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ON EDGE STATES IN SUPERCONDUCTOR WITH TIME INVERSION SYMMETRY BREAKING

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Superconducting states with different internal topology are discussed for the layered high-T_c materials. If the time inversion symmetry is broken, the superconductivity is determined not only by the symmetry of the superconducting state, but also by the topology of the ground state. The latter is determined the integer-valued momentum-space topological invariant N. The current carrying boundary between domains with different N ($N_2 \neq N_1$) is considered. The current is produced by fermion zero modes, which number per one layer is $2(N_2 - N_1)$.

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1. Introduction. Recently a new phase transition in high temperature superconductor was reported on in Ni-doped $Bi_2Sr_2CaCu_2O_8$[1]. It occurs at low temperature, $T \sim 200$ mK and seems to correspond to the opening of the gap in the superconductor which is gapless (contains lines of nodes) above transition. Since the gap usually appears in superconductor with lines of nodes if the time inversion ($\mathcal{T}$) symmetry is violated, this suggests that the spontaneous breaking of the $\mathcal{T}$-symmetry occurs in the new transition. Such transition can be caused by interaction with magnetic impurities.

Some evidence of the additional symmetry breaking with opening of the gap in a pure $Bi_2Sr_2CaCu_2O_8$ in the presence of magnetic field was reported in [2]. The $d_{x^2-y^2} + id_{xy}$ complex order parameter was suggested in [3] to describe this experiment. The possibility of the broken $\mathcal{T}$-symmetry was discussed also within the grain boundaries and twin boundaries [4, 5].

Since the broken $\mathcal{T}$-symmetry becomes popular now for the high temperature superconductors, we discuss some consequences of the broken $\mathcal{T}$-symmetry in $D = 2$ spatial dimension relevant for layered superconductors. These consequences are similar to that in thin films of superfluid ^{3}He-A and other 2D systems where $\mathcal{T}$-symmetry is broken. They are described in terms of the integer-valued topological invariant of the ground state [6 – 10], which gives rise to chiral edge states on the boundary of superconductor and in some cases to quantization of Hall conductivity. We dicuss here this topological invariant for the layered superconductors.

2. Internal topological invariant. In the BCS model of the high-T_c superconductor, the quasiparticle spectrum is obtained from the Bogoliubov – Nambu Hamiltonian

$$\mathcal{H} = \tau_3\epsilon(\mathbf{k}) + \tau_1\Delta_1(\mathbf{k}) + \tau_2\Delta_2(\mathbf{k}) \equiv \vec{\tau} \cdot \mathbf{m}(\mathbf{k}) \quad , \tag{2.1}$$

where τ_i are the 2×2 Pauli matrices in particle-hole space; $\mathbf{k}$ is the 2D vector of the in-plane linear momentum. In the simplest model

$$\epsilon(\mathbf{k}) = \frac{\mathbf{k}^2 - k_F^2}{2m} \ , \quad \Delta_1(\mathbf{k}) = d_1(n_x^2 - n_y^2) + s_1 \ , \quad \Delta_2(\mathbf{k}) = 2d_2 n_x n_y + s_2 \quad , \tag{2.2}$$

where $\hat{n} = (n_x, n_y) = \mathbf{k}/|\mathbf{k}|$. The gap function $\Delta_1(\mathbf{k})$ describes the original $d_{x^2-y^2} + s$ state of the high-T_c superconductor, where the admixture of the s state comes from the orthorhombic deviation of the crystal symmetry from the tetragonal. It is believed that s_1 is less than d_1, so the gap nodes do not disappear in the mixture, but are shifted from their symmetric positions in the tetragonal crystal.

