

Theory of high- T_c superconductivity based on the fermion-condensation quantum phase transition

M. Ya. Amusia, S. A. Artamonov⁺, V. R. Shaginyan⁺¹⁾

The Racah Institute of Physics, the Hebrew University, Jerusalem 91904, Israel

Physical-Technical Institute RAS, 194021 St. Petersburg, Russia

⁺Petersburg Nuclear Physics Institute RAS, 188350 Gatchina, Russia

Submitted 3 September 2001

A theory of high temperature superconductivity based on the combination of the fermion-condensation quantum phase transition and the conventional theory of superconductivity is presented. This theory describes maximum values of the superconducting gap which can be as big as $\Delta_1 \sim 0.1\varepsilon_F$, with ε_F being the Fermi level. We show that the critical temperature $2T_c \simeq \Delta_1$. If there exists the pseudogap above T_c then $2T^* \simeq \Delta_1$, and T^* is the temperature at which the pseudogap vanishes. A discontinuity in the specific heat at T_c is calculated. The transition from conventional superconductors to high- T_c ones as a function of the doping level is investigated. The single-particle excitations and their lineshape are also considered.

PACS: 71.27.+a, 74.20.Fg, 74.25.Jb

The explanation of the large values of the critical temperature T_c , of the maximum value of the superconducting gap Δ_1 and of the relation between Δ_1 and the temperature T^* at which the pseudogap vanishes are, as years before, among the main problems in the physics of high-temperature superconductivity. To solve them, one needs to know the single-particle spectra of corresponding metals. Recent studies of photoemission spectra discovered an energy scale in the spectrum of low-energy electrons in copper oxides, which manifests itself as a kink in the single-particle spectra [1–4]. As a result, the spectra in the energy range (–200–0) meV can be described by two straight lines intersecting at the binding energy $E_0 \sim (50 - 70)$ meV [2, 3]. The existence of the energy scale E_0 could be attributed to the interaction between electrons and the collective excitations, for instance, phonons [4]. On the other hand, the analysis of the experimental data on the single-particle electron spectra demonstrates that the perturbation of the spectra by phonons or other collective states is in fact very small, therefore, the corresponding state of electrons has to be described as a strongly collectivized quantum state and was named “quantum protectorate” [5, 6]. Thus, the interpretation of the above mentioned kink as a consequence of electron-phonon interaction can very likely be in contradiction with the quantum protectorate concept. To describe the single-particle spectra and the kink, the assumption can be used that the electron system

of high- T_c superconductor has undergone the fermion-condensation quantum phase transition (FCQPT). This transition serves as a boundary separating the normal Fermi liquid from the strongly correlated liquid of a new type [7, 8] and fulfills the quantum protectorate requirements [9]. The FCQPT appears in many-electron systems at relatively low density, when the effective interaction constant becomes sufficiently large. In ordinary electron liquid, this constant is directly proportional to the dimensionless parameter $r_s \sim 1/p_F a_B$, where a_B is the Bohr radius and p_F is the Fermi momentum. The FCQPT appears at a certain value r_s , $r_s = r_{FC}$, and precedes formation of charge-density waves or stripes [10], which are observed in underdoped samples of copper oxides [11]. This is why the formation of the FCQPT in copper oxides can be considered as a quite determinate process stemming from general properties of a low-density electron liquid [9].

In this letter we address the mentioned above problems in the physics of high-temperature superconductivity and demonstrate that these problems can be resolved in a theory based on the combination of the FCQPT and the conventional theory of superconductivity. We show that the FCQPT manifests itself in large values of Δ_1 , T_c and T^* . We trace also the transition from conventional superconductors to high- T_c ones as a function of the parameter r_s , or as a function of the doping level.

