Pis'ma v ZhETF, vol. 75, iss. 4, pp. 199 – 201 © 2002 February 25

Cherenkov radiation by particles traversing the background radiation

I. M. Dremin[1)]

P. N. Lebedev Physical Institute RAS, 119991 Moscow, Russia

Submitted 15 January 2002

High energy particles traversing the Universe through the cosmic microwave backgroung radiation can, in principle, emit Cherenkov radiation. It is shown that the energy threshold for this radiation is extremely high and its intensity would be too low due to the low density of the "relic photons gas" and very weak interaction of two photons.

PACS: 13.85.Tp, 41.60.Bq, 98.70.Vc

Recently, the particles with energies exceeding 10^{20} eV were observed in cosmic ray studies. Their sources have not yet been identified, but they originate most likely outside of our Galaxy. One could hope that some knowledge about the distance from these sources would be possible to obtain, in principle, from studies of Cherenkov radiation by such particles since its intensity is proportional to the distance covered by a particle.

Let us consider Cherenkov radiation by a high energy particle traversing the "gas" of relic photons with the temperature 2.73 K. Even though the density ν of such photons is very low in the Universe $\nu \approx \approx 500\,\text{photons/cm}^3$, the particle path could be rather large up to tens Mpc (1 Mpc $\approx 3 \cdot 10^{24}$ cm), and one could hope to register its Cherenkov radiation because the intensity is proportional to the path length.

The necessary conditions for Cherenkov radiation to be observed are the excess of the index of refraction n over 1, i.e.

$$\Delta n = n - 1 > 0 \tag{1}$$

and the real emission angle, given by the formula

$$\cos\theta = 1/\beta n, \tag{2}$$

where $\beta = v/c = \sqrt{1 - m^2/E^2}, m, E$ are the particle mass and energy. For small values of m/E and Δn one gets

$$\theta \approx \sqrt{2\Delta n - m^2/E^2}. \tag{3}$$

Herefrom, the condition for the energy threshold E_{th} is written as

$$E \geq E_{th} = m/\sqrt{2\Delta n}. \tag{4}$$

It is easily seen that the threshold can become very high for small Δn.

The number of Cherenkov photons emitted by a particle with the electric charge e in the interval of frequencies $d\omega$ from the path length dl is given by the common expression [1]

$$dN/d\omega dl = 2\alpha\Delta n, \tag{5}$$

where the fine structure constant $\alpha = e^2 \approx 1/137$. Thus all physical characteristics of the process are determined by the value Δn. The intensity of the radiation (5) decreases with the threshold energy (4) increase:

$$dN/d\omega dl = \frac{\alpha m^2}{E_{th}^2}. \tag{6}$$

Surely, it is possible to use the notion of the medium (and, consequently, the macroscopic approach) only in the case of extremely long-wave radiation for the very diluted gas of relic photons. At the same time, it is well known that in usual media the value of Δn is uniquely related to the polarization operator of the medium. If this relation is valid also in the case treated here, one must consider the polarization operator of the light-light scattering or the real part of the forward elastic light-light scattering amplitude $\text{Re}F(\omega, 0^0)$. For the index of refraction slightly different from one[2)], this relationship is given in the quantum scattering theory by the common formula [2, 3]:

$$\Delta n = \frac{2\pi\nu}{\omega^2}\text{Re}F(\omega, 0^0), \tag{7}$$

where the elastic scattering amplitude of the two gamma-quanta $F(\omega)$ has been normalized to the total cross section $\sigma_{\gamma\gamma}(\omega)$ according to the optical theorem

$$\text{Im}F(\omega, 0^0) = \frac{\omega}{4\pi}\sigma_{\gamma\gamma}(\omega). \tag{8}$$

1) e-mail: dremin@lpi.ru

2) The more general formula of Lorentz-Lorentz (see [2], p. 693) can be applied without this restriction.

Using these formulas, one easily gets

$$\Delta n = \nu\sigma_{\gamma\gamma}\rho/2\omega, \tag{9}$$

where $\rho = \mathrm{Re}F(\omega, 0^0)/\mathrm{Im}F(\omega, 0^0)$. Herefrom and from (5), it follows that

$$dN/d(\ln\omega)dl = \alpha\nu\sigma_{\gamma\gamma}\rho, \tag{10}$$

and

$$E_{th} = m\sqrt{\omega}/\sqrt{\nu\sigma_{\gamma\gamma}\rho}, \tag{11}$$

i.e. the radiation threshold increases with the energy of the registered quanta.

