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On the energy loss of high energy quarks in a finite-size quark-gluon plasma

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We study within the light-cone path integral approach the induced gluon emission from a fast quark passing through a finite-size QCD plasma. We show that the leading log approximation used in previous studies fails when the gluon formation length becomes of the order of the length of the medium traversed by the quark. Calculation of the energy loss beyond the leading log approximation gives the energy loss which grows logarithmically with quark energy contrary to the energy independent prediction of the leading log approximation.

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In resent years much attention has been attracted to the problem of the induced gluon radiation from fast partons in a hot QCD medium (for a review, see [1]). It is of great interest in connection with the current experiments at SPS, RHIC, and future experiments at LHC on $A+A$ collisions since jet quenching due to the parton energy loss can be a good probe of formation of a hot quark-gluon plasma (QGP).

Evaluation of the gluon emission from a fast parton in a medium requires the understanding of the non-abelian analogue of the Landau-Pomeranchuk-Migdal (LPM) effect [2, 3]. There are two approaches to the LPM effect in QCD: the so-called BDMS approach [4] (see also [1, 5]) based on the Feynman diagrammatic formalism, and the light-cone path integral (LCPI) approach developed in our paper [6] (see also [7–10]). The BDMS approach neglects the mass effects, and applies for large suppression of the radiation rate as compared to the Bethe-Heitler one. The LCPI approach applies for arbitrary strength of suppression. For large suppression these approaches are equivalent [4, 1, 11]. The probability of gluon emission in the BDMS and LCPI approaches is expressed through the solution of a two-dimensional Schrödinger equation with an imaginary potential. This equation describes evolution of the color singlet $\bar{q}gq$ system in the medium. The potential is proportional to the cross section for scattering of the $\bar{q}gq$ system on a medium constituent. For the QGP the constituents can be modeled as Debye-screened colored Coulomb scattering centers [12].

In [4] the quark energy loss, ΔE, has been evaluated analytically treating interaction of the $\bar{q}gq$ system with the Debye-screened centers in the Leading Log Approximation (LLA) which is equivalent to the harmonic oscillator approximation for the Hamiltonian of the $\bar{q}gq$ system. For a quark produced inside a finite-size QGP the BDMS prediction is

$$\Delta E_{\mathrm{BDMS}} = \frac{C_F \alpha_s}{4}\frac{L^2\mu^2}{\lambda_g}\tilde{v}\,, \qquad (1)$$

where L is the length of QGP traversed by the quark, μ is the Debye screening mass, λ_g is the mean free path of the gluon in QGP, C_F is the color Casimir for the quark, and the factor $\tilde{v}$ grows smoothly with L, at $L \gg \lambda_g$ $\tilde{v} \approx \log(L/\lambda_g)$.

The energy independent ΔE (1) differs from that obtained recently by Gyulassy, Levai, and Vitev (GLV) [13]. Calculating the Feynman diagrams for the single scattering (the first order ($N=1$) in opacity) they have obtained

$$\Delta E_{\mathrm{GLV}} = \frac{C_F \alpha_s}{4}\frac{L^2\mu^2}{\lambda_g}\log\frac{E}{\mu}\,. \qquad (2)$$

Since the ΔE_{BDMS} should include the $N = 1$ contribution the contradiction between (1) and (2) at $E \to \infty$ seems to be surprising[1]. By now there has not been given any explanation of this fact, except the argument of the authors of Ref.[13] that it can be connected with the neglect of the finite kinematic bounds in the analysis [4]. However, it is clear that it cannot be important at $E \to \infty$.

[1] Strictly speaking, the derivation of the BDMS formalism given in Ref.[4] is valid only when the number of rescatterings is large. However, since the formulas obtained are equivalent to those of the LCPI [6] approach which is free from this restriction, it is clear that the BDMS prediction should contain the $N=1$ term.

