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Nonlinear $k_\perp$-factorization: a new paradigm for hard QCD processes in a nuclear environment

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Large thickness of heavy nuclei brings in a new hard scale into the pQCD description of hard processes in a nuclear environment. This new scale breaks the conventional linear $k_\perp$-factorization which must be replaced by a new concept of the nonlinear nuclear $k_\perp$-factorization. Here we review a recent progress in the theory of nonlinear $k_\perp$-factorization. Our focus is on the rôle of diffractive interactions, the variation of the pattern of $k_\perp$-factorization for single-jet processes from deep inelastic scattering to hadron-nucleus collisions and universality classes for dijet production off nuclei.

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1. Introduction. The linear $k_\perp$-factorization is a fundamental ingredient of the pQCD description of high energy hard processes off free nucleons. A large thickness of a target nucleus introduces a new scale – the so-called saturation scale Q_A^2, – which breaks the linear $k_\perp$-factorization theorems for hard scattering in a nuclear environment. In this paper we review the recent work by the ITEP-Jülich-Landau collaboration in which a new concept of the nonlinear $k_\perp$-factorization has been introduced [1]. We illustrate this new concept on several examples of single-let to dijet production in deep inelastic scattering (DIS) off heavy nuclei and proton(deuteron)-nucleus collisions studied at Relativistic Heavy Ion Collider (RHIC). The nonlinear $k_\perp$-factorization emerges as a generic feature of the pQCD approach to hard processes in nuclear environment [2 – 5]. The concrete realizations depend strongly on the relevant pQCD subprocesses and we define the universality classes of nonlinear $k_\perp$-factorization for production of hard dijets. Our approach is based on the equivalence between the parton fusion description of the shadowing introduced in 1975 [6] and the unitarization on the color dipole-nucleus interaction [7]. The major technical problem in the unitarization program is the non-Abelian evolution of color dipoles in a nuclear environment and we present a closed-form solution based on the multiple-scattering theory [8, 9, 1].

2. The $k_\perp$-factorization for DIS off free nucleons. Our starting point is the color dipole factorization for DIS at small x

$$\sigma_T(x,Q^2) = \langle\gamma^*|\sigma(x,\mathbf{r})|\gamma^*\rangle = \int_0^1 dz \int d^2\mathbf{r}\,\Psi^*_{\gamma^*}(z,\mathbf{r})\sigma(x,\mathbf{r})\Psi_{\gamma^*}(z,\mathbf{r}). \qquad (1)$$

Here z and $(1-z)$ is the energy partition between q & $\bar q$ and $\mathbf{r}$ = size of the color dipole. There is an exact equivalence between color dipole and $k_\perp$-factorization [7, 10, 11]:

$$\sigma(x,\mathbf{r}) = \alpha_S(r)\int \frac{d^2\boldsymbol{\kappa}4\pi[1-\exp(i\boldsymbol{\kappa}\mathbf{r})]}{N_c\kappa^4}\frac{\partial G_N}{\partial\log\kappa^2}, \qquad (2)$$

$$f(x,\boldsymbol{\kappa}) = \frac{4\pi}{N_c\sigma_0(x)}\frac{1}{\kappa^4}\frac{\partial G_N(x,\boldsymbol{\kappa})}{\partial\log\kappa^2}. \qquad (3)$$

where $\sigma_0(x) = \sigma(x,\mathbf{r})\big|_{r\to\infty}$. The x-dependence of $\sigma(x,\mathbf{r})$ is governed by the color dipole BFKL equation [12]. The unintegrated gluon density $f(x,\boldsymbol{\kappa})$ furnishes a universal description of $F_{2p}(x,Q^2)$ and of the final states. For instance, the linear $k_\perp$-factorization for forward dijet cross section reads [13]

$$\frac{(2\pi)^2 d\sigma_N}{dz d^2\mathbf{p}_+ d^2\boldsymbol{\Delta}} = \frac{\alpha_S(\mathbf{p}^2)}{2}f(x,\boldsymbol{\Delta})\times \times|\Psi(z,\mathbf{p}_+)-\Psi(z,\mathbf{p}_+-\boldsymbol{\Delta})|^2, \qquad (4)$$

where $\Psi(z,\mathbf{p})$ is the $q\bar q$ wave function of the photon and $\boldsymbol{\Delta} = \mathbf{p}_+ + \mathbf{p}_-$ is the jet-jet decorrelation momentum.

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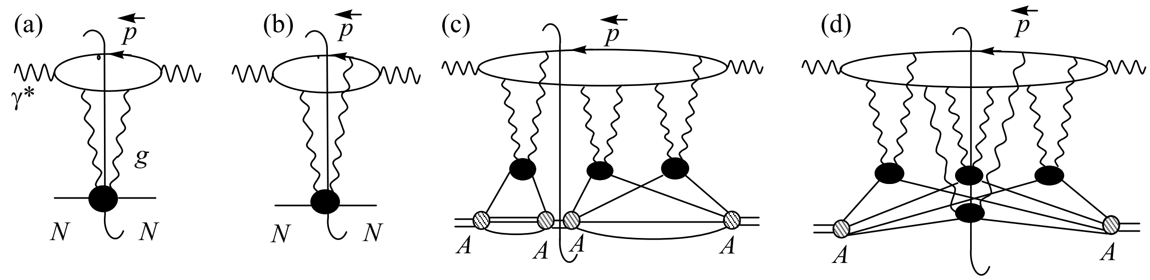

Fig.1. The typical unitarity cuts and dijet final states in DIS : (a), (b) – free-nucleon target, (c) – coherent diffractive DIS off a nucleus, (d) – truly inelastic DIS with multiple color excitation of the nucleus

From the unitarity point of view, Eq. (4) describes the unitarity cuts of diagrams Fig.1a,b for the forward Compton scattering amplitude. The point that the acoplanarity momentum $\mathbf{\Delta}$ comes from the transverse momentum of the exchanged gluon is obvious.