The impressive strong limit is imposed on the energy threshold for Cherenkov radiation in the gas of relic photons. In electrodynamical processes, the total cross section $\sigma_{\gamma\gamma}$ reaches its maximum value of about $1.6\,\mu$b at the total energy of the two photons in their center of mass system $\omega_c \approx 3m_e$ where the electron mass is $m_e \approx 0.5$ MeV (see [4, 5]). At lower energies, below the threshold for the creation of the electron-positron pair (in particular, for a visible light), only elastic scattering is important, the real part of the amplitude dominates and the cross section decreases at $\omega \to 0$ proportionally to ω_c^6 (see the discussion at the end of the paper). At high energies, as seen from the formula (9), the value of Δn tends to zero at $\omega \to \infty$, and, therefore, the decline from the linear dispersion law $\omega = k$ can become noticeable only in the region near the maximum of the cross section $\sigma_{\gamma\gamma}$.

The c.m.s. energy of the two-photon system varies from 0 (when photons move in the same direction in the laboratory system) to its maximum value

$$\omega_c = 2\sqrt{\omega\omega_r}, \tag{12}$$

when the relic photon with energy ω_r moves in the laboratory system in the opposite direction to the emitted quantum with energy ω. For the relic photons $\omega_r \approx 2.4 \cdot 10^{-4}$ eV. Thus the energy $\omega_c \approx 3m_e$ can be only achieved if the particle emits quanta with the energy

$$\omega = \omega_c^2/4\omega_r \approx 2 \cdot 10^{15}\mathrm{eV}. \tag{13}$$

First, let us consider the high energy quanta which can create the electron-positron pairs in the cosmic microwave background radiation. In this case of very energetic radiation quanta $\Delta n < 10^{-48}$, if one inserts high-energy values of $\rho \sim 0.1$. Then for the proton with mass m=1 GeV one gets an estimate

$$E_{th} > 10^{33}\,\mathrm{eV}. \tag{14}$$

The particles with such high energy have not yet been observed anywhere. Such a low index of refraction and, correspondingly, high value of the energy threshold are determined by the low density of relic photons in the Universe and by the small total cross section of the photon-photon interaction.

The estimate of the upper limit of the intensity of the high-energy Cherenkov radiation at the path length L

$$N < \alpha\nu\sigma_{\gamma\gamma}L \tag{15}$$

is obtained from the formula (10), if one takes into account that $\Delta\omega/\omega < 1$ and $\rho < 1$. If the path, wherefrom the radiation is collected, is equal to $L \sim 1$ Mpc $\sim 3{\cdot}10^{24}$ cm, and the maximum value of $\sigma_{\gamma\gamma}$ is about $1.6\,\mu$b (see [4, 5]) then for $\nu \approx 500\,\mathrm{photons/cm^3}$ the following upper limit on the number of emitted quanta is imposed according to (15)

$$N < 3 \cdot 10^{-5}. \tag{16}$$

Thus, the intensity of the high-energy Cherenkov radiation in the gas of relic photons is too low to be observable even if a proton passes through the Universe along hundreds Mpc. For a primary nucleus, one should insert in the formula (5) its total charge, and, therefore, the intensity of radiation increases in proportion to the nucleus charge squared. However, the total threshold energy increases proportionally to its mass number so that the threshold is much higher for nuclei compared to protons. At the same time, let us note that according to formula (4) the threshold energy per nucleon of a nucleus is only determined by the value of Δn, and, therefore, it is the same for all nuclei.

Some contribution to the polarization operator of the light-light scattering is, in principle, provided by the hadronic component of the photon structure function as well. It can be accounted in the framework of the vector dominance model when the quanta are transformed into virtual ρ-mesons which interact resonancely. The real part of the scattering amplitude is with necessity positive in one of the halves of the resonance peak as clearly follows from the Breit-Wigner formula. The cross section $\sigma_{\gamma\gamma}$ slightly increases. Nevertheless, this does not lead either to the diminished threshold value in the formula (14), or to the increased intensity of Cherenkov radiation because of the increase of ω due to ω_c increasing up to the values of the order of the ρ-meson mass. The hadronic contribution to the ratio $\rho = \mathrm{Re}F/\mathrm{Im}F$ can be also positive at higher energies as it is well known from experiments in all studied hadronic reactions in complete accordance with predictions of dispersion relations. However, the threshold of the radiation again

increases in this case due to the further increase of the required values of ω_c.

The estimates have shown that the derived above conclusions are "protected" by several orders of magnitude as is already seen from the formulas (14) and (16), and therefore they are robust.