At $T = 0$, the ground state energy $E_{gs}[\kappa(\mathbf{p}), n(\mathbf{p})]$ of two-dimensional electron liquid is a functional of the order parameter of the superconducting state $\kappa(\mathbf{p})$ and

¹⁾e-mail: vrshag@thd.pnpi.spb.ru

of the occupation numbers $n(\mathbf{p})$ and is determined by the known equation of the weak-coupling theory of superconductivity

$$E_{gs} = E[n(\mathbf{p})] + \int \lambda_0 V(\mathbf{p}_1, \mathbf{p}_2) \kappa(\mathbf{p}_1) \kappa^*(\mathbf{p}_2) \frac{d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi)^4}. \quad (1)$$

Here $E[n(\mathbf{p})]$ is the ground-state energy of normal Fermi liquid, $n(\mathbf{p}) = v^2(\mathbf{p})$ and $\kappa(\mathbf{p}) = v(\mathbf{p})\sqrt{1-v^2(\mathbf{p})}$. It is assumed that the pairing interaction $\lambda_0 V(\mathbf{p}_1, \mathbf{p}_2)$ is weak. Minimizing E_{gs} with respect to $\kappa(\mathbf{p})$ we obtain the equation connecting the single-particle energy $\varepsilon(\mathbf{p})$ to $\Delta(\mathbf{p})$,

$$\varepsilon(\mathbf{p}) - \mu = \Delta(\mathbf{p}) \frac{1 - 2v^2(\mathbf{p})}{2\kappa(\mathbf{p})}. \quad (2)$$

The single-particle energy $\varepsilon(\mathbf{p})$ is determined by the Landau equation, $\varepsilon(\mathbf{p}) = \delta E[n(\mathbf{p})]/\delta n(\mathbf{p})$ [12], and μ is chemical potential. The equation for superconducting gap $\Delta(\mathbf{p})$ takes form

$$\begin{aligned} \Delta(\mathbf{p}) &= - \int \lambda_0 V(\mathbf{p}, \mathbf{p}_1) \kappa(\mathbf{p}_1) \frac{d\mathbf{p}_1}{4\pi^2} = \\ &= - \frac{1}{2} \int \lambda_0 V(\mathbf{p}, \mathbf{p}_1) \frac{\Delta(\mathbf{p}_1)}{\sqrt{(\varepsilon(\mathbf{p}_1) - \mu)^2 + \Delta^2(\mathbf{p}_1)}} \frac{d\mathbf{p}_1}{4\pi^2}. \end{aligned} \quad (3)$$

If $\lambda_0 \rightarrow 0$, then, the gap $\Delta(\mathbf{p}) \rightarrow 0$, and Eq. (2) reduces to the equation proposed in [7]

$$\varepsilon(\mathbf{p}) - \mu = 0, \text{ if } 0 < n(\mathbf{p}) < 1; p_i \leq p \leq p_f. \quad (4)$$

At $T = 0$, Eq.(4) defines a particular state of Fermi liquid with the fermion condensate (FC) for which the modulus of the order parameter $|\kappa(\mathbf{p})|$ has finite values in the L_{FC} range of momenta $p_i \leq p \leq p_f$, and $\Delta_1 \rightarrow 0$ in the L_{FC} . Such a state can be considered as superconducting, with infinitely small value of Δ_1 so that the entropy of this state is equal to zero. It is obvious, that this state, being driven by the quantum phase transition, disappears at $T > 0$ [9]. When $p_i \rightarrow p_F \rightarrow p_f$, Eq.(4) determines the point r_{FC} at which the FCQPT takes place. It follows from Eq. (4) that the system brakes into two quasiparticle subsystems: the first subsystem in the L_{FC} range is occupied by the quasiparticles with the effective mass $M_{FC}^* \rightarrow \infty$, while the second one is occupied by quasiparticles with finite mass M_L^* and momenta $p < p_i$. If $\lambda_0 \neq 0$, Δ_1 becomes finite, leading to finite value of the effective mass M_{FC}^* in L_{FC} , which can be obtained from Eq.(2) [9]

$$M_{FC}^* \simeq p_F \frac{p_f - p_i}{2\Delta_1}. \quad (5)$$

As to the energy scale, it is determined by the parameter E_0 :

$$E_0 = \varepsilon(\mathbf{p}_f) - \varepsilon(\mathbf{p}_i) \simeq 2 \frac{(p_f - p_F)p_F}{M_{FC}^*} \simeq 2\Delta_1. \quad (6)$$

Thus, a system with the FC has the single-particle spectrum of a universal form and possesses quantum protectorate features at $T \ll T_f$, with T_f being a temperature, at which the effect of the FCQPT disappears.