3. Collective unintegrated nuclear glue. DIS off a nucleus at $x \lesssim x_A$ can be described in terms of the color dipole-nucleus cross-section [7]

$$\sigma_A(\mathbf{r}) = 2\int d^2\mathbf{b}\left\{1 - \exp[-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b})]\right\}, \qquad (5)$$

where $T(\mathbf{b}) = \int dr_z n_A(r_z, \mathbf{b})$ is the optical the thickness of a nucleus at the impact parameter $\mathbf{b}$ and $n_A(r_z, \mathbf{b})$ is the nuclear matter density. This dipole cross section sums in the compact form the Glauber-Gribov multiple-scattering diagrams and is a basis for the quantitative description of nuclear shadowing in DIS [14]. On the other hand, nuclear shadowing can be understood in terms of the fusion of parton fields of nucleons which spatially overlap in an ultra-relativistic nuclei [6] and one needs to quantify the idea of a fusion of partons.

Taking for the guidance Eq. (4), we would like to define the collective unintegrated nuclear glue in terms of the final state observables. This requires an understanding of unitarity properties of the dipole-nucleus cross section (5). In Fig.1 we show the two typical unitarity cuts for a nuclear target: the diagram of Fig.1c corresponds to the so-called coherent diffractive final state in which the target nucleus remains in the ground state. It is remarkable that although a deposition of dozen MeV energy will break any heavy nucleus, at $x \lesssim x_A$ such a coherent diffraction makes $\approx 50\%$ of the total cross section of DIS off heavy nucleus [15]. To the lowest order in pQCD, the coherent diffractive final state consists of the back-to-back dijet with vanishing transverse momentum transfer to the target nucleus and the large transverse momentum of dijets comes entirely from gluons exchanged with target nucleons [16]. Consequently, one can take the partial wave of the diffraction amplitude, i.e., the nuclear profile function, for the definition of the collective nuclear glue per unit area in the impact parameter space, $\phi(\mathbf{b}, x, \boldsymbol{\kappa})$ [16, 1]:

$$\Gamma_A(\mathbf{b}, \mathbf{r}) = [1 - \exp(-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b}))] = = \int d^2\boldsymbol{\kappa}\phi(\mathbf{b}, x, \boldsymbol{\kappa})\{1 - \exp[i\boldsymbol{\kappa}\mathbf{r}]\}. \qquad (6)$$

A useful expansion is

$$\phi(\mathbf{b}, x, \boldsymbol{\kappa}) = \sum_{j=1}^{\infty} w_j(\mathbf{b}) f^{(j)}(x, \boldsymbol{\kappa}),$$
$$w_j(\mathbf{b}) = \frac{1}{j!}\left[\frac{1}{2}\sigma_0(x)T(\mathbf{b})\right]^j \exp\left[-\nu_A(x, \mathbf{b})\right], \qquad (7)$$

where $\nu_A(x, \mathbf{b}) = \frac{1}{2}\sigma_0(x)T(\mathbf{b})$, w_j is the probability to find j overlapping nucleons at impact parameter $\mathbf{b}$ in a Lorentz-contracted nucleus and $f^{(j)}(x, \boldsymbol{\kappa})$ is a collective glue of j overlapping nucleons:

$$f^{(j)}(x, \boldsymbol{\kappa}) = \int \prod_{i=1}^{j} d^2\boldsymbol{\kappa}_i f(x, \boldsymbol{\kappa}_i)\delta(\boldsymbol{\kappa} - \sum_{i=1}^{j}\boldsymbol{\kappa}_i)$$
$$f^{(0)}(x, \boldsymbol{\kappa}) = \delta(\boldsymbol{\kappa}). \qquad (8)$$

The plateau at small momenta of gluons,

$$\phi(\mathbf{b}, x, \boldsymbol{\kappa}) \approx \frac{1}{\pi}\frac{Q_A^2(\mathbf{b})}{(\boldsymbol{\kappa}^2 + Q_A^2(\mathbf{b}))^2},$$
$$Q_A^2(\mathbf{b}, x) \approx \frac{4\pi^2}{N_c}\alpha_S(Q_A^2)G(x, Q_A^2)T(\mathbf{b}), \qquad (9)$$

is a signal of the saturation effect.

The above defined collective nuclear glue $\phi(\mathbf{b}, x, \boldsymbol{\kappa})$ gives precisely the same description of the amplitude of diffraction off nucleus as $f(\boldsymbol{\kappa})$ does for the free-nucleon target [16 – 18]. However, the diffraction cross section which makes $\approx 50\%$ of DIS off nucleus, is a quadratic, nonlinear functional of the collective nuclear glue. Specifically, the diffractive single-jet cross section reads

$$\frac{d\sigma_D}{d^2\mathbf{b}d^2\mathbf{p}dz} = \frac{1}{(2\pi)^2} \times$$
$$\times \left| \int d^2\boldsymbol{\kappa}\phi(\mathbf{b},x,\boldsymbol{\kappa}) \left[\Psi(z,\mathbf{p}) - \Psi(z,\mathbf{p}-\boldsymbol{\kappa})\right] \right|^2 . \quad (10)$$

4. The non-Abelian intranuclear evolution of color dipoles and truly inelastic DIS. One must study, then, the factorization properties of truly inelastic DIS of nuclei, which leaves several target nucleons in the color excited state, see Fig.1d. The salient feature of such processes is a non-Abelian intranuclear evolution of color dipoles [8, 9, 1]. We start directly with the dijet spectrum. The *ab initio* calculation of the nuclear distortion of the two-parton density matrix the Fourier transform of which gives the spectrum of dijets, can be reduced, upon the closure over nuclear excitations, to the problem of intranuclear propagation of the color-singlet 4-parton states as illustrated in Fig.2:

$$\frac{(2\pi)^4 d\sigma_{in}}{dz d^2\mathbf{p}_+ d^2\mathbf{p}_-} = \int d^2\mathbf{b}_+{}' d^2\mathbf{b}_-{}' d^2\mathbf{b}_+ d^2\mathbf{b}_- \times$$
$$\times \exp[-i\mathbf{p}_+(\mathbf{b}_+ - \mathbf{b}_+{}') - i\mathbf{p}_-(\mathbf{b}_- - \mathbf{b}_-{}')] \times$$
$$\times \Psi^*(z,\mathbf{b}_+{}' - \mathbf{b}_-{}')\Psi(z,\mathbf{b}_+ - \mathbf{b}_-) \times$$
$$\times \Big\{ S_{4A}(\mathbf{b}_+{}',\mathbf{b}_-{}',\mathbf{b}_+,\mathbf{b}_-) -$$
$$- S_{4A}^{(\mathrm{Diffr})}(\mathbf{b}_+{}',\mathbf{b}_-{}',\mathbf{b}_+,\mathbf{b}_-) \Big\}, \quad (11)$$

