Pis'ma v ZhETF, vol. 82, iss. 8, pp. 524 – 528

Heavily-chirped solitary pulses in the normal dispersion region: new solutions of the cubic-quintic complex Ginzburg-Landau equation

E. Podivilov[1)], *V. L. Kalashnikov*[+]

Institute for Automation and Electrometry RAS, 630090 Novosibirsk, Russia

[+]*Institut für Photonik, TU Wien, A-1040 Vienna, Austria*

Submitted 11 July 2005

A new type of the heavily-chirped solitary pulse solutions of the nonlinear cubic-quintic complex Ginzburg-Landau equation has been found. The methodology developed provides for a systematic way to find the approximate but highly accurate analytical solutions of this equation with the generalized nonlinearities within the normal dispersion region. It is demonstrated that these solitary pulses have the extra-broadened parabolic-top or finger-like spectra and allow compressing with more than hundredfold growth of the pulse peak power. The obtained solutions explain the energy scalable regimes in the fiber and solid-state oscillators operating within the normal dispersion region and promising to achieve the micro-joules femtosecond pulses at MHz repetition rates.

PACS: 05.45.Yv, 42.65.Tg, 42.65.Re, 42.81.Dp

The complex nonlinear Ginzburg-Landau equation (CGLE) is a basic nonlinear equation taking into account linear and nonlinear dispersion as well as linear and nonlinear dissipation that allows describing a variety of phenomena from nonlinear optical waves to Bose-Einstein condensation and field theory [1]. In a non-dissipative limit, the cubic CGLE gives the nonlinear Schrödinger equation (NLSE), which is completely integrable in one dimension [2]. An isolated solitary-wave solution of NLSE is called a soliton and is of interest for telecommunication, nonlinear and laser optics. The solitary pulse solutions of CGLE have been found, as well [3]. However, in the case of more general nonlinearity within the normal dispersion region (NDR), where there exists no the bright soliton of NLSE and the solitary pulse solutions of the cubic CGLE can suffer an explosive instability, the exact or approximated solitary-wave partial solutions of CLGE are known only for some restrictions imposed on the pulse phase [4, 5].

Numerical simulations have been revealed also a variety of the pulsing solutions of CGLE in NDR [6, 7]. Such breather-like regimes have, as a counterpart, the so-called self-similar pulses (SSP) propagating under the group-delay dispersion and the nonlinearity varying along propagation axis [8]. As it was demonstrated, SSP in NDR tolerates the strong nonlinearities that allows the compressible high-energy pulses from a fiber laser [9, 10]. This regime offers producing the high-energy femtosecond pulses directly from fiber [9] as well as solid-state [11 – 13] oscillators. Such oscillators are required for both scientific (e.g. high harmonics generation in gases) and technological (e.g. effective material modification at MHz pulse repetition rate) purposes.

Here we report a new type of an approximated but high-accuracy heavily-chirped solitary pulse (CSP) solution of the cubic-quintic CGLE. A closed analytical form of such solution allows tracing all basic characteristics of CSP in dependence on variation of the CGLE parameters. One can suppose, that CSP is highly interesting for laser and fiber optics. This results from its energy-scalability in NDR and possibility of compression down to femtosecond pulse duration. An energy-scalability of the compressible CSP is based on a simple physical ground. The pulse energy can increase with both its peak power and duration. However, the peak power growth is restricted from above by the nonlinear processes such as self-focusing, optical damage, etc. A single way to the higher energy is to increase the pulse duration T. For the soliton propagating in the anomalous dispersion region this means an inevitable decrease of its spectral width $\Delta \simeq 1/T$. On the contrary, it is possible to increase the CSP duration without loss of its spectral width in NDR: $\Delta \simeq 4f/T$, where $f \gg 1$ is the chirp. Then CSP can be compressed down to $\simeq 1/\Delta$ that results in its peak power growth ($\simeq f$ times). In contrast to the compressible chirped SSP [14], which is disrupted in a spectrally dissipative system [15], CSP propagates under the spectral filtering. This suits perfectly to an oscillator generating the spectrally-wide ($\approx$100 nm) pulses in NDR [12].

