z)} = \frac{d(z)}{\tau^2}[\frac{d(z)}{d^2}c(z)\tau(z)] \equiv \frac{d(z)}{\tau^2}\alpha(z'). \tag{9}$$

We introduce here a new variable $z'(z)$ defined through

$$\frac{dz'}{dz} = \frac{d(z)}{\tau(z)^2}; \quad z'(0) = 0. \tag{10}$$

After substitution of (9) and (10) into Eq.(8) we get

$$iQ_{z'} + Q_{\xi\xi} - a^2\xi^2 Q + \alpha(z')(|Q|^2 Q - a^2\xi^2 Q) = 0. \tag{11}$$

Variable z' rapidly oscillates and slowly grows during the amplification period in the case of the anomalous residual dispersion. In the case of periodic τ and ν the function $z'(z)$ presents a sum of a periodic function with zero mean value and a linearly growing part (due to residual dispersion). It is interesting to note that as a particular case (for a specific dispersion map), our general theory reproduces results obtained in [18]. Namely, for the special dispersion profile having a form

$$d(z) = \frac{a\,c(z)}{\sqrt{a^2 + 4\nu_0^2}} \cosh[2\sqrt{a^2 + 4\nu_0^2}\,y(z)]; \quad \frac{dy}{dz} = c(z);$$

$$\cosh[2\sqrt{a^2 + 4\nu_0^2}\,y(0)] = \frac{\sqrt{a^2 + 4\nu_0^2}}{a}, \tag{12}$$

the function $\alpha(z')$ in Eq.(11) becomes a constant. For this specific choice of the dispersion profile Eq.(1) is transformed exactly to the NLSE with additional quadratic potential [18]. However, a pulse chirp is not recovered after amplification period in such a system and additional dispersion compensating element should be added at the end of each sections.

Now we demonstrate that in the general case of an arbitrary dispersion map (under conditions specified above), the average evolution of a pulse dynamics in Eq.(1) is given by the NLSE with additional parabolic potential. The averaging procedure in a form of the Lie-transform [16] or the method used in [19] can be applied directly to the transformed Eq.(11). In this Letter we present only the result in the leading order, because already in this order a remarkable new properties of average soliton arise. Averaging is over one cycle of the variation of z' corresponding to one amplification period. We use also the following useful relations (here $\oint$ denotes the integration over on cycle in z')

$$\oint dz' = \int \frac{d(z)}{\tau^2} dz; \quad \oint \alpha(z') dz' = \int \frac{c(z)}{\tau} dz. \tag{13}$$

After straightforward calculations, in the leading order, the averaged equation describing slow evolution of the chirped pulse due to nonlinearity and residual dispersion reads

$$i\frac{\partial U}{\partial z'} + \frac{\partial^2 U}{\partial \xi^2} - (1 + \frac{r_1}{r_2})a^2\xi^2 U + \frac{r_1}{r_2}|U|^2 U = 0; \; r_1 = \langle \frac{c}{\tau} \rangle; \; r_2 = \langle \frac{d}{\tau^2} \rangle. \tag{14}$$

Here $\langle f \rangle$ denotes averaging over one amplification period in z. Obtained averaged equation is the main result of the Letter. This equation possesses steady-state solution in the form of a soliton with Gaussian wings [14, 18]. Energy of such soliton is above the energy of the corresponding NLSE soliton [20]. Taking into account that the pulse structure in the original variables is given by transformation

(3), it is seen that an asymptotic pulse is highly chirped. Thus, our approach explains the main features of the breathing pulses observed in numerical simulations and experiments. Comprehensive investigation of a soliton solution of Eq.(14) will be published elsewhere. Higher-order corrections to Eq.(14) can be found using Lie-transform technique developed in [16] . Eq.(14) is a Hamiltonian system:

$$i\frac{\partial U}{\partial z'} = \frac{\delta H}{\delta U^*}; \; H = \int |U_\xi|^2 d\xi + (1+\frac{r_1}{r_2})a^2 \int \xi^2 |U|^2 d\xi - \frac{r_1}{2r_2}\int |U|^4 d\xi. \qquad (15)$$

Simple scaling analysis of the Hamiltonian indicates that a ground soliton solution of Eq.(14) is stable. It should be pointed out that if an input pulse differs from the shape of the soliton solution of Eq.(14), a radiation is emitted during formation of the breathing soliton. Interaction of the soliton with the radiation leads to occurrence of slowly decreasing oscillations studied in the case of the NLSE in [21]. As was found in [14] by numerical simulations of original Eq.(1), in the general case an asymptotic state presents breathing soliton interacting with a radiative pedestal.

In conclusions, an averaged equation is derived in the leading order to describe an asymptotic breathing dynamics of a chirped optical pulse in the transmission systems with dispersion compensation. This average equation is the NLSE with additional quadratic potential. It is demonstrated that the breathing pulse shape is an intermediate state between the NLSE sech-type soliton and a Gaussian pulse.

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