where we subtracted the diffractive contribution. To the standard dilute-gas nucleus approximation, the Glauber-Gribov theory gives

$$S_{4A}(\mathbf{b}_+{}',\mathbf{b}_-{}',\mathbf{b}_+,\mathbf{b}_-) =$$
$$= \exp\{-\frac{1}{2}\sigma_4(\mathbf{b}_+{}',\mathbf{b}_-{}',\mathbf{b}_+,\mathbf{b}_-)T(\mathbf{b})\}, \quad (12)$$

where $\sigma_4(\mathbf{b}_+{}',\mathbf{b}_-{}',\mathbf{b}_+,\mathbf{b}_-)$ is the coupled-channel operator in the space of singlet-singlet $|11\rangle$ or octet-octet $|88\rangle$ 4-body dipoles. The derivation of this operator is a major technical task, see Ref. [1] for more details.

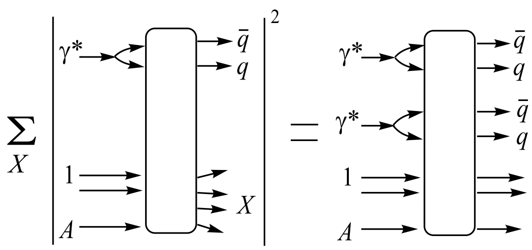

Fig.2. Unitarity diagram for the dijet spectrum in terms of the 4-parton scattering amplitude

The integration over the transverse momenta of the antiquark jet gives the following single-jet spectrum in truly inelastic DIS [1]:

$$(2\pi)^2 \frac{d\sigma_{in}}{d^2\mathbf{b}d^2\mathbf{p}dz} =$$
$$= \int d^2\boldsymbol{\kappa}\phi(\mathbf{b},x,\boldsymbol{\kappa}) \left|\Psi(\mathbf{p}) - \Psi(\mathbf{p}-\boldsymbol{\kappa})\right|^2 -$$
$$- \left| \int d^2\boldsymbol{\kappa}\phi(\mathbf{b},x,\boldsymbol{\kappa}) \Big[\Psi(\mathbf{p}) - \Psi(\mathbf{p}-\boldsymbol{\kappa})\Big] \right|^2 . \quad (13)$$

It is also the nonlinear functional of the collective nuclear glue. However, in the total inclusive single-jet spectrum the diffractive and truly inelastic nonlinear terms exactly cancel each other, and the single particle spectrum takes the linear $k_\perp$-factorizable form given by the integral form of Eq. (4) [19].

It is a highly nontrivial finding: the whole multitude of diffractive and truly inelastic unitarity cuts for DIS off nuclei sums up to the same unitarity cuts as shown in Figs. 1a and 1b, in which the unintegrated glue is replaced by the collective nuclear glue as defined in [16, 1]. All distortions of the transverse momentum distribution of the struck quark can exactly be reabsorbed into the collective nuclear glue, which by itself is a highly nonlinear functional of the free nucleon glue. This is not a universal feature of hard scattering off nuclei, though: such a linear $k_\perp$-factorization for single-jet spectrum in DIS is a special consequence of the incident photon being the color-singlet parton. Even in DIS, the property of linear $k_\perp$-factorization shall break for dijets.

5. The fate of $k_\perp$-factorization for dijets in nuclear DIS. The couple-channel non-Abelian evolution problem for the dijet spectrum in nuclear DIS is readily solvable to the large-N_c approximation. Here the incident color-singlet dipole first propagates the slice $[0,\beta]$ from the front face of a nucleus, then at some depth $0 < \beta < 1$ excites into the color-octet state and the further non-Abelian evolution in the remaining slice $[\beta,1]$ consists of color rotations within the space of octet dipoles, Fig.3.

The resulting nuclear dijet spectrum is a manifestly nonlinear functional of the collective nuclear glue and here emerges a new concept of the nonlinear $k_\perp$- factorization:

$$\frac{(2\pi)^2 d\sigma_A(\gamma^* \to Q\overline{Q})}{d^2\mathbf{b}dz d^2\mathbf{p}_- d^2\boldsymbol{\Delta}} = \frac{1}{2}T(\mathbf{b}) \int_0^1 d\beta \int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa} \times$$
$$\times f(x,\boldsymbol{\kappa})\Phi(1-\beta,\mathbf{b},x,\boldsymbol{\Delta}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa})\Phi(1-\beta,\mathbf{b},x,\boldsymbol{\kappa}_1) \times$$
$$\times \left|\Psi(\beta;z,\mathbf{p}_- - \boldsymbol{\kappa}_1) - \Psi(\beta;z,\mathbf{p}_- - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa})\right|^2 +$$
$$+ \delta^{(2)}(\boldsymbol{\Delta}) \left|\Psi(1;z_g,\mathbf{p}_-) - \Psi(z_g,\mathbf{p}_-)\right|^2, \quad (14)$$

where $\Phi(\beta,\mathbf{b},x,\boldsymbol{\kappa})$ is the collective nuclear glue for the slice $[0,\beta]$ of a nucleus defined by

$$\exp\left[-\frac{1}{2}\beta\sigma(x,\mathbf{r})T(\mathbf{b})\right] = \int d^2\boldsymbol{\kappa}\Phi(\beta,\mathbf{b},x,\boldsymbol{\kappa})\exp(i\boldsymbol{\kappa}\mathbf{r}) \tag{15}$$

and

$$\Psi(\beta;z,\mathbf{p}) = \int d^2\boldsymbol{\kappa}\Phi(\beta,\mathbf{b},x,\boldsymbol{\kappa})\Psi(z,\mathbf{p}+\boldsymbol{\kappa}) \tag{16}$$

is the wave function of the incident dipole distorted by the coherent Initial State Interaction (ISI) in the slice $[0,\beta]$ of a nucleus. The diffractive component, $\propto \delta^{(2)}(\boldsymbol{\Delta})$, gives exactly back-to-back dijets (for finite-