1)e-mail: Podivilov@iae.nsk.su

We base an analysis on the (1+1) dimensional cubic-quintic CGLE [3]:

$$\frac{dA}{dz} = \left(-\sigma + (\alpha + i\beta)\frac{d^2}{dt^2} + (\kappa - i\gamma)P - \kappa\zeta P^2\right) A, \tag{1}$$

where t and z are the local time and the propagation distance, respectively; $A(t,z)$ is the slowly varying field amplitude, so that $|A|^2$ means the power P. Parameter σ is the net-loss, which in an oscillator equals to a difference of the net-passive loss and the saturated gain. Parameters α and β are the square of the spectral filter bandwidth and the group-delay dispersion (GDD) coefficient, respectively. Parameters κ and γ are the self-amplitude modulation (SAM) and the self-phase modulation (SPM) coefficients, respectively. And at last, parameter ζ describes the saturation of SAM and confines the maximum power.

We shall search an approximated solutions of Eq. (1) for positive parameters and in the limit of $\alpha/\beta < \kappa/\gamma \ll 1$. This limit is well grounded for an oscillator operating in NDR. As an example, $\alpha <5$ fs^2 and $\beta \approx 100$ fs^2 for a Ti:sapphire oscillator of Refs. [12, 13, 16]. Simultaneously, $\kappa/\gamma \approx 0.04$ that is a typical value for the so-called Kerr-lens mode-locking regime [16, 17].

To find a stationary solution of Eq. (1), we use ansatz $A(t,z) = \sqrt{P(t)}\exp(i\phi(t) - iqz)$ that results in:

$$\begin{gathered}\gamma P(t) = q - \beta\Omega^2(t),\\ \beta\left(\frac{d\Omega}{dt} + \frac{\Omega(t)}{P(t)}\frac{dP}{dt}\right) = \kappa P(t)(1 - \zeta P(t)) - \sigma - \alpha\Omega^2(t).\end{gathered} \tag{2}$$

Here $\Omega(t) \equiv d\phi(t)/dt$ is the instant frequency. In Eqs. (2) the terms $\propto d^2\sqrt{P(t)}/dt^2$ are omitted in the framework of the adiabatic approximation ($\beta \simeq 10^{-4}$ ps^2 whereas $T \simeq 1$ ps for CSP [12]). In first Eq. (2) a smallness of the terms $\propto \alpha/\beta$ is used.

Since the power $P(t)$ is a positive defined value, Eqs. (2) give $\Omega(t)^2 < \Delta^2 \equiv q/\beta$ and the peak power $P_0 \equiv P(0) = q/\gamma = \beta\Delta^2/\gamma$. Regularity of the desired solution demands $d\Omega/dt < \infty$ that imposes a restriction on P_0:

$$\gamma P_0 = \beta\Delta^2 = \frac{3\gamma}{4\zeta}\left(1 - c/2 \mp \sqrt{(1-c/2)^2 - 4\sigma\zeta/\kappa}\right). \tag{3}$$

The parameter $c = \alpha\gamma/\beta\kappa$ means a contribution of spectral dissipation (dispersion) relatively to that of SAM (SPM). It is confined from above: $c < c_{\max} = 2 - 4\sqrt{\sigma\zeta/\kappa}$. This defines the maximum peak power: $P_0^{\max} = (3/4\zeta)(1 - c_{\max}/2) = (3/2)\sqrt{\sigma/\kappa\zeta}$. For a given c, the parameter σ is also confined from above: $\sigma < (\kappa/4\zeta)(1 - c/2)^2$.

One can see from Eq. (3), that the solution for P_0 with "+"-sign before a square root has not a finite asymptotic for $\zeta \to 0$. Hence, this solution has not a counterpart among the solutions of the cubic CGLE and therefore we shall not consider it henceforth.

The regularity condition simplifies also Eq. (2):

$$\begin{aligned}\frac{d\Omega}{dt} &= \frac{\beta\zeta\kappa}{3\gamma^2}(\Delta^2 - \Omega^2)(\Omega^2 + \Omega_L^2),\\ \beta\Omega_L^2 &= \frac{\gamma}{\zeta}(1 + c) - \frac{5}{3}\gamma P_0.\end{aligned} \tag{4}$$

The implicit solution of differential Eq. (4) with a zero asymptotic of the local power $P(t)$ at $t \to \pm\infty$ is:

$$t = \tau\left[\operatorname{arctanh}\left(\frac{\Omega}{\Delta}\right) + \frac{\Delta}{\Omega_L}\arctan\left(\frac{\Omega}{\Omega_L}\right)\right], \tag{5}$$

where $\tau = 3\gamma^2/(\zeta\beta\kappa\Delta(\Delta^2 + \Omega_L^2))$. Eq. (5) gives a time-profile of the instant frequency $\Omega(t)$ and, as a results of Eq. (2), a time-profile of the power $P(t)$. In the limit of the cubic CGLE ($\zeta \to 0$) this solution turns into the well-known solitary pulse solution $A \propto \cosh(t/\tau)^{i\ f - 1}$ [3]. The duration $T = 1.76\tau$ and the chirp f are close to the exact ones to an accuracy $\approx \alpha/\beta,\ \kappa/\gamma \ll 1$.