We assume that the range L_{FC} is small, $(p_f - p_F)/p_F \ll 1$, and $2\Delta_1 \ll T_f$ so that the order parameter $\kappa(\mathbf{p})$ is governed mainly by the FC [9]. To solve Eq. (2) analytically, we take the Bardeen-Cooper-Schrieffer (BCS) approximation for the interaction [13]: $\lambda_0 V(\mathbf{p}, \mathbf{p}_1) = -\lambda_0$ if $|\varepsilon(\mathbf{p}) - \mu| \leq \omega_D$, the interaction is zero outside this region, with ω_D being the characteristic phonon energy. As a result, the gap becomes dependent only on the temperature, $\Delta(\mathbf{p}) = \Delta_1(T)$, being independent of the momentum, and Eq. (2) takes the form

$$\begin{aligned} 1 &= N_{FC} \lambda_0 \int_0^{E_0/2} \frac{d\xi}{\sqrt{\xi^2 + \Delta_1(0)^2}} + \\ &+ N_L \lambda_0 \int_{E_0/2}^{\omega_D} \frac{d\xi}{\sqrt{\xi^2 + \Delta_1(0)^2}}. \end{aligned} \quad (7)$$

Here we set $\xi = \varepsilon(\mathbf{p}) - \mu$ and introduce the density of states N_{FC} in the L_{FC} , or E_0 , range. As it follows from Eq.(5), $N_{FC} = (p_f - p_F)p_F/2\pi\Delta_1(0)$. The density of states N_L in the range $(\omega_D - E_0/2)$ has the standard form $N_L = M_L^*/2\pi$. If the energy scale $E_0 \rightarrow 0$, Eq.(7) reduces to the BCS equation. On the other hand, assuming that $E_0 \leq 2\omega_D$ and omitting the second integral in the right hand side of Eq.(7), we obtain

$$\begin{aligned} \Delta_1(0) &= \frac{\lambda_0 p_F (p_f - p_F)}{2\pi} \ln(1 + \sqrt{2}) = \\ &= 2\beta \varepsilon_F \frac{p_f - p_F}{p_F} \ln(1 + \sqrt{2}), \end{aligned} \quad (8)$$

where the Fermi energy $\varepsilon_F = p_F^2/2M_L^*$, and dimensionless coupling constant $\beta = \lambda_0 M_L^*/2\pi$. Taking the usual values of the dimensionless coupling constant $\beta \simeq 0.3$, and $(p_f - p_F)/p_F \simeq 0.2$, we get from Eq.(7) the large value of $\Delta_1(0) \sim 0.1\varepsilon_F$, while for normal metals one has $\Delta_1(0) \sim 10^{-3}\varepsilon_F$. Taking into account the omitted integral, we obtain

$$\Delta_1(0) \simeq 2\beta \varepsilon_F \frac{p_f - p_F}{p_F} \ln(1 + \sqrt{2}) \left(1 + \beta \ln \frac{2\omega_D}{E_0} \right). \quad (9)$$

It is seen from Eq.(9) that the correction due to the second integral is small, provided $E_0 \simeq 2\omega_D$. Below we show that $2T_c \simeq \Delta_1(0)$, which leads to the conclusion that there is no isotope effect since Δ_1 is independent of ω_D . But this effect is restored as $E_0 \rightarrow 0$. Assuming $E_0 \sim \omega_D$ and $E_0 > \omega_D$, we see that Eq.(7) has no standard solutions $\Delta(p) = \Delta_1(0)$ because $\omega_D < \varepsilon(p \simeq p_f) - \mu$ and the interaction vanishes at these momenta. The only way to obtain solutions is to restore the condition $E_0 < \omega_D$. For instance, we can define the momentum $p_D < p_f$ such that

$$\Delta_1(0) = 2\beta\varepsilon_F \frac{p_D - p_F}{p_F} \ln(1 + \sqrt{2}) = \omega_D, \quad (10)$$

while the other part in the L_{FC} range can be occupied by a gap Δ_2 of the different sign, $\Delta_1(0)/\Delta_2 < 0$. It follows from Eq.(10) that the isotope effect is presented. A more detailed analysis will be published elsewhere.

