

## THE BFKL-REGGE PHENOMENOLOGY OF DEEP INELASTIC SCATTERING

*N.N.Nikolaev<sup>0,+</sup>, B.G.Zakharov<sup>+</sup>, V.R.Zoller\**

<sup>0</sup>*Institut für Kernphysik, Forschungszentrum Jülich,  
D-52425 Jülich, Germany*

<sup>+</sup>*Landau Institute for Theoretical Physics,  
117334 Moscow, Russia*

<sup>\*</sup>*Institute for Theoretical and Experimental Physics,  
117218 Moscow, Russia*

Submitted 3 July 1997

We calculate the Regge trajectories of the subleading BFKL singularities and eigenfunctions for the running BFKL pomeron in the color dipole representation. We obtain a viable BFKL-Regge expansion of the proton structure function  $F_{2p}(x, Q^2)$  in terms of several rightmost BFKL singularities. We find large subleading contributions to  $F_{2p}(x, Q^2)$  in the HERA kinematical region which explains the lack of a predictive power of GLDAP-extrapolations of  $F_{2p}(x, Q^2)$  to a domain of small  $x$ . We point out the relation of our early finding of the precocious BFKL asymptotics to the nodal structure of subleading BFKL eigenfunctions.

PACS: 12.38.-t, 13.60.-r

Extrapolation of the proton structure function from the accessible region of  $x$  and  $Q^2$  to small  $x$  remains the topical issue ever since the first SLAC experiments on deep inelastic scattering (DIS). The GLDAP evolution [1] has been a standard instrument in these studies. It was soon realized that GLDAP extrapolations are not unique, and equally good fits to the large- $x$ -data did invariably, and wildly, diverge with each other and new experimental data, when extrapolated beyond the studied range of  $x$ . For instructive comparison of extrapolations of the pre-HERA fits to the proton structure function with the new data from HERA down to  $x \sim 10^{-5}$  see [2]. The lack of predictive power at small  $x$  is not surprising: the GLDAP evolution describes the future for large  $Q^2$  starting with assumptions on the parton densities at past,  $Q_0^2$ . One needs to assume the whole function of  $x$  and different past defines a different future.

At very small  $x$ , the assumptions of the GLDAP evolution break down and it is superseded by the  $\log(1/x)$  BFKL evolution [3]. The BFKL evolution is meant to predict the future at small  $x$  from the past defined as a function of  $Q^2$  at a sufficiently small  $x_0$ . At an asymptotically small  $x$  the BFKL prediction,  $F_2(x, Q^2) = F_{\mathbb{P}}(Q^2)x^{-\Delta_{\mathbb{P}}}$ , is unique modulo to the total normalization factor. In the BFKL approach, the  $x$ -dependence of the structure function at moderately small  $x$  depends on the spectrum of eigenvalues and on the eigenfunctions of the BFKL operator. The purpose of the present study<sup>1)</sup> is an evaluation of contributions from subleading BFKL singularities for the running BFKL pomeron in the color dipole formulation [5-7].

The BFKL equation for the interaction cross section  $\sigma(\xi, r)$  of the colour dipole  $r$  with the target reads (hereafter  $\xi = \log(1/x)$ )  $\partial\sigma(\xi, r)/\partial\xi = \mathcal{K} \otimes \sigma(\xi, r)$ .

<sup>1)</sup> The preliminary results have been reported at the DIS'97 Workshop [4]

Here the kernel  $\mathcal{K}$  is related to the wave function squared of the color-singlet  $q\bar{q}g$  state with the Weizsäcker-Williams soft gluon. The gluon fields are calculated with the running QCD coupling and perturbative gluons are infrared regularized imposing a finite propagation radius  $R_c \simeq 0.2 - 0.3$  fm. The BFKL pole  $\mathbb{P}_n$  corresponds to the Regge-behaving solution of the BFKL equation,

$$\sigma_n(\xi, r) = \sigma_n(r) \exp(\Delta_n \xi) = \sigma_n(r) \left( \frac{x_0}{x} \right)^{\Delta_n}, \quad (1)$$

where the eigenfunction  $\sigma_n(r)$  and the eigenvalue (intercept)  $\Delta_n$  are determined from

$$\mathcal{K} \otimes \sigma_n = \Delta_n \sigma_n(r). \quad (2)$$

The behavior of eigenfunctions at  $r \rightarrow 0$  and/or  $\alpha_S(r) \rightarrow 0$  has been found in [6, 7]:

$$\sigma_n(r) = r^2 \left[ \frac{1}{\alpha_S(r)} \right]^{\gamma_n - 1}, \quad \gamma_n \Delta_n = \frac{4}{3}. \quad (3)$$