Let us estimate the absorption of Cherenkov radiation at cosmic distances. For a plane wave e^{ikr} it is given by a factor

$$\exp[-\omega L \mathrm{Im} n(\omega)] \approx \exp[-\nu\sigma_{\gamma\gamma}L/2], \qquad (17)$$

wherefrom the absorption length L_{abs} is estimated as

$$L_{abs} = 2/\nu\sigma_{\gamma\gamma} > 2\cdot 10^{27}\,\mathrm{cm}. \qquad (18)$$

The absorption length of the gamma-quantum in the relic radiation gas depends on the energy of the gamma-quantum only due to the cross section energy dependence and it is so large that the damping of Cherenkov radiation can be neglected for the distances shorter than thousands Mpc. Correspondingly, the energy losses which determine the redshift are very small at such "short" distances $L \ll L_{abs}$:

$$\frac{\Delta\omega}{\omega} = 1 - \exp\left(-\frac{\nu\sigma_{\gamma\gamma}L}{2}\right) \approx \frac{\nu\sigma_{\gamma\gamma}L}{2} \ll 1. \qquad (19)$$

Thus, the final conclusion is that the high-energy Cherenkov radiation by particles traversing the gas of relic photons is impossible to observe, first of all, because until now there have been detected no particles with such high energies in the Nature exceeding the required energy threshold. Even if such particles were registered, the intensity of the high-energy Cherenkov radiation would, possibly, be too low to detect it.

The principal possibility of the Cherenkov radiation by a charged particle in the intergalactic space is, however, not excluded if there exists "an intergalactic medium component" with higher density and larger cross section of interaction with photons.

Another possibility is related to studies of quanta with energies lower than the threshold for creation of the electron-positron pair, i.e. for $\omega < 10^{15}$ eV. Even though the cross section decreases, the ratio ρ becomes very large. To estimate the index of refraction for such quanta, the classical consideration with Maxwell equations and dispersion relations [6] or the effective lagrangian with 4-photon interaction [7], describing the deviation from the classical theory by quantum effects, have been used. More "optimistic" estimates of $\Delta n \sim \sim 10^{-41}$ follow, e.g., from Fig.2 of Ref.[6], where, in our notations, the variables k and z read

$$k = m_e^4/16\pi^2\alpha^2\nu\omega_r \approx 8\cdot 10^{40};$$
$$z = m_e^2/\omega\omega_r = 4m_e^2/\omega_c^2. \qquad (20)$$

This increase of Δn is because of much larger values of ρ. This would lead to lower threshold $E_{th} > 10^{29}$ eV. Nevertheless, this value of the threshold energy is still very high. The intensity of radiation could, however, become quite noticeable $N \sim 10$. In Ref. [7] the value of Δn at low energies was estimated as $4.7\cdot 10^{-43}$. Let us remind that the notion of the medium in classical approach can be used when the wavelength of the impinging photon is larger than typical distances inside the medium.

At the same time, Δn increases [7] with the temperature T in equilibrium as T^4. Therefore, at the early stage of the Universe, when the cosmic microwave background radiation separated from the matter at the temperature about 3000 K, the index of refraction was $\Delta n \sim 10^{-30}$, i.e. the threshold energy was $E_{th} \sim 10^{24}$ eV. This estimate is closer to "realistic" but still rather high energies.

Thus the final conclusion about extremely high energy threshold for Cherenkov radiation in the cosmic microwave background radiation is left intact.

In laboratory, Cherenkov radiation by a particle traversing the "photon medium" can be observed for high energy electron beams colliding with optical laser bunches. The corresponding estimates are: $\Delta n \sim \sim 6.2\cdot 10^{-10}$, $E_{th} = \gamma_{th} m_e \sim 14$ GeV, $\theta_{max} \sim \gamma_{th}^{-1} \sim 3.5\cdot 10^{-5}$. It can be used for beam energy measurements, for tests of laser bunch parameters and understanding of the "photon medium" properties (see hep-ph/0202060).

I am grateful to L. G. Tkatchev who pointed out this problem to me and to I. V. Andreev, B. M. Bolotovsky, A. D. Erlykin, E. L. Feinberg, I. F. Ginzburg, V. L. Ginzburg and V. A. Maisheev for discussions and comments.

This work is supported by the RFBR grant # 00-02-16101.

1. I. E. Tamm and I. M. Frank, Doklady AN SSSR **14**, 107 (1937); I. E. Tamm, J. Phys. USSR **1**, 439 (1939).
2. M. L. Goldberger and K. M. Watson, *Collision Theory*, J. Wiley & Sons, New York, 1964; Russian translation published by Mir, Moscow, 1967, pp. 689, 693, 712.
3. I. M. Dremin, JETP Lett. **30**, 140 (1979); Sov. J. Nucl. Phys. **33**, 726 (1981).
4. R. Karplus and M. Neuman, Phys. Rev. **83**, 776 (1951).
5. B. De Tollis, Nuovo Cim. **35**, 1182 (1965).
6. V. A. Maisheev, ZhETF **112**, 2016 (1997); JETP **85**, 1102 (1997).
7. M. Thoma, Europhys. Lett. **52**, 498 (2000).