The spectral amplitude is $E(\omega) = \int dt\sqrt{P(t)} \times \exp(i\phi(t))\exp(-i\omega t)$. For CSP one can assume in the limit of $\gamma/\kappa \gg 1$, that the phase $\phi(t)$ is a rapidly varying function. Then $E(\omega)$ can be calculated by the method of stationary phase [18]. The resulting spectral power is:

$$p(\omega) \equiv |E(\omega)|^2 \simeq \frac{6\pi\gamma}{\zeta\kappa}\frac{\theta(\Delta^2 - \omega^2)}{\omega^2 + \Omega_L^2}, \tag{6}$$

where $\theta(x)$ is the Heaviside function. Eq. (6) allows finding the pulse energy:

$$\mathcal{E} = \int_{-\infty}^{\infty} dt P(t) = \int_{-\Delta}^{\Delta}\frac{d\omega}{2\pi}p(\omega) = \frac{6\gamma}{\zeta\kappa\Omega_L}\arctan\left(\frac{\Delta}{\Omega_L}\right). \tag{7}$$

One can see from Eqs. (3)–(5), (7), that the dimensionless values $\Delta' = \Delta\sqrt{\beta\zeta/\gamma}$, $\Omega_L' = \Omega_L\sqrt{\beta\zeta/\gamma}$, $\tau' = \tau\kappa/\sqrt{\beta\zeta\gamma}$ and $\mathcal{E}' = \mathcal{E}\ \kappa\sqrt{\zeta/\gamma\beta}$ completely define the CSP spectral and time profiles as well as its energy. These values depend on only two dimensionless parameters c and $b = \sigma\zeta/\kappa$.

Typical example of the variations of Δ', Ω_L' and normalized pulse width T' with c are shown in Fig.1 for $b = 11/64$. One can see, that the spectrum half-width

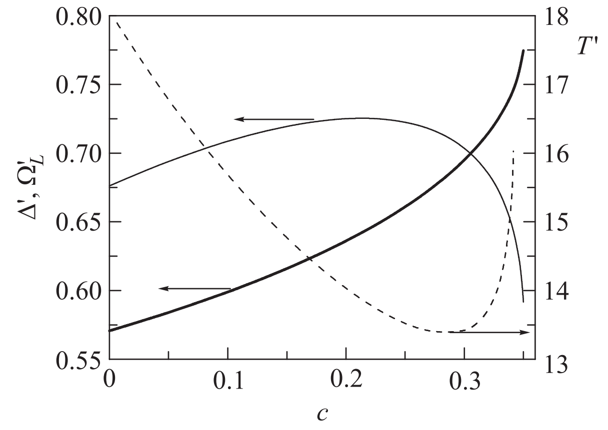

Fig.1. Variation of dimensionless spectral (Δ', solid thick curve) and Lorentzian (Ω'_L, thin curve) half-widths as well as CSP duration (T', dashed curve) with $c = \alpha\gamma/\beta\kappa$; $b = 11/64$

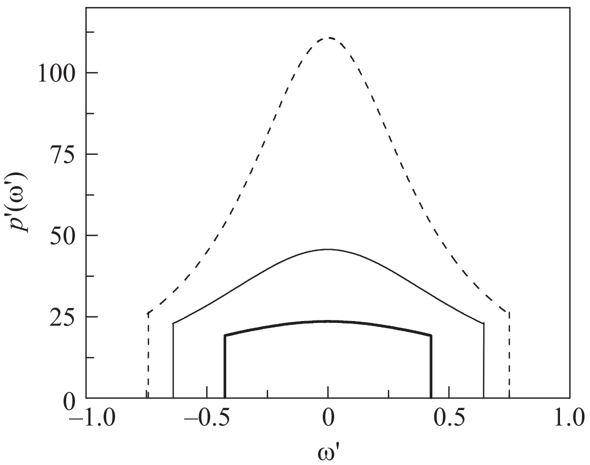

Fig.2. Normalized spectral powers of CSP. $b = 0.1$ (thick), 0.186 (thin) and 0.215 (dashed); $c = 0.1$

Δ' (thick curve) increases monotonically with c. In particular, this can be interpreted as the spectrum broadening with GDD approaching to zero in agreement with the experimental data of Ref. [13]. The Lorentzian half-width reaches a maximum and then shortens with the c growth (thin curve). As a result, a main part of $p(\omega)$ concentrates within the interval of $|\ \omega\ |< \Omega_L$ that provides a higher peak power P_0 close to the boundary of CSP existence. In agreement with [13], CSP duration decreases with c except to the narrow region close to $c_{\max}$ (dashed curve).