At $T \rightarrow T_c$, Eqs. (5) and (6) are replaced by the equation, which is valid also at $T_c \leq T \ll T_f$ [9]

$$M_{FC}^* \simeq p_F \frac{p_f - p_i}{4T_c}, \quad E_0 \simeq 4T_c; \\ \text{if } T_c \leq T: \quad M_{FC}^* \simeq p_F \frac{p_f - p_i}{4T}, \quad E_0 \simeq 4T. \quad (11)$$

Equation (7) is replaced by its conventional finite temperature generalization

$$1 = N_{FC}\lambda_0 \int_0^{E_0/2} \frac{d\xi}{\sqrt{\xi^2 + \Delta_1(T)^2}} \tanh \frac{\sqrt{\xi^2 + \Delta_1(T)^2}}{2T} + \\ + N_L\lambda_0 \int_{E_0/2}^{\omega_D} \frac{d\xi}{\sqrt{\xi^2 + \Delta_1(T)^2}} \tanh \frac{\sqrt{\xi^2 + \Delta_1(T)^2}}{2T}. \quad (12)$$

Putting $\Delta_1(T \rightarrow T_c) \rightarrow 0$, we obtain from Eq. (12)

$$2T_c \simeq \Delta_1(0), \quad (13)$$

with $\Delta_1(0)$ being given by Eq.(9). By comparing Eqs.(5), (11) and (13), we see that M_{FC}^* and E_0 are almost temperature independent at $T \leq T_c$. Now a few remarks are in order. One can define T_c as the temperature when $\Delta_1(T_c) \equiv 0$. At $T \geq T_c$, Eq. (12) has only the trivial solution $\Delta_1 \equiv 0$. On the other hand, T_c can be defined as a temperature at which the superconductivity vanishes. Thus, we deal with two different definitions, which can lead to different temperatures. It was shown [14, 15] that in the case of the d-wave superconductivity, taking place in the presence of the FC, there exist a nontrivial solutions of Eq.(12) at $T_c \leq T \leq T^*$ corresponding to the pseudogap state. It happens when the

gap occupies only such a part of the Fermi surface, which shrinks as the temperature increases. Here T^* defines the temperature at which $\Delta_1(T^*) \equiv 0$ and the pseudogap state vanishes. The superconductivity is destroyed at T_c , and the ratio $2\Delta_1/T_c$ can vary in a wide range and strongly depends upon the material's properties, as it follows from consideration given in [14, 15]. Therefore, provided there exists the pseudogap above T_c , then T_c is to be replaced by T^* , and Eq.(13) takes the form

$$2T^* \simeq \Delta_1(0). \quad (14)$$

The ratio $2\Delta_1/T_c$ can reach very high values. For instance, in the case of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{6+\delta}$, where the superconductivity and the pseudogap are considered to be of the common origin, $2\Delta_1/T_c$ is about 28, while the ratio $2\Delta_1/T^* \simeq 4$, which is also valid for various cuprates [16]. Thus, Eq.(14) gives good description of the experimental data. We remark that Eq.(7) gives also good description of the maximum gap Δ_1 in the case of the d-wave superconductivity [14, 15], because the different regions with the maximum absolute value of Δ_1 and the maximal density of states can be considered as disconnected [17]. Therefore, the gap in this region is formed by attractive phonon interaction which is approximately independent of the momenta. According to the model proposed in [9], the doping level x is related to the parameter r_s in the following way: $(x_{FC} - x) \sim (r_s - r_{FC}) \sim (p_f - p_i)/p_F$. The value x_{FC} matches r_{FC} when defining the point at which the FCQPT takes place. We assume that the dopant concentration x_{FC} corresponds to the highly overdoped regime at which slight deviations from the normal Fermi liquid are observed [18]. Then, from Eqs.(8) and (9) it follows that Δ_1 is directly proportional to $(x_{FC} - x)$. From Eq.(14) one finds that the function $T^*(x)$ presents a straight line crossing the abscissa at the point $(x_{FC} \simeq x)$, while in the vicinity of this point T^* merges with T_c and both of them tends to zero.