The function $\Delta_2(\mathbf{k})$ appears in the state with broken $\mathcal{T}$-symmetry. The $\mathcal{T}$-symmetry can be broken in two ways: with conservation of the symmetry $\mathcal{I}$ with respect to reflection in $\langle 100\rangle$ crystal axis (cf. [11]), or with conservation of the combined symmetry $\mathcal{IT}$. In the $\mathcal{I}$-symmetric state one has $d_2 = 0$, while in the $\mathcal{IT}$-symmetric state $s_2 = 0$. However, we consider here the more general case, when both $\mathcal{I}$ and $\mathcal{IT}$ are broken and one has a mixture of is and id components. Our goal is to show that the properties of the system at $T = 0$ are determined by the ground state topology rather than by symmetry.

In the broken $\mathcal{T}$-symmetry state the gap in the energy spectrum should appear

$$\Delta = \min |E(\mathbf{k})| \neq 0 \quad , \quad E^2(\mathbf{k}) = m^2(\mathbf{k}) = \epsilon^2(\mathbf{k}) + \Delta_1^2(\mathbf{k}) + \Delta_2^2(\mathbf{k}) \tag{2.3}$$

There are exceptions from this rule: at some values of the parameters satisfying the condition

$$(s_1/d_1)^2 + (s_2/d_2)^2 = 1 \tag{2.4}$$

the gap Δ becomes zero at some points $\mathbf{k}$. This equation thus determines the surfaces in the phase space of the parameters s_1, d_1, s_2, d_2, which separate the regions of the gapped superconducting states. On these surfaces the superconductivity is gapless.

The gapped superconducting states on different sides of the surface of gapless superconductivity have the same symmetry. But these states are different in terms of the internal topology, determined by the topological invariant of the ground state [10]:

$$N = \frac{1}{4\pi} \int d^2k \; \hat{m} \cdot \left(\frac{\partial \hat{m}}{\partial k_x} \times \frac{\partial \hat{m}}{\partial k_y} \right) \quad , \quad \hat{m} = \mathbf{m}/|\mathbf{m}|. \tag{2.5}$$

In our simple model this invariant is

$$N = 0 \quad , \quad \text{if} \quad \left(\frac{s_2}{d_2}\right)^2 + \left(\frac{s_1}{d_1}\right)^2 > 1 \tag{2.6}$$

and

$$N = 2 \;\text{sign}\; (d_1 d_2) \quad , \quad \text{if} \quad \left(\frac{s_2}{d_2}\right)^2 + \left(\frac{s_1}{d_1}\right)^2 < 1 \; . \tag{2.7}$$

The sign of N is determined by the relative sign of real and imaginary components of the d-wave order parameter. The transition between the states with different integer-valued invariant N occurs via the gapless intermediate state (see similar phenomenon in the thin films of ^{3}He-A, where the $\mathcal{T}$-symmetry is also broken [10]).

In general the order parameter is more complicated, moreover the system of interacting fermions cannot be described by the effective Hamiltonian of the type (2.1). The topological invariant of the ground state nevertheless exists, but it is expressed in terms of the Green's functions [6, 10].

3. Chiral edge states in domain walls. Because of the 2-fold degeneracy of the $\mathcal{T}$-symmetry broken states, there can be domain walls: surfaces in the real space separating the domains with opposite parity. If $N \neq 0$, the degenerate superconducting states have opposite value of N. Because of the jump of N across the domain wall, such wall must contain the fermion zero modes. These are the gapless branches $E(k_\parallel)$, where $k_\parallel$ is the linear momentum along the wall, they cross zero energy when $k_\parallel$ varies. These fermion zero modes correspond to the chiral gapless edge states in the Quantum Hall Effect (see [9] for the p-wave pairing in ^{3}He-A film and [3] for the pure d-wave case with $s_1 = s_2 = 0$). Close to zero energy the spectrum of the a-th mode is linear:

$$E_a(k_\parallel) = c_a(k_\parallel - k_a). \tag{3.1}$$

There is an index theorem which determines the algebraic number ν of the fermion zero modes, ie the number of modes crossing zero with positive slope, minus the number of modes with negative slope,

$$\nu = \sum_a \text{sign } c_a. \tag{3.2}$$

According to index theorem the number ν per one CuO_2 layer for a wall separating the superconducting states with $N = N_1$ and $N = N_2$ is

$$\nu = 2(N_2 - N_1) \quad . \tag{3.3}$$

If one considers the boundary between superconductor and insulator, the invariant N on the insulating side should be put zero.