Useful clues come from the experience with solutions of the eigenvalue problem for the Schrödinger equation. First, the leading eigenfunction  $\sigma_0(r)$ , found numerically in [5-7], is node free. Consequently, the subleading solutions with eigenfunctions  $\Delta_n < \Delta_0$  must have nodes. Second, the asymptotics (3) must hold for all eigenfunctions in the region of  $r$  beyond all nodes. Third, because of the infrared regularization  $\sigma_n(r)$  saturate at  $r \gtrsim R_c$ . This suggests that for eigenfunctions with  $n$  nodes one must try  $\sigma_n(r) = \mathcal{P}_n(z) \sigma_0(r)$  where  $\mathcal{P}_n(z)$  is a polynomial of a suitably chosen variable  $z \sim [1/\alpha_S(Q^2)]^\gamma$ . Then we apply the variational principle and minimize the functional

$$\Phi(\sigma_n) = \int \frac{dr}{r} \left| \frac{\mathcal{K} \otimes \sigma_n(r) - \Delta_n \sigma_n(r)}{\sigma_0(r)} \right|^2 \quad (4)$$

in the space of trial functions with the polynomial prefactor  $\mathcal{P}_n(z)$ . The success of such an unorthodox variational principle for excited states depends on the guessed trial function, examples of astonishingly successful applications can be found in [8].

With the so obtained eigenfunctions  $\sigma_n(r)$  we can calculate the contributions  $F_2^{(n)}(Q^2)$  to the proton structure function using the color dipole factorization [9] and can perform the BFKL-Regge expansion of the proton structure function

$$F_{2p}(x, Q^2) = \sum_n A_n F^{(n)}(Q^2) \left( \frac{x_0}{x} \right)^{\Delta_n}. \quad (5)$$

Our principal findings on solutions of the BFKL equation are as follows.

The running BFKL equation gives rise to a series of moving poles in the complex  $j$ -plane. The intercepts  $\Delta_n$  (Fig.1) very closely, to better than 10%, follow the law

$$\Delta_n = \frac{\Delta_0}{(n+1)} \quad (6)$$

suggested earlier by Lipatov from the quasiclassical consideration [10]. The found eigenfunctions plotted as  $\mathcal{E}_n(r) = \sigma_n(r)/r$  (Fig.2) to a crude approximation are similar to Lipatov's quasi-classical eigenfunctions, which are  $\mathcal{E}_n(r) \sim \cos[\phi(r)]$  for  $n \gg 1$  [10]. For a related numerical analysis of the running BFKL equation see

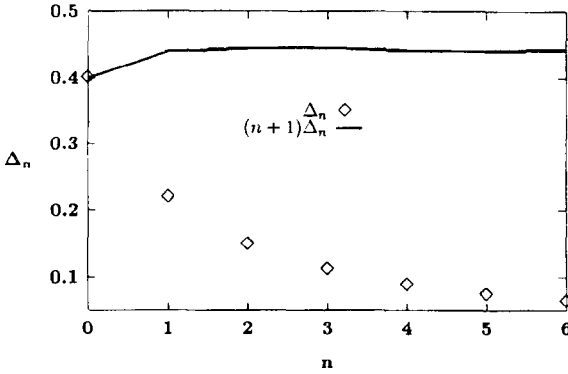


Fig.1 BFKL eigenvalues

[11]. Within our specific infrared regularization the node of  $\sigma_1(r)$  is located at  $r \simeq 0.05 - 0.06$  fm. With growing  $n$ , the location of the first node moves to a somewhat larger  $r$ , and the first nodes accumulate at  $r \sim 0.1 - 0.15$  fm.

The slope  $\alpha'_n$  of the Regge trajectory for the pole  $\mathbb{P}_n$  can be found using the technique of Ref.[12, 13]. We find  $\alpha'_n \simeq 0.07 - 0.06 \text{ GeV}^{-2}$  with weak dependence on  $n$ .