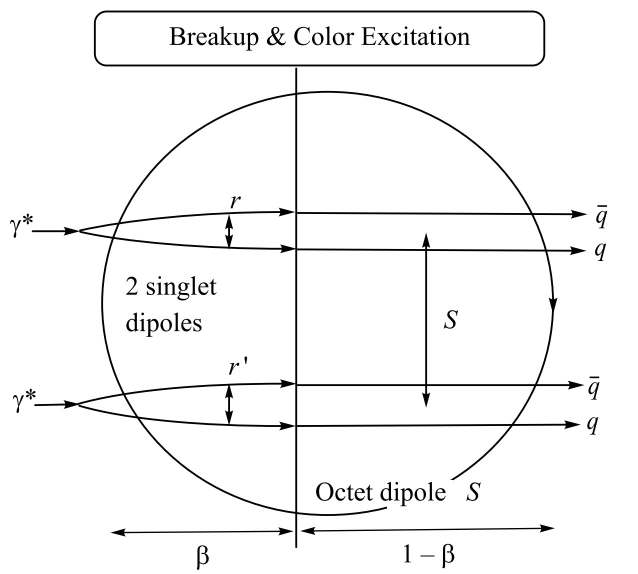

Fig.3. The color excitation of the dipole in the large-N_c approximation

size nuclei the $\boldsymbol{\Delta}$-dependence is controlled by a slightly modified nuclear form factor with the width $\boldsymbol{\Delta}^2 \lesssim R_A^{-2}$, see Ref. [16]). The first component in (14) describes truly inelastic DIS. Here the slice $[\beta,1]$ in which the dipole is in the color-octet state gives the Final State Interactions (FSI). The nuclear dijet spectrum is of fifth order in gluon field densities: a quartic functional of the collective nuclear glue for the two slices of a nucleus and a linear one of the free-nucleon glue $f(x,\boldsymbol{\kappa})$. The rôle of different glues is noteworthy: the latter one describes the hard singlet-to-octet transition; the former ones the coherent ISI and incoherent FSI's. The distinct physics of ISI and FSI can requires invoking the collective glue for slices of the nucleus, the $k_\perp$-factorization for the truly inelastic DIS cannot be described by the classical gluon field of the whole nucleus. The nonlinear $k_\perp$-factorization result (14) must be contrasted to the free-nucleon spectrum (4); it entails a nuclear enhancement of the decorrelation of dijets from truly inelastic DIS, the semihard dijets, $|\mathbf{p}_\pm|^2 \lesssim Q_A^2(\mathbf{b},x)$, are completely decorrelated.

6. The master formula for nuclear dijets. The above discussion of leading quark-antiquark dijets in DIS, $\gamma^* \to Q\overline{Q}$ can readily be extended to quark-antiquark and quark-gluon dijets in subprocesses $g^*g \to Q\overline{Q}$, $q^*g \to qg$. Here we discuss the case when the beam and final state partons interact coherently over the whole nucleus, which at RHIC amounts to dijets in the largest rapidity bins of the proton fragmentation region, $x = (Q^2 + M_{JJ}^2)/2mE_a \lesssim x_A = 1/R_A m_p \approx 0.1A^{-1/3}$, where R_A is the radius of the target nucleus of mass number A, E_a is energy of the beam parton a in the target rest frame and m_p is the proton mass [6, 14]. To the lowest order in pQCD, all the above processes are of the form $ag \to bc$ and, in the laboratory frame, can be viewed as an excitation of the perturbative $|bc\rangle$ Fock state of the physical projectile $|a\rangle$ by one-gluon exchange with the target nucleon. Our focus on excitation of the lowest Fock states at $x \lesssim x_A$ is justified by the kinematical constraints in pA collisions at RHIC; understanding the very rich pattern of nonlinear $k_\perp$-factorization found in this regime is a must for the further studies of the small-x evolution of jet phenomena.

The derivation of the master formula for the dijet spectrum, based on the technique developed in [8, 9, 1], is found in [3]:

$$\begin{aligned}\frac{d\sigma(a^*\to bc)}{dz_b d^2\mathbf{p}_b d^2\mathbf{p}_c} &= \frac{1}{(2\pi)^4}\int d^2\mathbf{b}_b d^2\mathbf{b}_c d^2\mathbf{b}'_b d^2\mathbf{b}'_c \times \\ &\exp[-i\mathbf{p}_b(\mathbf{b}_b-\mathbf{b}'_b) - i\mathbf{p}_c(\mathbf{b}_c-\mathbf{b}'_c)] \times \\ &\times \Psi(z_b,\mathbf{b}_b-\mathbf{b}_c)\Psi^*(z_b,\mathbf{b}'_b-\mathbf{b}'_c) \times \\ &\times \Big\{S^{(4)}_{\bar{b}\bar{c}cb}(\mathbf{b}'_b,\mathbf{b}'_c,\mathbf{b}_b,\mathbf{b}_c) + S^{(2)}_{\bar{a}a}(\mathbf{b}',\mathbf{b}) - \\ &- S^{(3)}_{\bar{b}\bar{c}a}(\mathbf{b},\mathbf{b}'_b,\mathbf{b}'_c) - S^{(3)}_{\bar{a}bc}(\mathbf{b}',\mathbf{b}_b,\mathbf{b}_c)\Big\}.\end{aligned} \tag{17}$$

It generalizes the DIS equation (11). If $\mathbf{b}_a = \mathbf{b}$ is the projectile's impact parameter, then $\mathbf{b}_b = \mathbf{b} + z_c\mathbf{r}$, $\mathbf{b}_c = \mathbf{b} - z_b\mathbf{r}$, where $z_{b,c}$ stand for the fraction of the lightcone momentum of the projectile a carried by partons b and c, $\Psi(z,\mathbf{r})$ stands for the lightcone wave function of the $|bc\rangle$ Fock state of the projectile, its connection to the parton-splitting functions is found in [3]. All $S^{(n)}$ describe a scattering of color-singlet systems of n partons, as indicated in Fig.4. This is the crucial point – in the course of our derivation of the dijet spectra and single-jet spectra we only deal with infrared-safe observables. $S^{(2)}$ and $S^{(3)}$ are readily calculated in terms of the 2-parton and 3-parton dipole cross sections [7, 11, 9], general rules for the multiple scattering theory calculation of the coupled-channel $S^{(4)}$ are found in [1, 5].