In the present paper we resolve the above puzzle of the discrepancy between the BDMS and GLV predictions. We demonstrate that the absence of the logarithmic energy dependence in (1) is connected with the fact that the LLA fails when the gluon formation length becomes of the order of L. In this case the spectrum is dominated by the $N=1$ scattering which simply vanishes in the LLA. We show that if one uses the actual imaginary potential the energy loss grows logarithmically with quark energy. However, the denominator in the argument of the logarithm is not the Debye mass as it is in (2).

We will work in the LCPI formalism [6]. The probability distribution of the induced gluon emission from a quark produced at $z=0$ can be written as [10]

$$\frac{dP}{dx}=\int\limits_0^\infty dz\, n(z)\frac{d\sigma_{\rm eff}^{BH}(x,z)}{dx}\,, \tag{3}$$

where x is the gluon fractional momentum, n is the number density of the medium, and

$$\frac{d\sigma_{\rm eff}^{BH}(x,z)}{dx}=\mathrm{Re}\int d\boldsymbol{\rho}\,\Psi^*(\boldsymbol{\rho},x)\sigma_3(\rho,x)\Psi_m(\boldsymbol{\rho},x,z)\,. \tag{4}$$

Here σ_3 is the cross section for interaction of the $\bar{q}gq$ system with a scattering center. The relative transverse separations in the $\bar{q}gq$ system are $\boldsymbol{\rho}_{g\bar{q}}=(1-x)\boldsymbol{\rho}$, $\boldsymbol{\rho}_{q\bar{q}}=-x\boldsymbol{\rho}$; $\Psi(\boldsymbol{\rho},x)$ is the light-cone wave function for the $q\to gq$ transition in vacuum, and $\Psi_m(\boldsymbol{\rho},x,z)$ is the quark light-cone wave function in the medium at the longitudinal coordinate z (we omit spin and color indices). The wave functions (modulo a color factor) read

$$\Psi(\boldsymbol{\rho},x)=P(x)\left(\frac{\partial}{\partial\rho'_x}-is_g\frac{\partial}{\partial\rho'_y}\right)\times$$
$$\times\int\limits_0^\infty d\xi\exp\left(-\frac{i\xi}{L_f}\right)\mathcal{K}_0(\boldsymbol{\rho},\xi|\boldsymbol{\rho}',0)\Big|_{\boldsymbol{\rho}'=0}\,, \tag{5}$$

$$\Psi_m(\boldsymbol{\rho},x,z)=P(x)\left(\frac{\partial}{\partial\rho'_x}-is_g\frac{\partial}{\partial\rho'_y}\right)\times$$
$$\times\int\limits_0^z d\xi\exp\left(-\frac{i\xi}{L_f}\right)\mathcal{K}(\boldsymbol{\rho},z|\boldsymbol{\rho}',z-\xi)\Big|_{\boldsymbol{\rho}'=0}\,, \tag{6}$$

where $P(x)=i\sqrt{\alpha_s/2x}[s_g(2-x)+2s_qx]/2M(x)$, $s_{q,g}$ denote parton helicities, $\mathcal{K}$ is the Green's function for the two-dimensional Hamiltonian

$$\hat{H}(z)=-\frac{1}{2M(x)}\left(\frac{\partial}{\partial\boldsymbol{\rho}}\right)^2-i\frac{n(z)\sigma_3(\rho,x)}{2}\,, \tag{7}$$

and

$$\mathcal{K}_0(\boldsymbol{\rho}_2,z_2|\boldsymbol{\rho}_1,z_1)=$$
$$=\frac{M(x)}{2\pi i(z_2-z_1)}\exp\left[\frac{iM(x)(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2}{2(z_2-z_1)}\right] \tag{8}$$

is the Green's function for the Hamiltonian (7) with $v(\boldsymbol{\rho},z)=0$, $M(x)=Ex(1-x)$, and $L_f=2Ex(1-x)/\epsilon^2$ with $\epsilon^2=m_g^2(1-x)+m_q^2x$. The gluon mass m_g plays the role of infrared cutoff removing the contribution from long wave gluons which cannot propagate in the QGP. It is natural to take $m_g\sim\mu$. However, for large suppression which occurs at $E\to\infty$ the parton masses can simply be neglected.