Now we turn to the calculations of the gap and the specific heat at the temperatures $T \rightarrow T_c$. It is worth noting that this consideration is valid provided $T^* = T_c$, otherwise the considered below discontinuity is smoothed out over the temperature range $T^* - T_c$. For the sake of simplicity, we calculate the main contribution to the gap and the specific heat coming from the FC. The function $\Delta_1(T \rightarrow T_c)$ is found from Eq. (12) upon expanding the right hand side of the first integral in powers of Δ_1 and omitting the contribution from the second integral on the right hand side of Eq. (12). This procedure leads to the following equation

$$\Delta_1(T) \simeq 3.4T_c \sqrt{1 - T/T_c}. \quad (15)$$

Thus, the gap in the spectrum of the single-particle excitations has quite usual behavior. To calculate the specific heat, the conventional expression for the entropy S [13] can be used

$$S = 2 \int [f(\mathbf{p}) \ln f(\mathbf{p}) + (1 - f(\mathbf{p})) \ln(1 - f(\mathbf{p}))] \frac{d\mathbf{p}}{(2\pi)^2}, \quad (16)$$

where

$$f(\mathbf{p}) = \frac{1}{1 + \exp[E(\mathbf{p})/T]}; \quad (17)$$

$$E(\mathbf{p}) = \sqrt{(\varepsilon(\mathbf{p}) - \mu)^2 + \Delta_1^2(T)}.$$

The specific heat C is determined by

$$C = T \frac{dS}{dT} \simeq 4 \frac{N_{FC}}{T^2} \int_0^{E_0} f(E)(1 - f(E)) \times$$

$$\times \left[E^2 + T \Delta_1(T) \frac{d\Delta_1(T)}{dT} \right] d\xi + 4 \frac{N_L}{T^2} \times$$

$$\times \int_{E_0}^{\omega_D} f(E)(1 - f(E)) \left[E^2 + T \Delta_1(T) \frac{d\Delta_1(T)}{dT} \right] d\xi. \quad (18)$$

When deriving Eq.(18) we again use the variable ξ and the densities of states N_{FC} , N_L , just as before in connection to Eq.(7), and use the notation $E = \sqrt{\xi^2 + \Delta_1^2(T)}$. Equation (18) predicts the conventional discontinuity δC in the specific heat C at T_c because of the last term in the square brackets of Eq.(18). Upon using Eq.(15) to calculate this term and omitting the second integral on the right hand side of Eq.(18), we obtain

$$\delta C \simeq \frac{3}{2\pi} (p_f - p_i) p_F. \quad (19)$$

In contrast to the conventional result when the discontinuity is a linear function of T_c , δC is independent of the critical temperature T_c because the density of state varies inversely with T_c as it follows from Eq.(11). Note, that deriving Eq.(19) we take into account the main contribution coming from the FC. This contribution vanishes as soon as $E_0 \rightarrow 0$ and the second integral of Eq.(18) gives the conventional result.

Consider the lineshape $L(q, \omega)$ of the single-particle spectrum which is a function of two variables. Measurements carried out at a fixed binding energy $\omega = \omega_0$, where ω_0 is the energy of a single-particle excitation, determine the lineshape $L(q, \omega = \omega_0)$ as a function of the momentum q . We have shown above that M_{FC}^* is finite and constant at $T \leq T_c$. Therefore, at excitation