The fermion zero modes in the domain wall have the same origin as the fermion zero modes in spectrum of the Caroli – de Gennes – Matricon bound states in the vortex core, where the $\mathcal{T}$-symmetry is also broken [12]. In the latter case the chiral fermions are orbiting around the vortex axis, which corresponds to the motion along the closed domain boundary [13]. For the circular domain wall of radius R the edge state have an angular momentum $Q = k_\parallel R$, and from Eq.(3.1) one obtains the spectrum of the low-energy bound states in terms of the angular momentum Q:

$$E_a(Q) = \omega_a(Q - Q_a) \tag{3.4}$$

Here $\omega_a = c_a/R$ is the angular velocity of the rotation along the closed trajectory. This equation represents the general spectrum of fermions bound to the vortex core [14 – 16]. The number of the fermion zero modes in the vortex core, the branches $E_a(Q)$ which as functions of Q cross zero energy, is also determined by the index theorem [17]:

$$\nu = \sum_a \text{sign } \omega_a = 2N \quad , \tag{3.5}$$

but now N is the winding number of the vortex.

4. Current carrying by edge states. The breaking of the $\mathcal{T}$-symmetry leads to the ground-state current carried by the occupied negative energy states in the domain wall. The topological characteristics of the fermionic charge (current) accumulated by the general texture in ^{3}He-A, where $\mathcal{T}$-symmetry is broken, was discussed in [18]. In our case the current is concentrated within the domain wall and this current along the closed wall leads to the angular momentum of the domain. The magnitude of the angular momentum can vary because of the fermionic charge accumulated by the superconducting state. This accumulation occurs due to spectral flow in the fermion zero modes (see [15]).

In the superconducting state is axisymmetric and has no additional breaking of spatial parity, the angular momentum of superconductor per electron is quantized at $T = 0$:

$L_z = = \frac{1}{2}\hbar N$. This momentum does not depend on details of the gap structure and is determined only by the topological invariant N. In our case simple model the axisymmetric state with nonzero $N = 2$ occurs if $s_1 = s_2 = 0$ and $d_1 = d_2$. However when one deforms the superconducting state from the axisymmetric to the relevant one, the momentum L_z will is substantially modified by the deformation, if the spectral flow takes place during such a deformation [15]. But typically L_z remains to be of the same order.

One can estimate the change of the edge current under the deformation of superconducting state from the most symmetric one. The mass current along the domain wall changes in the following way:

$$\Delta J_M = \frac{e}{8\pi\hbar}\sum_a k_a^2 \text{ sign } c_a \quad . \tag{4.1}$$

This gives the following change of the electric current

$$\Delta J_e = \frac{e}{8\pi\hbar}\sum_a \frac{k_a^2}{m} \text{ sign } c_a \quad . \tag{4.2}$$

The Eq.(4.1) is obtained from the following consideration. The change in the current is caused by the spectral flow which takes place during the change of the parameters of the system, (s_1, s_2, d_1, d_2). If δn_a is the number of levels at a-th branch, which cross zero energy under the deformation of the state, the variation of the current is

$$\delta J_M = \sum_a k_a \delta n_a \quad , \tag{4.3}$$

since each level, when it crosses zero, carries the momentum k_a with it. On the other hand the variation of the number of levels is the variation of the momentum divided by the phase space volume $2\pi\hbar$:

$$\delta n_a = \frac{\delta k_a}{2\pi\hbar} \text{ sign } c_a \quad . \tag{4.4}$$

This gives

$$\delta J_M = \frac{1}{8\pi\hbar}\sum_a \delta(k_a^2) \text{ sign } c_a \quad . \tag{4.5}$$

An extra 1/2 is introduced to compensate the duplication of degrees of freedom, when both particle and hole states are considered in the Bogoliubov description in Eq.(2.1). The Eq.(4.5) leads to Eq.(4.1).