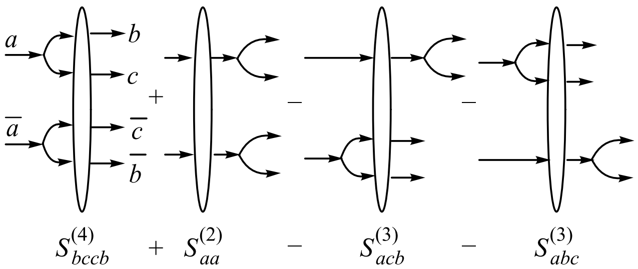

Fig.4. The S-matrix structure of the two-body density matrix for excitation $a \to bc$

7. The fate of $k_\perp$-factorization for single-jet spectra in pA collisions. A short digression from dijets to single jets: the integration over the transverse momentum of the unobserved parton in the master formula (17) is straightforward. The unobserved parton and its antiparton will enter at the same impact parameter and multiparton color singlet states simplify to the two-parton ones. The non-Abelian evolution simplifies to the Abelian one for color-singlet dipoles made out of partons in the relevant color representations. Still, the non-Abelian features of QCD manifest themselves in the breaking of linear $k_\perp$-factorization.

As we mentioned above, a remarkable recovery of linear $k_\perp$-factorization (9) for the single-jet spectrum in DIS is rather an exception due to the abelianization in the case of a colorless projectile – the photon. The radiation of gluons from quarks, $q^* \to gq$, illustrates nicely the salient features of breaking of linear $k_\perp$-factorization for the single-jet spectrum [3]. It is directly relevant to jet production in the proton hemisphere of pA collisions at RHIC [20, 21].

Here we again show the large-N_c results. The spectrum of gluons for the free-nucleon target reads

$$\frac{(2\pi)^2 d\sigma_A(q^*\to gq)}{dz_g d^2\mathbf{p}_g} = \frac{1}{2}\int d^2\boldsymbol{\kappa} f(x,\boldsymbol{\kappa}) \times \times \left\{ \left|\Psi(z_g,\mathbf{p}_g) - \Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa})\right|^2 + + \left|\Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa}) - \Psi(z_g,\mathbf{p}_g+z_g\boldsymbol{\kappa})\right|^2 \right\}, \qquad (18)$$

where $\Psi(z_g,\mathbf{p}_g)$ is the wave function of the qg Fock state of the photon, its explicit form in terms of the parton splitting functions is found in [3]. The same spectrum for the nuclear target is of the two-component form

$$\frac{(2\pi)^2 d\sigma_A(q^*\to gq)}{dz_g d^2\mathbf{p}_g d^2\mathbf{b}} = = S[\mathbf{b}, \frac{C_A}{C_F}\sigma_0(x)] \int d^2\boldsymbol{\kappa}\phi(\mathbf{b},x,\boldsymbol{\kappa}) \times \times \left\{ |\Psi(z_g,\mathbf{p}_g) - \Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa})|^2 + |\Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa}) - \Psi(z_g,\mathbf{p}_g+z_g\boldsymbol{\kappa})|^2 \right\} + + \int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 \phi(\mathbf{b},x_A,\boldsymbol{\kappa}_1)\phi(\mathbf{b},x,\boldsymbol{\kappa}_2) \times \times |\Psi(z_g,\mathbf{p}_g+z_g\boldsymbol{\kappa}_1) - \Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)|^2. \qquad (19)$$

The first component is an exact counterpart of the free-nucleon spectrum: It is linear $k_\perp$-factorizable, but is suppressed by the nuclear absorption factor $S[\mathbf{b},\sigma_0(x)] = \exp[-\frac{1}{2}\sigma_0(x)T(\mathbf{b})]$. For central interactions of the main experimental interest, the gluon spectrum is entirely dominated by the second component which is a non-linear functional of the collective nuclear glue. This illustrates clearly a breaking of linear $k_\perp$-factorization for single jets, a full compendium of nonlinear $k_\perp$-factorization results for single jest from all possible pQCD subprocesses is found in [3].

For soft gluons , $z_g \ll 1$, the result (19) simplifies to

$$\frac{(2\pi)^2 d\sigma_A(q^*\to gq)}{dz_g d^2\mathbf{p}_g d^2\mathbf{b}} = = \int d^2\boldsymbol{\kappa}\phi_{gg}(\mathbf{b},x_A,\boldsymbol{\kappa})|\Psi(z_g,\mathbf{p}_g) - \Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa})|^2. \qquad (20)$$

In takes the linear $k_\perp$-factorization form in terms of $\phi_{gg}(\mathbf{b},x_A,\boldsymbol{\kappa}) = \Big(\phi\otimes\phi\Big)(\mathbf{b},x_A,\boldsymbol{\kappa})$ which has a meaning of the collective nuclear glue defined in terms of the intranuclear propagation of the gluon-gluon color dipole. This illustrates nicely the important point that the collective nuclear glue is a density matrix in the color space rather than a single scalar function [1].

One more point is noteworthy: there is a conspicuous difference between the z_g-dependence of the free-nucleon and nuclear spectra. This amounts to the $\mathbf{p}_g$-dependence of the Landau–Pomeranchuk–Migdal effect; the same applies to the spectrum of leading quarks and nuclear quenching of forward jets in pA collisions [3].