energies $\omega \leq E_0$ the system behaves like an ordinary superconducting Fermi liquid with the effective mass given by Eq.(5) [9]. At $T_c \leq T$ the low energy effective mass M_{FC}^* is finite and is given by Eq.(11). Once again, at the energies $\omega < E_0$, the system behaves as a Fermi liquid, the single-particle spectrum is well defined, while the width of single-particle excitations is of the order of T [9, 19]. This behavior was observed in experiments on measuring the lineshape at a fixed energy [1]. It is pertinent to note that recent measurements of the lineshape suggest that quasiparticle excitation even in the $(\pi, 0)$ region of the Brillouin zone of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ ($\text{Bi}2212$) are much better defined than previously believed from earlier $\text{Bi}2212$ data [20]. The lineshape can also be determined as a function $L(q = q_0, \omega)$ at a fixed $q = q_0$. At small ω , the lineshape resembles the one considered above, and $L(q = q_0, \omega)$ has a characteristic maximum and width. At energies $\omega \geq E_0$, quasiparticles with the mass M_L^* come into play, leading to a growth of the function $L(q = q_0, \omega)$. As a result, the function $L(q = q_0, \omega)$ possesses the known peak-dip-hump structure [21] directly defined by the existence of the two effective masses M_{FC}^* and M_L^* [9]. To have more quantitative and analytical insight into the problem we use the Kramers-Krönig transformation to construct the imaginary part $\text{Im}\Sigma(\mathbf{p}, \varepsilon)$ of the self-energy $\Sigma(\mathbf{p}, \varepsilon)$ starting with the real one $\text{Re}\Sigma(\mathbf{p}, \varepsilon)$ which defines the effective mass [22]

$$\frac{1}{M^*} = \left(\frac{1}{M} + \frac{1}{p_F} \frac{\partial \text{Re}\Sigma}{\partial p} \right) / \left(1 - \frac{\partial \text{Re}\Sigma}{\partial \varepsilon} \right). \quad (20)$$

Here M is the bare mass, while the relevant momenta p and energies ε are subjected to the conditions: $|p - p_F|/p_F \ll 1$, and $\varepsilon/\varepsilon_F \ll 1$. We take $\text{Re}\Sigma(\mathbf{p}, \varepsilon)$ in the simplest form which accounts for the change of the effective mass at the energy scale E_0 :

$$\text{Re}\Sigma(\mathbf{p}, \varepsilon) = -\varepsilon \frac{M_{FC}^*}{M} + \left(\varepsilon - \frac{E_0}{2} \right) \frac{M_{FC}^* - M_L^*}{M} \times$$

$$\times [\theta(\varepsilon - E_0/2) + \theta(-\varepsilon - E_0/2)]. \quad (21)$$

Here $\theta(\varepsilon)$ is the step function. Note that in order to ensure a smooth transition from the single-particle spectrum characterized by M_{FC}^* to the spectrum defined by M_L^* the step function is to be substituted by some smooth function. Upon inserting Eq.(21) into Eq.(20) we can check that inside the interval $(-E_0/2, E_0/2)$ the effective mass $M^* \simeq M_{FC}^*$, and outside the interval $M^* \simeq M_L^*$. By applying the Kramers-Krönig transfor-

mation to $\text{Re}\Sigma(\mathbf{p}, \varepsilon)$, we obtain the imaginary part of the self-energy,

$$\begin{aligned} \text{Im}\Sigma(\mathbf{p}, \varepsilon) &\sim \varepsilon^2 \frac{M_{FC}^*}{\varepsilon_F M} + \frac{M_{FC}^* - M_L^*}{M} \times \\ &\times \left(\varepsilon \ln \left| \frac{\varepsilon + E_0/2}{\varepsilon - E_0/2} \right| + \frac{E_0}{2} \ln \left| \frac{\varepsilon^2 - E_0^2/4}{E_0^2/4} \right| \right). \end{aligned} \quad (22)$$

We can see from Eq.(22) that at $\varepsilon/E_0 \ll 1$ the imaginary part is proportional to ε^2 ; at $2\varepsilon/E_0 \simeq 1$ $\text{Im}\Sigma \sim \varepsilon$; at $E_0/\varepsilon \ll 1$ the main contribution to the imaginary part is approximately constant. This is the behavior that gives rise to the known peak-dip-hump structure. Then, it is seen from Eq.(22) that when $E_0 \rightarrow 0$ the second term on the right hand side tends to zero, the single-particle excitations become better defined resembling that of a normal Fermi liquid, and the peak-dip-hump structure eventually vanishes. On the other hand, the quasiparticle amplitude $a(\mathbf{p})$ is given by [22]

$$\frac{1}{a(\mathbf{p})} = 1 - \frac{\partial \text{Re}\Sigma(\mathbf{p}, \varepsilon)}{\partial \varepsilon}. \quad (23)$$