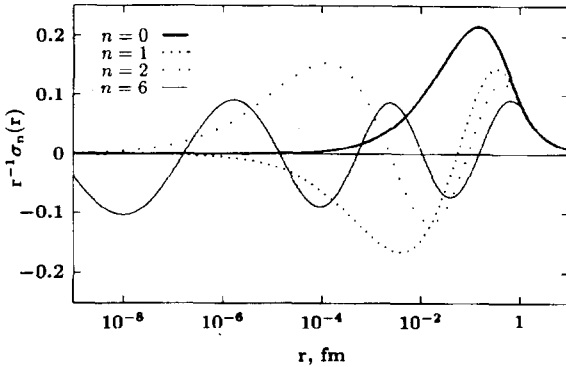


Fig.2 BFKL eigenfunctions

The structure functions  $F_2^{(n)}(Q^2)$  for the  $\mathbb{P}_n$  poles are shown in Fig.3. At large  $Q^2$ , far beyond the nodal region,  $F_2^n(x, Q^2) \propto (x_0/x)^{\Delta_n} [1/\alpha_S(Q^2)]^{4/3\Delta_n}$ . Since the relevant variable is a power of the inverse gauge coupling, which varies with  $Q^2$  very slowly, the nodes of  $F_2^{(n)}(Q^2)$  are spaced by 2-3 orders of magnitude in the  $Q^2$ -scale and only the first two of them are in the accessible range of  $Q^2$ . The first nodes of  $F_2^{(n)}(Q^2)$  are located at  $Q^2 \sim 20 - 60 \text{ GeV}^2$ . Hence, only the leading structure function contributes significantly in this region. This explains the precocious BFKL asymptotics for  $Q^2 \sim 60 \text{ GeV}^2$  found earlier from the numerical solution of the color dipole running BFKL equation [14].

An interesting finding is that the Born approximation for the dipole cross section,  $\sigma_B(r)$ , gives a very good quantitative description of the proton structure function at  $x_0 \simeq 0.03$  [14]. The  $r$ -dependence of  $\sigma_B(r)$  is quite different from that of the leading eigenfunction  $\sigma_0(r)$ , and expansion of  $\sigma_B(r)$  in BFKL eigenfunctions shows that the contribution of subleading terms with  $n > 0$  makes up to  $\sim 60\%$  of  $\sigma_B(r)$ . Consequently, the  $Q^2$ -dependence of the proton structure function at

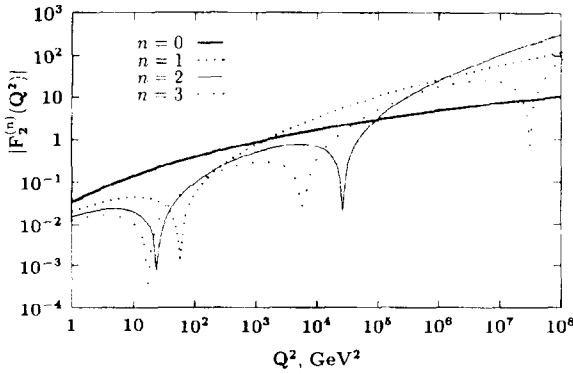


Fig.3 Modulus of BFKL structure functions

$x = x_0$ , and the subsequent  $x$ -dependence towards smaller  $x$ , is controlled to a large extent by the subleading contributions  $F_2^{(n)}(Q^2)$ .

At small  $x$  only  $Q^2 \lesssim 10^3 \text{ GeV}^2$  are accessible. In this range the structure functions with  $n \geq 3$  are hardly distinguishable. Besides, the splitting of intercepts with  $n \geq 3$  is much smaller than for  $n = 0, 1, 2$ . Hence, the Regge expansion (5) can be truncated at  $n = 2$  and  $F_2^{(2)}(Q^2)$  comprises contributions of all poles with  $n \geq 2$ .

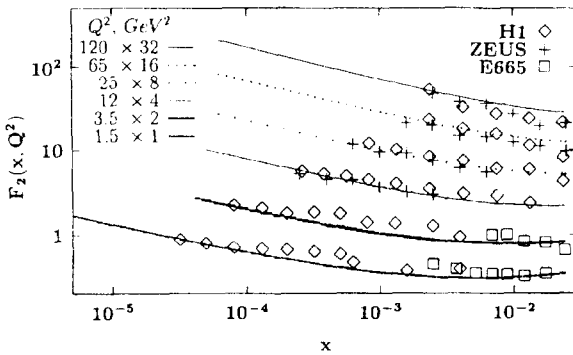


Fig.4 Three-pole approximation vs. the data of H1, ZEUS and E665

The BFKL equation allows one to determine the intercepts and structure functions  $F_2^{(n)}(Q^2)$ . In the expansion we put  $A_0 = 1$ . Then the overall normalization of eigenfunctions, for instance,  $\sigma_n(r \gg R_c) = 0.89 \text{ mb}$ , and  $A_1 = 0.39$  and  $A_2 = 0.33$  are two adjustable parameters which are fixed from the boundary condition at  $x = x_0$ . With the proper account of the valence [15] and non-perturbative [14] corrections to (5) we arrive at the three-pole-approximation which provides a viable description of the experimental data [16] in a wide kinematical range (Fig.4). Notice that in the pre-nodal region of  $Q^2 \lesssim 20 \text{ GeV}^2$  the leading and subleading structure functions are very close in shape and the experimental data in such a limited range of  $Q^2$  are absolutely insufficient for the determination of  $A_n$ , which explains the failure of the early GLDAP fits.