8. The fate of $k_\perp$-factorization for nuclear dijets from pA collisions. A comparison of the $k_\perp$-factorization properties of single-jet spectra in various pQCD processes suggests a very rich pattern of non-linear $k_\perp$-factorization Here we report closed-form analytic results for nuclear dijet spectra from $q^* \to gq$ and $g^* \to Q\overline{Q}$ subprocesses. We show the leading order terms of the $1/N_c$ expansion, the higher order terms can be derived following the technique of Ref. [1]. The large-N_c properties of the excitation $q^* \to q\bar{g}$ are similar to those of excitation $\gamma^* \to q\bar{q}$ in DIS, while those of $g^* \to Q\overline{Q}$ are quite different. The free-nucleon quark-gluon dijet cross section,

$$\frac{2(2\pi)^2 d\sigma_N(q^* \to gq)}{dz_g d^2\mathbf{p}_g d^2\mathbf{\Delta}} = f(x,\mathbf{\Delta}) \times \times \Big[|\Psi(z_g,\mathbf{p}_g) - \Psi(z_g,\mathbf{p}_g - \mathbf{\Delta})|^2 + + |\Psi(z_g,\mathbf{p}_g - \mathbf{\Delta}) - \Psi(z_g,\mathbf{p}_g - z_g\mathbf{\Delta})|^2\Big], \qquad (21)$$

is simply the differential form of the single-jet spectrum, Eq. (82) of Ref. [3].

The extension to nuclear targets is straightforward. The set of color singlet 4-parton states $qg\bar{q}'g'$ which enter the master formula (17) includes $|3\bar{3}\rangle$, $|6\bar{6}\rangle$ and $|15\overline{15}\rangle$ states. The amplitude of excitation of the $|6\bar{6}\rangle$ and $|15\overline{15}\rangle$ states from the initial state $|3\bar{3}\rangle$ is suppressed $\propto 1/N_c$, which is compensated for in the dijet cross section by the number of color states in $|6\bar{6}\rangle$ and $|15\overline{15}\rangle$. At large N_c one of the $|6\bar{6}\rangle \pm |15\overline{15}\rangle$ states decouples from the initial state $|3\bar{3}\rangle$ [5]. The nuclear dijet spectrum takes the form

$$\frac{(2\pi)^2 d\sigma_A(q^* \to qg)}{d^2\mathbf{b}dz d^2\mathbf{p}_g d^2\mathbf{\Delta}} = \frac{1}{2}T(\mathbf{b}) \times \times \int_0^1 d\beta \int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 d^2\boldsymbol{\kappa} f(x,\boldsymbol{\kappa}) \times \times \Phi(\beta,\mathbf{b},x,\boldsymbol{\kappa}_2)\Phi(1-\beta,\mathbf{b},x,\mathbf{\Delta}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa}) \times \times \Phi(\frac{C_A}{C_F}(1-\beta),\mathbf{b},x,\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_2) \times \times \Big|\Psi(\beta;z_g,\mathbf{p}_g-\boldsymbol{\kappa}_1) - \Psi(\beta;z_g,\mathbf{p}_g-\boldsymbol{\kappa}_1-\boldsymbol{\kappa})\Big|^2 + + \phi(\mathbf{b},x,\mathbf{\Delta})\Big|\Psi(1;z_g,\mathbf{p}_g-\mathbf{\Delta}) - \Psi(z_g,\mathbf{p}_g - z_g\mathbf{\Delta})\Big|^2 + + \delta^{(2)}(\mathbf{\Delta})S[\mathbf{b},\sigma_0(x)]\Big|\Psi(1;z_g,\mathbf{p}_g) - \Psi(z_g,\mathbf{p}_g)\Big|^2. \qquad (22)$$

The contribution from the coherent diffractive excitation of color-triplet qg dipoles, $\propto \delta^{(2)}(\mathbf{\Delta})$, is suppressed by the nuclear attenuation because of the initial parton q^* being a colored one. The second term in (22) can be associated with excitation of the color-triplet qg states. It looks like satisfying linear $k_\perp$-factorization in terms of $\phi(\mathbf{b},x,\mathbf{\Delta})$ but it does not: one of the wave functions, $\Psi(1;z_g,\mathbf{p}_g)$, is coherently distorted over the whole thickness of the nucleus, see Eq. (15). Finally, the first component of the nuclear spectrum (22) describes excitation of the color sextet and 15-plet qg states. Notice the emergence of the ratio of Casimir operators C_A/C_F in one of the collective nuclear densities, which is still another demonstration that that the collective nuclear glue is a density matrix in the color space [1]. The free-nucleon Eq. (21) is recovered to the impulse approximation.

Excitation of open charm is driven by gluon-gluon collisions. The free-nucleon dijet cross section from $g^* \to Q\overline{Q}$ is simply the differential form of the single-jet spectrum derived in [3]:

$$\frac{2(2\pi)^2 d\sigma_N(g^* \to Q\overline{Q})}{dz d^2\mathbf{p}_- d^2\mathbf{\Delta}} = f(x,\mathbf{\Delta}) \times \times \Big[|\Psi(z,\mathbf{p}_-) - \Psi(z,\mathbf{p}_- - z\mathbf{\Delta})|^2 + + |\Psi(z,\mathbf{p}_- - \mathbf{\Delta}) - \Psi(z,\mathbf{p}_- - z\mathbf{\Delta})|^2\Big]. \qquad (23)$$

Here one starts with the color-octet $Q\overline{Q}$ dipole, intranuclear interactions are color rotations in the space of the octet states and transitions to the color-singlet $Q\overline{Q}$ dipoles are $1/N_c$ suppressed [1]. The same suppression holds for coherent diffraction. The non-Abelian evolution of the $Q\overline{Q}Q'\overline{Q}'$ state becomes the single-channel problem and the resulting nuclear dijet cross section equals

$$\frac{(2\pi)^2 d\sigma_A(g^* \to Q\overline{Q})}{dz d^2\mathbf{p}_- d^2\mathbf{b} d^2\mathbf{\Delta}} = = \int d^2\boldsymbol{\kappa}\Phi(1;\mathbf{b},x,\boldsymbol{\kappa})\Phi(1;\mathbf{b},x,\mathbf{\Delta}-\boldsymbol{\kappa}) \times \times |\Psi(z,\mathbf{p}_- - \boldsymbol{\kappa}) - \Psi(z,\mathbf{p}_- - z\mathbf{\Delta})|^2 = = S[\mathbf{b},\sigma_0(x)]\phi(\mathbf{b},x,\mathbf{\Delta}) \times \times \Big\{|\Psi(z,\mathbf{p}_-) - \Psi(z,\mathbf{p}_- - z\mathbf{\Delta})|^2 + + |\Psi(z,\mathbf{p}_- - \mathbf{\Delta}) - \Psi(z,\mathbf{p}_- - z\mathbf{\Delta})|^2\Big\} + + \int d^2\boldsymbol{\kappa}\phi(\mathbf{b},x,\boldsymbol{\kappa})\phi(\mathbf{b},x,\mathbf{\Delta}-\boldsymbol{\kappa}) \times \times |\Psi(z,\mathbf{p}_- - \boldsymbol{\kappa}) - \Psi(z,\mathbf{p}_- - z\mathbf{\Delta})|^2. \qquad (24)$$