It follows from Eq.(20) that the quasiparticle amplitude $a(\mathbf{p})$ rises as the effective mass M_{FC}^* decreases. Since $M_{FC}^* \sim (p_f - p_i) \sim (x_{FC} - x)$ [9], we are led to a conclusion that the amplitude $a(\mathbf{p})$ rises as the doping level rises, and the single-particle excitations become better defined in highly overdoped samples. It is worth noting that such a behavior was observed experimentally in so highly overdoped Bi2212 that the gap size is about 10 meV [18]. Such a small size of the gap testifies that the region occupied by the FC is small since $E_0/2 \simeq \Delta_1$.

In conclusion, we have shown that the theory of high temperature superconductivity based on the fermion-condensation quantum phase transition and on the conventional theory of superconductivity permits to describe high values of T_c , T^* and of the maximum value of the gap, which may be as big as $\Delta_1 \sim 0.1\varepsilon_F$. We have also traced the transition from conventional superconductors to high- T_c and demonstrated that in the highly overdoped cuprates the single-particle excitations be-

come much better defined, resembling that of a normal Fermi liquid.

This work was supported in part by the Russian Foundation for Basic Research, project # 01-02-17189.

1. T. Valla et al., *Science* **285**, 2110 (1999); T. Valla et al., *Phys. Rev. Lett.* **85**, 828 (2000).
2. P. V. Bogdanov et al., *Phys. Rev. Lett.* **85**, 2581 (2000).
3. A. Kaminski et al., *Phys. Rev. Lett.* **86**, 1070 (2001).
4. A. Lanzara et al., *Nature* **412**, 510 (2001).
5. R. B. Laughlin and D. Pines, *Proc. Natl. Acad. Sci. USA* **97**, 28 (2000).
6. P. W. Anderson, cond-mat/0007185; cond-mat/0007287.
7. V. A. Khodel and V. R. Shaginyan, *Pis'ma Zh.ETF* **51**, 488 (1990) [*JETP Lett.* **53**, 51, 553 (1990)].
8. G. E. Volovik, *Pis'ma ZhETF* **53**, 208 (1991) [*JETP Lett.* **53**, 222 (1991)].
9. M. Ya. Amusia and V. R. Shaginyan, *Pis'ma ZhETF* **73**, 268 (2001) [*JETP Lett.* **73**, 232 (2001)]; *Phys. Rev.* **B63**, 224507 (2001).
10. V. A. Khodel, V. R. Shaginyan, and M. V. Zverev, *Pis'ma ZhETF* **65**, 242 (1997) [*JETP Lett.* **65**, 253 (1997)].
11. G. Grüner, *Density Waves in Solids* Addison-Wesley, Reading, MA, 1994.
12. L. D. Landau, *ZhETF* **30**, 1058 (1956) [*Sov. Phys. JETP* **3**, 920 (1956)].
13. J. Bardeen, L. N. Cooper, and J. R. Schrieffer, *Phys. Rev.* **108**, 1175 (1957).
14. V. R. Shaginyan, *Pis'ma ZhETF* **68**, 491 (1998) [*JETP Lett.* **68**, 527 (1998)].
15. S. A. Artamonov and V. R. Shaginyan, *Zh.ETF* **119**, 331 (2001) [*JETP* **92**, 287 (2001)].
16. M. Kugler et al., *Phys. Rev. Lett.* **86**, 4911 (2001).
17. A. A. Abrikosov, *Phys. Rev.* **B52**, R15738 (1995); A. A. Abrikosov, cond-mat/9912394.
18. Z. Yusof et al., cond-mat/01044367.
19. J. Dukelsky et al., *Z. Phys.* **102**, 245 (1997).
20. D. L. Feng et al., cond-mat/0107073.
21. D. S. Dessau et al., *Phys. Rev. Lett.* **66**, 2160 (1991).
22. A. B. Migdal, *Theory of Finite Fermi Systems and Applications to Atomic Nuclei*, Benjamin, Reading, MA, 1977.