The effective pomeron intercept

$$\Delta_{eff} = - \frac{\partial \log F_{2p}(x, Q^2)}{\partial \log x} \quad (7)$$

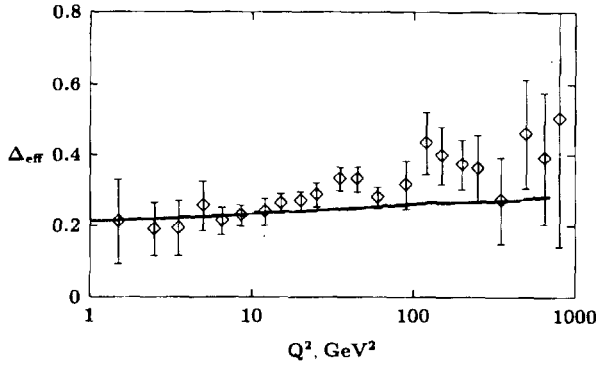


Fig.5 Effective intercept vs. H1 determination [17]

gives an idea of the role of the subleading singularities. The intercept  $\Delta_{eff}$  calculated with the experimental kinematic constraints is much smaller than  $\Delta_0 = 0.4$  which is expected to dominate asymptotically. The agreement of our numerical estimates with the H1 determination [17] (Fig.5) is quite satisfactory.

B.G.Zakharov and V.R.Zoller thank J.Speth for the hospitality in the Institut für Kernphysik, Forschungszentrum Jülich, where this work was finished. This work was partly supported by the DFG grant 436-RUS-17/13/97 and INTAS grant 93-239ext.

- 
1. V.N.Gribov and L.N.Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 (1972); Yu.L.Dokshitzer, *Sov. Phys. JETP* **46**, 641 (1977); G.Altarelli and G.Parisi, *Nucl. Phys. B* **126**, 297 (1977).
  2. H1 Collab., I.Abt et al., *Nucl. Phys. B* **44**, 515 (1993).
  3. V.S.Fadin, E.A.Kuraev, and L.N.Lipatov, *Sov. Phys. JETP* **44**, 443 (1976); *ibid* **45**, 199 (1977); Yu.Yu.Balitsky and L.N.Lipatov, *Sov. J. Nucl. Phys.* **28**, 822 (1978).
  4. V.R.Zoller, *Talk at 5th International Workshop on Deep Inelastic Scattering and QCD (DIS'97), Chicago, April 1997*, Report No. KFA-IKP(TH)-97-12.
  5. N.N.Nikolaev, B.G.Zakharov, and V.R.Zoller, *JETP Letters* **59**, 8 (1994).
  6. N.N.Nikolaev, B.G.Zakharov, and V.R.Zoller, *JETP* **105**, 1498 (1994).
  7. N.N.Nikolaev and B.G.Zakharov, *Phys. Lett. B* **327**, 157 (1994).
  8. G.Karl and V.Novikov, *Phys. Rev. D* **51**, 5069 (1995).
  9. N.N.Nikolaev and B.G.Zakharov, *Z. Phys. C* **49**, 607 (1991); *Z. Phys. C* **53**, 331 (1992).
  10. L.N.Lipatov, *Sov. Phys. JETP* **63**, 904 (1986).
  11. R.E.Hancock and D.A.Ross, *Nucl. Phys. B* **383**, 575 (1992).
  12. N.N.Nikolaev, B.G.Zakharov, and V.R.Zoller, *JETP Lett.* **60**, 678 (1994).
  13. N.N.Nikolaev, B.G.Zakharov, and V.R.Zoller, *Phys. Lett. B* **366**, 337 (1996).
  14. N.N.Nikolaev and B.G.Zakharov, *Phys. Lett. B* **333**, 250 (1994).
  15. M.Glück, E.Reya, and A.Vogt, *Z. Phys. C* **67**, 433 (1995).
  16. H1 Collab., T.Ahmed et al., *Nucl. Phys. B* **439**, 471 (1995); ZEUS Collab., M.Derrick et al., *Z. Phys. C* **69**, 607 (1996); E665 Collab., M.R.Adams et al., *Phys. Rev. D* **54**, 3006 (1996).
  17. H1 Collab., S.Aid et al., *Nucl. Phys. B* **470**, 3 (1996).