Interestingly, the nuclear dijet spectrum (24) is precisely the differential version of the single-quark spectrum, Eq. (31) of Ref. [3], if in the nonlinear term one makes an identification $\mathbf{\Delta} = \boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2$. It satisfies the quadratic-nonlinear $k_\perp$-factorization it terms of the collective glue defined for the whole nucleus, which must be contrasted to the fifth order nonlinearity for the leading quark-antiquark dijets in DIS and the sixth order nonlinearity for qg dijets from $q^* \to qg$. Kinematically, it looks like the subprocess $g^* g_{1A} g_{2A} \to Q\overline{Q}$ with two uncorrelated collective nuclear gluons g_A, but it cannot readily be associated with specific Feynman diagrams in terms of g_A.

Now the demonstrate that how the diverse nonlinear $k_\perp$-factorization results for different pQCD processes fall into universality classes.

9. Nonlinear $k_\perp$-factorization for dijets: the universality classes. 9.1. *Excitation of higher color representations from partons in the lower representations.* Excitation of color-octet states in DIS, and of sextet and 15-plet states in qA interactions, belong to this universality class. The two reactions have much similarity. In both cases the nonlinear $k_\perp$-factorization formulas contain the free-nucleon gluon density $f(x,\boldsymbol{\kappa})$, which describes the transition from the qg color dipole from the lower – triplet for qg and singlet for DIS – to higher – sextet and 15-plet for qg and octet in DIS – color dipoles. In both cases, the number of states in higher representations is by the factor N_c^2 larger than in the lower representation. In $q\bar{q}$ excitation in DIS the corresponding contribution to the dijet spectrum is the fifth order functional of gluon densities. In the qg case it is the sixth order functional of gluon densities. Of these, two powers of the collective nuclear glue enter implicitly via the coherent ISI distortions of the wave function $\Psi(\beta;z,\mathbf{p})$ in the slice of the nuclear matter before excitation of color dipoles in the higher representation.

The principal difference between DIS and qA interactions is in the nuclear thickness dependence of the distortion factors. Namely, the factor

$$\Phi((1-\beta),\mathbf{b},\boldsymbol{\kappa}_2)\Phi((1-\beta),\mathbf{b},\boldsymbol{\kappa}_1)$$

in DIS, Eq. (14), is the symmetric function of the collective nuclear gluon momenta $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2=\boldsymbol{\Delta}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa}$ which flow from the nucleus to the quark and antiquark (or vice versa), respectively. It describes equal, and uncorrelated, distortion of the outgoing quark and antiquark waves by pure FSI. The independence of the two distortion factors is a feature of the large N_c approximation.

For qg dijets in qA collisions the overall distortion factor in (22) is of the form

$$\Phi(\beta;\mathbf{b},\boldsymbol{\kappa}_3)\Phi(\frac{C_A}{C_F}(1-\beta);\mathbf{b},\boldsymbol{\kappa}_2)\Phi(1-\beta;\mathbf{b},\boldsymbol{\kappa}_1).$$

The FSI distortions in the slice $(1-\beta)$ of the nucleus are given by the two last factors, of which $\Phi(1-\beta;\mathbf{b},\boldsymbol{\kappa}_1)$ is a broadening due to final-state rescatterings of the quark, while second FSI factor, $\Phi(\frac{C_A}{C_F}(1-\beta);\mathbf{b},\boldsymbol{\kappa}_2)$ describes the FSI distortion of the outgoing gluon wave. To the large-N_c approximation the rescatterings of the quark and gluon are uncorrelated.

The coherent ISI distortion of the wave functions in the slice $[0,\beta]$ of the nucleus in DIS and qA collisions is identical. However, in qA collisions this coherent distortion is accompanied by an incoherent ISI distortions of the incident quark wave described by $\Phi(\beta;\mathbf{b},\boldsymbol{\kappa}_3)$. In DIS the incoherent ISI distortions are absent because the photon is a color-singlet particle. We can anticipate that gluon-nucleus collisions with excitation of gluon-gluon dijets in higher color representations will belong to this universality class.

9.2. *Excitation of final state dipoles in exactly the same color state as the incident parton: coherent diffraction.* To this universality class belong the exactly back-to-back dijets. Another experimental signature of the coherent diffraction is a retention of the target nucleus in the ground state and large rapidity gap between the hadronic debris of the diffractive dijet and the recoil nucleus. It is most important for DIS where coherent diffraction dissociation of the photon into $q\bar{q}$ dijets makes for heavy nuclei $\approx 50\%$ of the total DIS rate [15]. The origin of the coherent diffraction is a coherent nuclear distortion of the wave function of the $q\bar{q}$ Fock state over the whole thickness of the nucleus.

In the coherent diffractive excitation of qg dipoles in qA collisions the qg dipole must propagate in exactly the same color state as the incident quark. Here the nuclear suppression factor $\mathsf{S}[\mathbf{b},\sigma_0(x)]$ has the meaning of

$$\mathsf{S}[\mathbf{b},\sigma_0(x)]=\left(\mathsf{S}[\mathbf{b},\frac{1}{2}\sigma_0(x)]\right)^2 \qquad (25)$$

and the factor $\mathsf{S}[\mathbf{b},\frac{1}{2}\sigma_0(x)]$ in the diffractive amplitude corresponds to the intranuclear attenuation of the quark wave with the total cross section

$$\sigma_{qN}=\frac{1}{2}\sigma_0(x). \qquad (26)$$

Coherent diffractive excitation of color-octet gluon-gluon dijets in gluon-nucleus collisions is expected to exhibit similar properties.

Coherent diffractive excitation of $Q\bar{Q}$ dipoles in gA collisions is allowed, but it is suppressed at large N_c by the condition that the $Q\bar{Q}$ dipole must propagate in exactly the same color state as the incident gluon.