Importance of the parameter b is explained by the fact, that σ affects the CSP stability against the noise excitation. From Eq. (1), the minimum requirement for the pulse stability is $\sigma > 0$. The numerical simulations [16] have been demonstrated that the chirped pulse in NDR is stable except for the narrow regions close to the edges of the CSP existence region. In an oscillator, parameter σ is not quite independent value because it depends on the pulse energy due to effect of the gain saturation [19]. In this case the energy dependence of b can change noticeable the variation of CSP parameters with c.

A distinguishing feature of CSP is its spectral profile. Fig.2 shows the normalized powers $p' = p\kappa/\beta$ in dependence on the normalized frequency $\omega' = \omega\sqrt{\beta\zeta/\gamma}$. From Eq. (6), it is the Lorentzian function truncated at $\pm\Delta$. In fact, such truncation is slightly smoothed as the stationary phase approximation losses its validity close to $\pm\Delta$. Nevertheless, the spectral intensity vanishes very rapidly at the spectrum edges in the experiment, as well [9, 12, 13].

For a small b ($\sigma < 9\kappa/64\zeta$, $c_{\max} > 1/2$), the Lorentzian spectral half-width Ω_L is larger than Δ. As a result, the spectrum has a flat-top profile (thick curve) and a comparatively small area (energy). Also, this case corresponds to $\zeta \to 0$ (limit of the cubic CGLE). Within the region of a large b ($\sigma > 3\kappa/16\zeta$, $c_{\max} < 2 - \sqrt{3}$), $\Omega_L < \Delta$ and the spectrum has a finger-like profile and a maximum energy (dashed curve). In the intermediate region $9\kappa/64\zeta < \sigma < 3\kappa/16\zeta$, there exists some $c^* \equiv c = \sqrt{1 - 16b/3}$ providing $\Omega_L = \Delta$ (Fig.1) and forming a transitional parabolic-top spectrum (gray curve). As such spectrum is the most distant from the boundaries of the CSP existence, it possesses a maximum stability.

The chirp $Q(\omega)$ is defined as:

$$Q(\omega) = (1/2)\frac{d^2\psi(\omega)}{d\omega^2} \simeq \frac{3\gamma^2}{2\beta\kappa\zeta}\frac{1}{(\Delta^2 - \omega^2)(\Omega_L^2 + \omega^2)}, \tag{8}$$

where $\psi(\omega) = -\phi(t^*(\omega)) + \omega t^*(\omega)$ is the phase in spectral representation, the stationary point $t^*(\omega)$ is defined as a right-hand side of Eq. (5) with Ω replaced with ω. Fig.3 shows dependence of the normalized chirp $Q' = Q\kappa/\beta\zeta$ on ω'. The flat-top and parabolic-top spectra have a chirp minimum in the center (solid curves). At $c = c^*$ (or $b = 3(1 - c^2)/16$ for a fixed c) where the parabolic-top spectrum exists, the chirp variates most slowly with ω near the central frequency (thin curve). In contrast to the flat-top spectrum, this results from disappearance of the squared dependence on ω:

$$Q(\omega) \simeq \frac{3\gamma^2}{2\beta\kappa\zeta}\frac{1}{\Delta^4 - \omega^4}.$$

In this point the CSP peak power and its spectral half-width are:

$$\gamma P_0 = \beta\Delta^2 = \frac{3\gamma}{8\zeta}(1 + c^*).$$

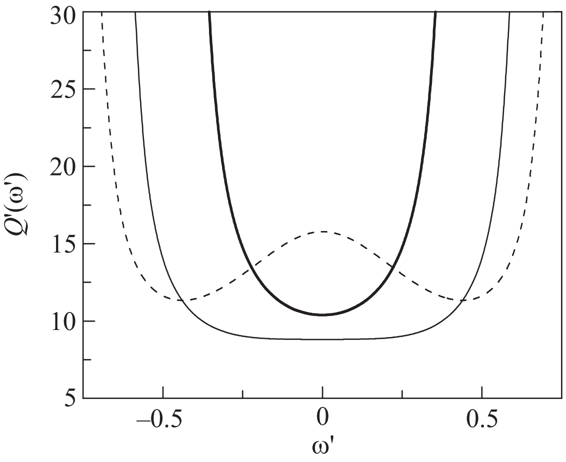

Fig.3. Normalized chirps $Q'(\omega')$ corresponding to the spectra in Fig.2

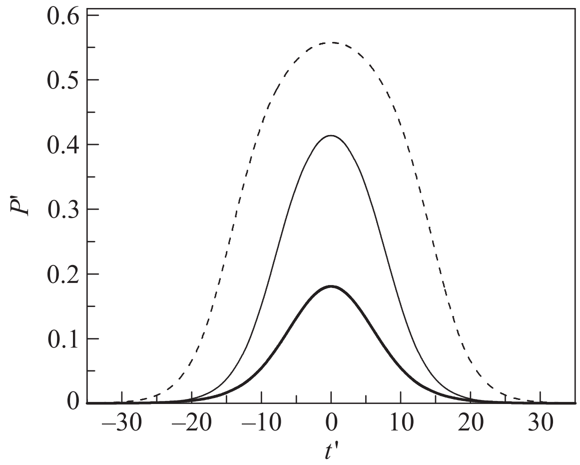

Fig.4. Normalized time profiles of CSP corresponding to the spectra in Fig.2

In contrast to the flat-top and parabolic-top spectra, the finger-like one has a local chirp maximum in the spectrum center (dashed curve).

For all spectral types of CSP the chirp increases rapidly at the spectrum edges. The frequency-dependence of the chirp limits its compressibility. A compressor with the dispersion $\simeq -Q(0)$ will produce the almost chirp-free pulse with the duration $\simeq 2/\Delta^* > 2/\Delta$. Here the reduced spectral half-width $\Delta^* < \Delta$ corresponds to the spectral region around $\omega = 0$, where the frequency dependence of the chirp is weak. Therefore the parabolic-top spectrum possessing a most weak frequency-dependence of the chirp is most compressible. In agreement with the experiment [12, 13], CSP with the picosecond duration can be compressed by the factor $T\Delta \simeq 3\gamma/\kappa \simeq 100$ that allows producing the femtosecond pulses with over-10 MW peak power at the MHz repetition rate.

The normalized time profiles are shown in Fig.4. CSP with flat-top spectrum has the solitonic-like time profile (thick curve). It is reasonable, because the limit of $\zeta \to 0$ (i.e. $b \to 0$) corresponds to the exact solitonic solution of the cubic CGLE. The finger-like spectrum close to the boundary of the CSP existence corresponds to the parabolic-like (or even flat-top) time profile (dashed curve). Such profile agrees with the numerical results of Refs. [4, 9] and is caused by the saturation of SAM around the pulse peak. The parabolic-top spectrum corresponds to the transitional time profile (thin curve).

The above described method of the approximate integration of the cubic-quintic CGLE can be applied successfully to the various modifications of CGLE. For example, CGLE with the SAM term $\kappa P/(1+\zeta P)$ is also integrable in the considered limits. The spectra in this case are also parabolic-top with the profiles:

$$p(\omega) \propto (\Delta^2 + A - \omega^2)/(\Delta^2 + B - \omega^2),$$

where $|\ \omega\ | < \Delta$; A, B and Δ are the positive-definite functions of the CGLE parameters.

In conclusion, the new method of integration of the nonlinear CGLE was proposed in the limits of domination of GDD over the spectral dissipation as well as SPM over SAM. These limits are valid for both fiber and solid-state oscillators operating within NDR. The proposed method was realized for the cubic-quintic CGLE, but it is applicable also to CGLE with a more general nonlinearity. The approximated but highly accurate heavily-chirped solitary pulse solution of CGLE was obtained. It was found, that CSPs have the flat-top, parabolic-top and finger-like spectra, which agree with the latest experimental data obtained from the high-energy oscillators. The parabolic-top spectrum corresponds to the most stable and compressible CSP. The wide spectrum of CSP in combination with its strong chirp allow compressing the pulse from picosecond down to femtosecond duration. As the micro-joule energy is reachable for CSP, the over-10 MW peak power is available from an oscillator operating at the MHz repetition rate.

Authors are grateful to Dr. A. Apolonski and Dr. A. Chernykh for valuable discussions concerning both experimental and theoretical aspects of the NDR regime. V.L.K acknowledges support from the Max-Planck Gesellschaft.

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