The three-body cross section can be written as [14]

$$\sigma_3(\rho,x)=\frac{C_A}{2C_F}[\sigma_2((1-x)\rho)+\sigma_2(\rho)-\frac{1}{N_c^2}\sigma_2(x\rho)]\,, \tag{9}$$

where $C_A=N_c$ is the octet color Casimir, $\sigma_2(\rho)$ is the dipole cross section for scattering of a $\bar{q}q$ pair on a color center. For the parametrization $\sigma_2(\rho)=C_2(\rho)\rho^2$ the factor C_2 is

$$C_2(\rho)=\frac{C_TC_F\alpha_s^2}{\rho^2}\int d\mathbf{q}\frac{[1-\exp(i\mathbf{q}\boldsymbol{\rho})]}{(q^2+\mu^2)^2}\,. \tag{10}$$

Here C_T is the color Casimir of the scattering center. In the region $\rho\ll 1/\mu$ which dominates the spectrum for strong suppression (10) takes the form

$$C_2(\rho)\approx\frac{C_FC_T\alpha_s^2\pi}{2}\log\left(\frac{1}{\rho\mu}\right)\,. \tag{11}$$

The LLA consists in replacing $C_2(\rho)$ by $C_2(\rho_{\rm eff})$, where $\rho_{\rm eff}$ is the typical value of ρ. This seems to be a reasonable procedure since $C_2(\rho)$ has only a slow logarithmic dependence on ρ. Then $\sigma_3(\rho,x)=C_3(x)\rho^2$, where $C_3(x)=C_2(\rho_{\rm eff})A(x)$ with $A(x)=[1+(1-x)^2-x^2/N_c^2]C_A/2C_F$, and the Hamiltonian (7) takes the oscillator form with the frequency $\Omega(x)=\sqrt{-iC_3(x)n/M(x)}$. The value of $\rho_{\rm eff}$ is connected with the gluon formation length, l_f, by the Schrödinger diffusion relation $\rho_{\rm eff}^2\sim l_f/2M$. l_f is simply the typical scale of ξ in (5), (6) when the wave functions are substituted in (4).

Let us discuss the gluon emission at qualitative level. We begin by estimating $\rho_{\rm eff}$ and l_f. Let us first estimate these quantities for gluon emission from a quark in an infinite medium. We will denote them as $\bar{\rho}_{\rm eff}$ and $\bar{l}_f$. They should also be related by the Schrödinger diffusion relation. On the other hand, the absorption effects for the $\bar{q}gq$ system should become strong at the scale $\bar{l}_f$. It means that $\bar{l}_f nC_3\bar{\rho}_{\rm eff}^2/2 \sim 1$. From these conditions one gets $\bar{\rho}_{\rm eff} \sim [E_a x(1-x)nC_3]^{-1/4}$ and $\bar{l}_{\rm eff} \sim 2\sqrt{E_a x(1-x)/nC_3}$. These estimates are valid when $\bar{\rho}_{\rm eff} \lesssim 1/\epsilon$ and $\bar{l}_f \lesssim L_f$.

Now we turn to the gluon emission from a quark produced inside a finite-size medium. In this case in the high-energy limit qualitatively two different situations are possible. The first regime gets for the gluons with x such that $\bar{l}_f \lesssim L$. In this case the finite-size effects play a marginal role, and $\rho_{\rm eff} \sim \bar{\rho}_{\rm eff}$. The spectrum can roughly be calculated using the effective Bethe-Heitler cross section for the infinite medium. We call this regime the infinite medium regime. The second regime occurs for the gluons for which $\bar{l}_f \gtrsim L$. In this case $\rho_{\rm eff} \sim \rho_d(L)$, where $\rho_d(L) = \sqrt{L/2M}$ is simply the diffusion radius on the scale of the quark path length inside the medium. In this regime the effective Bethe-Heitler cross section is chiefly controlled by the finite-size effects. We will call this regime the diffusion regime. Thus we can write for the above two regimes

$$\rho_{\rm eff} \sim \min(\bar{\rho}_{\rm eff}, \rho_d(L), 1/\epsilon)\,. \qquad (12)$$