9.3. *Incoherent excitation of final state dipoles in the same lower color representation as the incident parton.* An example of this universality class is an inelastic excitation of color-triplet qg states in qA collisions followed by a color excitation of the target. Here both the incident parton and dijet belong to the fundamental, i.e., lower, representation of $SU(N_c)$. The intranuclear evolution of such a dipole is confined to rotations within the color-triplet state. This contribution is not suppressed at large N_c. The dijet cross section for this universality class looks like satisfying the linear $k_\perp$-factorization in terms of $\phi(\mathbf{b},x,\boldsymbol{\Delta})$. But this is not the case: one of the wave functions, $\Psi(1;z,\mathbf{p}_g)$, is coherently distorted over the whole thickness of the nucleus, so that this contribution is a cubic functional of the collective nuclear glue.

We can anticipate that gluon-nucleus collisions with excitation of color-octet gluon-gluon dijets will belong to this universality class, although one has to account for the existence of the two, F-coupled and D-coupled, octet states.

Although superficially it looks like a subclass of this universality class, the coherent diffraction is a distinct class for the property of the exact back-to-back dijets and the rapidity gap between the dijet and the recoil nucleus in the ground state.

9.4. *Excitation of final state dipoles in the same higher color representation as the incident parton.* In the realm of QCD with gluons in the adjoint representation and quarks in the fundamental representation, this universality class consists of the quark-antiquark dijets in gluon-nucleus collisions. Only in this case the initial parton (gluon) belongs to the higher (octet) color multiplet of the final $Q\bar{Q}$ state. At large N_c, the intranuclear evolution of $Q\bar{Q}$ will consist of color rotations within the space of color-octet states. The de-excitation from the color-octet to color-singlet $Q\bar{Q}$ dipoles is suppressed at large N_c. Consequently, the non-Abelian evolution of the $Q\bar{Q}Q'\bar{Q}'$ state becomes the single channel problem. The coherent diffraction excitation, in which the initial and final color states must be identical, is likewise suppressed. The emerging pattern of quadratic nonlinearity can be related to the large-N_c gluon behaving like the color-uncorrelated quark and antiquark.

The above classification exhausts reactions caused by incident photons, quarks and gluons. However, technically all the universality classes have a much broader basis. Indeed, instead of an incident gluon one can think of the projectile which is a compact lump of many partons in the highest possible color representation. For instance, in presence of extra gluons compact diquarks in the proton can be viewed as sextet partons.

10. Summary and outlook. Despite the manifest breaking of the linear $k_\perp$-factorization, the collective nuclear glue remains a useful concept and is an important ingredient of nonlinear $k_\perp$-factorization, which is a generic feature of the pQCD description of single-jet and dijet production in a nuclear environment. The pattern of nuclear $k_\perp$-factorization changes dramatically depending on color properties of the specific pQCD subprocess. A unique case is a leading quark jet in DIS off nuclei: here the collective nuclear glue – by itself a highly nonlinear functional of the free-nucleon glue – furnishes a linear $k_\perp$-factorization description of nuclear single-jet cross section which is an exact counterpart of the conventional linear $k_\perp$-factorization for free-nucleon target. The pQCD Bremsstrahlung of gluon jets which carry very small fraction of energy of the incident quark or gluon, possesses the same property in terms of the collective nuclear glue defined in terms of octet-octet color dipoles. However, in the generic case of single jets in the hadron induced reactions the linear $k_\perp$-factorization is badly broken.

We presented the closed-form analytic results for nuclear dijets and identified four major universality classes for the dijet cross sections. The variation of the degree of nonlinearity from one universality class to another is clearly related to color properties of pQCD excitation processes. However, the found four universality classes differ by more than the degree of the nonlinearity. The coherent diffractive mechanism and the excitation of quark-gluon dijets in the same color representation as the incident quark are explicitly calculable in terms of the collective nuclear glue of Eq. (5) which is defined for the whole nucleus. This is not the case for the excitation of leading quark-antiquark dijets in DIS and quark-gluon dijets in higher color multiplets. Here the hard excitation is described by the unintegrated gluon density in the free nucleon. The coherent initial state interaction, before the excitation of higher color multiplets at the depth β of the nucleus, must be described in terms of the unintegrated collective glue (15) defined for the slice $[0,\beta]$ of the nucleus. Coherent distortions of the qg wave function are complemented by incoherent broadening of the incident quark transverse momentum distribution in the same slice of the nucleus. Likewise, the final state interactions after the excitation of higher multiplets must be described in terms of the unintegrated collective glue defined for the slice $[\beta,1]$ of the nucleus. This reinforces the point [1] that hard processes in a nuclear environment can not be described in terms of a nuclear gluon density defined for the whole nucleus, as it was advocated, for instance, within the Color Glass Condensate approach [22]. Furthermore, besides the collective nuclear glue defined for color-singlet quark-antiquark dipole, there emerges a new nuclear gluon density which depends on the Casimir operators of higher quark-gluon color representations, i.e., gluon field of the nucleus must be described by a density matrix in the space of color representations.

The representation for the dijet cross section similar to our master formula (17) has been discussed recently by several authors [23–25], but these works stopped short of the solution of the coupled-channel intranuclear evolution for the for 4-parton state.

The nuclear coherency condition, $x \lesssim x_A \approx 0.1 \cdot A^{-1/3}$, restricts the applicability domain of the reviewed formalism to the forward part of the proton hemisphere of pA collisions at RHIC. These predictions could be tested after the detectors at RHIC II will be upgraded to cover the proton fragmentation region [26]. This restriction is only technical, however, it can be lifted and we would like to conclude with the statement that our formalism can readily be extended to the mid-rapidity dijets studied so far at RHIC [20]. Specifically, in close similarity to incoherent diffraction production of heavy quarkonia [27], one must distinguish coherency in the excitation of of the dijet from the incident parton and the coherency in the intranuclear propagation of produced partons. Only the former will be broken for the midrapidity jets; hopefully, it will be tractable within the technique of lightcone evolution developed in application to the LPM effect [28], this issue is being studied. Finally, one can extend the reported technique to a derivation of nonlinear $k_\perp$-factorization for correlation of properties of forward jets with excitation of the target nucleus, this analysis is in progress.

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