In some special cases one has $k_a = 0$ and the change in the edge current becomes zero. This may happen if the spatial parity $\mathcal{P}$ is not violated and for special orientations of the domain wall. In general case the momentum, for which the energy is zero, has a finite value, which is typically of order k_F. There are can be special orientations of the wall for which $k_a = \pm k_F$. In this case one obtains the quantized response of the current to the chemical potential μ: $\delta J_e = e\nu\delta\mu/4\pi\hbar$ [9]. However this is not a general result. In general $\Delta J_e \sim e\nu k_F^2/4\pi\hbar m$ if $\nu = \sum_a \text{sign } c_a \neq 0$.

We described different classes of superconductivity in CuO_2 planes with broken time inversion symmetry. The superconducting states may have the same symmetry but differ by the topological properties. The superconducting states are described by the integer-valued momentum-space topological invariant N. The boundary separating domains with different N ($N_2 \neq N_1$) contains fermion zero modes, which number is determined by $N_2 - N_1$. These fermions are current carrying and produce nonzero current along the domain wall in the ground states if $N_2 \neq N_1$.

The magnitude of the edge current is usually of order $ek_F^2/4\pi\hbar m$. It is large compared to the magnitude obtained by Laughlin (see Eq.(21) of [3]), who considered the pure d-wave case, which has $N = 2$. Laughlin's result can be obtained if one takes $k_a^2/2m = \Delta$ in Eq.(4.2), where Δ is the gap in the spectrum in Eq.(2.3). However there is no reason for such identification.

The domain wall fermions are also important in particle physics. For example the chiral fermions in our 3+1 dimensions can be reproduced by zero modes bound to domain wall in 4+1 dimensions [19].

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1. R.Movshovich, M.A.Hubbard, M.B.Salamon et al., Report No. cond-mat/9709061.
2. K.Krishana, N.P.Ong, Q.Li et al., Science **277**, 83 (1997).
3. R.B.Laughlin, Report No. cond-mat/9709004.
4. M.Sigrist, K.Kuboki, P.A.Lee, et al., Phys. Rev. **B 43**, 2835 (1996).
5. M.E.Zhitomirsky and M.B. Walker, Phys. Rev. Lett. **79**, 1734 (1997).
6. G.E.Volovik and V.M.Yakovenko, J. Phys. Cond. Matt. **1**, 5263 (1989).
7. V.M.Yakovenko, Phys. Rev. Lett. **65**, 251 (1990); Fizika (Zagreb) **21**, suppl. 3, 231 (1989); Report No. cond-mat/9703195.
8. V.M.Yakovenko, Phys. Rev. B **43**, 11353 (1991).
9. G.E.Volovik, Pis'ma ZhETF **55**, 363 (1992); [JETP Lett. **55**, 368 (1992)].
10. G.E.Volovik, *Exotic properties of superfluid* ^{3}He, Singapore: World Scientific, 1992.
11. J.F.Annett, N.Goldenfeld, and A.J.Leggett, in: *Physical Properties of High Temperature Superconductors*, Ed. D.M. Ginsberg, Singapore: World Scientific, 1996, vol.5, p. 375.
12. C.Caroli, P.G. de Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964).
13. G.E.Volovik, Report No. cond-mat/9709159.
14. N.B.Kopnin, Physica B **210**, 267 (1995).
15. G.E.Volovik, Pis'ma ZhETF **61**, 935 (1995) [JETP Lett. **61**, 958 (1995)].
16. N.B.Kopnin and G.E.Volovik, Rotating vortex core: An instrument for detecting the core excitations, Report No. cond-mat/9706082, submitted to Phys. Rev. B.
17. G.E. Volovik, Pis'ma ZhETF **57**, 233 (1993); [JETP Lett. **57**, 244 (1993)].
18. M. Stone and F. Gaitan, Ann. Phys. **178**, 89 (1987).
19. D.B. Kaplan and M. Schmaltz, Phys. Lett. **B 368**, 44 (1996).