Here we have taken into account that $\rho_{\rm eff} \lesssim 1/\epsilon$. In terms of x the infinite medium regime occurs at $x \lesssim \delta$ and $(1-x) \lesssim \delta$, and the diffusion regime gets at $\delta \lesssim x \lesssim (1-\delta)$, where

$$\delta \sim nC_3L^2/4E\,. \qquad (13)$$

For the sake of definiteness, below we discuss only the region $x \lesssim 0.5$. At $x \gtrsim \delta$ the probability of interaction of the $\bar{q}gq$ system with the medium (it is of the order of $n\sigma_3(\rho_d, x)L$) becomes small. Thus, it is clear that in the developed diffusion regime the spectrum is dominated by the $N=1$ scattering. It is surprising that this turns out to be in apparent contradiction with prediction of the LLA. The LLA spectrum can be obtained using in (6) the oscillator Green's function. For zero parton masses it gives

$$\frac{dP}{dx} = -\frac{2G(x)}{\pi}{\rm Re}\int\limits_0^L dz\int\limits_0^z d\xi\frac{\Omega^2}{\cos^2\Omega\xi} =$$

$$= \frac{2G(x)}{\pi}\ln|\cos\Omega L|\,, \qquad (14)$$

where $G(x) = \alpha_s C_F[1-x+x^2/2]/x$. This spectrum has been derived in [4]. Note that $|\Omega L| \sim 1$ at $x \sim \delta$. For the diffusion regime from (14) one gets

$$\left.\frac{dP}{dx}\right|_{x\gg\delta}^{\rm LLA} \approx \frac{G(x)C_3^2n^2L^4}{8\pi E^2x^2(1-x)^2}\,. \qquad (15)$$

Since the right-hand side of (15) $\propto n^2$ it is clear that it corresponds to the $N=2$ term. Thus one sees that the $N=1$ contribution is simply absent in the LLA.

The fact that the LLA fails in the diffusion regime can be directly seen from calculation of the $N=1$ contribution. To obtain it one should use in (6) the free Green's function (8). Then in the massless limit (4) gives

$$\left.\frac{d\sigma_{\rm eff}^{\rm BH}(x,z)}{dx}\right|_{N=1} = \frac{G(x)A(x)M(x)}{2\pi}\times$$

$$\times{\rm Im}\int\limits_0^z\frac{d\xi}{\xi^2}\int\limits_0^\infty d\rho^2\rho^2C_2(\rho)\exp\left(\frac{iM(x)\rho^2}{2\xi}\right)\,. \qquad (16)$$

For $C_2(\rho) = {\rm const}$ the ρ^2-integral in (16) has zero imaginary part, and the right-hand side of (16) is also zero. On the other hand, using (11) one gets from (16)

$$\left.\frac{d\sigma_{\rm eff}^{\rm BH}(x,z)}{dx}\right|_{N=1} = \frac{\alpha_s^2\pi C_TC_FG(x)A(x)z}{4Ex(1-x)}\,. \qquad (17)$$

Then (3) yields

$$\left.\frac{dP}{dx}\right|_{N=1} = \frac{\alpha_s^2\pi C_TC_FG(x)A(x)nL^2}{8Ex(1-x)}\,. \qquad (18)$$

Let us see why the LLA fails in momentum representation in which (4) reads

$$\frac{d\sigma_{\rm eff}^{\rm BH}(x,z)}{dx} = \frac{\alpha_s^2C_TC_FA(x)}{(2\pi)^2}\times$$

$$\times{\rm Re}\int d\mathbf{p}d\mathbf{q}\frac{[\Psi^*(\mathbf{p},x)-\Psi^*(\mathbf{p}-\mathbf{q},x)]\Psi_m(\mathbf{p},x,z)}{(q^2+\mu^2)^2}\,. \qquad (19)$$

In the massless limit from (19) one can obtain

$$\left.\frac{d\sigma_{\rm eff}^{\rm BH}(x,z)}{dx}\right|_{N=1} =$$

$$= \frac{\alpha_s^2C_TC_FG(x)A(x)}{2\pi}\int dp^2dq^2\frac{F(p,q)}{(q^2+\mu^2)^2}\,, \qquad (20)$$

$$F(p,q) = {\rm Re}\frac{1}{p^2}\left[1-\exp\left(-\frac{izp^2}{2M(x)}\right)\right]\int\limits_0^{2\pi}d\phi\frac{\mathbf{q}(\mathbf{q}-\mathbf{p})}{(\mathbf{q}-\mathbf{p})^2}\,, \qquad (21)$$

where ϕ is the angle between $\mathbf{q}$ and $\mathbf{p}$. The logarithmic situation with dominance of $q^2 \ll p^2$ would correspond to $F(p,q) \propto q^2$ at $q^2 \ll p^2$. However, the azimuthal ϕ integral in (21) equals $2\pi\theta(q^2 - p^2)$, and the process is dominated by hard t-channel exchanges with $q^2 > p^2 \sim 2M(x)/z$. After integrating over p^2 and q^2 in (20) one reproduces (17).

It must be emphasized that the LLA fails only in the diffusion regime. But it is a good approximation in the infinite medium regime when Ψ_m falls off rapidly at the scale much smaller than $\rho_d(L)$. It is also worth noting that the boundary (13) beyond which the diffusion regime occurs is obtained assuming that in the infinite medium regime LPM suppression is strong (it means that $\bar{\rho}_{\rm eff}(x \sim \delta) \ll 1/\epsilon$). It is possible that there the Bethe – Heitler (BH) situation takes place. One can easily show that in this case $\delta \sim L\mu^2/2E$. Thus, in general, the diffusion regime occurs for the gluons with energy $\omega \gtrsim \omega_{cr}$, where

$$\omega_{cr} \sim \max\left(\frac{nC_3L^2}{4}, \frac{L\mu^2}{2}\right). \qquad (22)$$

Let us now discuss the energy loss. It can be written as

$$\Delta E = \int\limits_{\mu}^{\omega_{cr}} d\omega\omega\frac{dP}{d\omega} + \int\limits_{\omega_{cr}}^{\omega_{\max}} d\omega\omega\frac{dP}{d\omega}. \qquad (23)$$

One can show that the first term in (23) does not depend on energy, and is of the order of $\Delta E_{\rm BDMS}$ (1) for both the LPM and Bethe-Heitler situations. At $E \to \infty$ the energy loss is dominated by the second term in (23) which grows logarithmically with E. Then, using (18) to the logarithmic accuracy one can obtain in the high-energy limit

$$\Delta E = \frac{C_F\alpha_s}{4}\frac{L^2\mu^2}{\lambda_g}\log\frac{E}{\omega_{cr}}. \qquad (24)$$

Here we have used $\alpha_s^2\pi C_F C_T A(0)n/2\mu^2 = 1/\lambda_g$. Note that since $L \gg 1/\mu$ from (22) it follows that always $\omega_{cr} \gg \mu$. The qualitative estimates (including the region $\omega \lesssim \omega_{cr}$) show that the appearance of ω_{cr} in the logarithm in (24) instead of μ in (2) for RHIC conditions ($L \sim 4$ fm) can suppresses the energy loss at $E \sim 10$ GeV by a factor of ~ 0.5. For SPS conditions ($L \sim 2$ fm) the suppression is not strong ($\sim 0.7 - 0.8$ at $E \sim 5$ GeV). The above estimates are obtained for the plasma temperature $T = 250$ MeV. Note that the absence of ω_{cr} in the GLV prediction (2) is connected with the neglect in [13] of the mass effects in evaluating the phase factor which controls the interference for gluon emission from different points of the quark trajectory.

The above analysis is valid for the gluon emission from a fast gluon as well. In this case in (24) C_F should be replaced by $2C_A$ (here the factor 2 comes from symmetry of the spectrum with respect to change $x \leftrightarrow (1-x)$).

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