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On projection (in)dependence of monopole condensate in lattice $SU(2)$ gauge theory

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We study the temperature dependence of the monopole condensate in different Abelian projections of the $SU(2)$ lattice gauge theory in thermodynamic limit. Using the Fröhlich-Marchetti monopole creation operator we show numerically that the monopole condensate depends strongly on the choice of the Abelian projection. Contrary to the claims in the literature we observe that in the Abelian Polyakov gauge and in the field strength gauge the monopole condensate does not vanish at the critical temperature in large-volume limit. Therefore the monopole condensate in these gauges is not an order parameter of the confinement-deconfinement phase transition.

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1. Introduction. The confinement of color in QCD is one of the most interesting phenomena in the modern quantum field theory. Numerical simulations of non-Abelian gauge theories on the lattice [1] show that the confinement of quarks is due to a formation of the chromoelectric string spanned between quarks and antiquarks. Despite the color confinement is not understood from the first principles, there are various effective (or, phenomenological) models which describe the emergence of the QCD string. According to the dual superconductor model [2], the vacuum of a non-Abelian gauge theory is a medium of Abelian monopoles. The monopole condensate – this is present in the confinement phase – squeezes the chromoelectric flux coming from the quarks and the QCD flux tube is thus formed due to the dual Meissner effect. This flux tube is an analogue of the Abrikosov vortex in an ordinary superconductor.

The key element of the dual superconductor picture is the Abelian monopole. In the pure gauge theory the monopole neither exists as a finite-energy solution to the classical equations of motion, nor motivated by topological structure of the theory. However, if one fixes a specific gauge, then the monopole positions can be identified with the certain class of singularities in the gluonic fields. To this end one fixes the Abelian gauge which reduces the non-Abelian gauge freedom up to an Abelian one [3]. The residual Abelian gauge group is necessarily compact and it is the compactness of the residual Abelian subgroup that guarantees the existence of the Abelian monopoles in the Abelian projection.

It is impossible to identify the Abelian monopoles in a general configuration of the gluonic fields using analytical tools only. Therefore the most investigations of the dual superconductor idea is performed by numerical simulations. The results of the simulations indicate that the Abelian degrees of freedom in an Abelian projection are in fact responsible for the confinement of quarks (for a review, see, *e.g.*, Refs. [4]). One of the striking features of the Abelian projection is the effect of the Abelian dominance [5]: the Abelian gauge fields provide a dominant contribution to the tension of the confining string. Moreover, the internal structure of the string energy, such as energy profile and the field distribution are very well described by the dual superconductor model [1]. In particular, the monopole condensate in the Maximal Abelian (MA) gauge is formed in the low temperature (confinement) phase and the condensate disappears in the high temperature (deconfinement) phase [6, 7].

On the other hand, almost all results supporting the dual superconductor scenario were obtained in the so called MA projection [8]. Besides the MA projection there are Abelian projections which are defined by a diagonalization of certain adjoint operators $X[U]$ with respect to the $SU(2)$ gauge transformations [3]. The most popular examples of such gauges are the Abelian Polyakov (AP) gauge and the Abelian field strength gauge (F_{12} gauge).

The important issue is to understand whether the dual superconductor nature of non-Abelian vacuum is universally realized in all Abelian gauges. In this paper we study the universality of the dual superconductor mechanism in the thermodynamical limit extending our preliminary work [9], where the universality of the monopole condensation was tested in a finite volume. Our study is motivated by conflicting reports of the projection independence of the dual superconductor mech-

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anism. A short review of the current literature on this subject can be found in Ref. [10], and below we briefly outline a few key observations for and against universality of the monopole mechanism.

The universality of the mechanism is supported by the facts that (i) Abelian and Monopole dominance were observed in more than one gauge [5, 11]; (ii) the London penetration length measured in the MA projection is the same as the one obtained without gauge fixing [12]; (iii) monopole condensation studied with the help of a monopole creation operator was observed not only in the MA gauge of $SU(2)$ gauge theory [6, 7] but also in other gauges [13].

The arguments against the universality include: (i) in the MA gauge the monopole trajectories percolate only in the confinement phase contrary to the case without any gauge fixing, in which the monopoles are percolating in any phase [14]; (ii) the flux tube in different Abelian projections looks differently [15]; (iii) chiral condensate is dominated by the contributions of the Abelian monopoles in the MA gauge [16, 17] contrary to F_{12} [16] and AP [17] gauges; (iv) one can show analytically that in the AP gauge the dual superconductor mechanism can not be realized [10], while the condensation of the monopoles may still occur.

In this paper we investigate the universality hypothesis using the monopole creation operator introduced by Fröhlich and Marchetti in Ref. [18]. In Section 2 we describe how to obtain the monopole condensate using this monopole creation operator. In Section 3 we calculate numerically the condensate in the MA, F_{12} and AP gauges in the thermodynamical limit. Our conclusions are summarized in the last Section.

2. Abelian monopole creation operator in $SU(2)$ model. We study the $SU(2)$ gauge theory with the standard Wilson action, $S[U] = -1/2(\sum_P \mathrm{Tr}\, U_P)$, where the sum goes over the plaquettes P and U_P is the $SU(2)$ plaquette variable composed of the link fields U_l, $U_P = U_1 U_2 U_3^\dagger U_4^\dagger$. The link field is parameterized in the standard way:

$$U_{x\mu} = \begin{pmatrix} \cos\phi_{x\mu}\, e^{i\theta_{x\mu}} & \sin\phi_{x\mu}\, e^{i\chi_{x\mu}} \\ -\sin\phi_{x\mu}\, e^{-i\chi_{x\mu}} & \cos\phi_{x\mu}\, e^{i\theta_{x\mu}} \end{pmatrix},$$

$0 \le \phi \le \pi/2$ and $-\pi < \theta, \chi \le \pi$.

In Abelian projection the residual gauge transformation matrices have the diagonal form $\Omega^{\mathrm{Abel}}(\omega) = \mathrm{diag}(e^{i\omega}, e^{-i\omega})$, where ω is an arbitrary scalar function. Under these transformations the diagonal field θ transforms as an Abelian gauge field, $\theta_{x\mu} \to \theta_{x\mu} + \omega_x - \omega_{x+\hat\mu}$, the off-diagonal field χ changes as a double charged matter field, $\chi_{x\mu} \to \chi_{x\mu} + \omega_x + \omega_{x+\hat\mu}$, the field ϕ remains intact. The $SU(2)$ plaquette action contains [19] various interactions between these fields as well as the action for the Abelian gauge field θ:

$$S[U] = -\sum_P \beta_P(\phi) \cos\theta_P + \dots \qquad (1)$$

Here $\theta_P = \theta_1 + \theta_2 - \theta_3 - \theta_4$ is a lattice analogue of the Abelian field strength tensor and $\beta_P(\phi)$ is an effective coupling constant dependent of the fields ϕ, Ref. [19].

Following Ref. [6] we apply the monopole creation operator of Fröhlich-Marchetti [18] to the Abelian part of the non-Abelian action (1). Effectively, this operator shifts the Abelian plaquette variable θ_P as follows:

$$\Phi_{mon} = \exp\Bigl\{\sum_P \beta_P(\phi)\Bigl[-\cos\theta_P + \cos(\theta_P + W_P)\Bigr]\Bigr\}, \qquad (2)$$

where $W_P = 2\pi\delta\Delta^{-1}(D_x - \omega_x)$, ω_x is a Dirac string attached to the monopole and the Dirac cloud D_x satisfies the equation $\delta^* D_x = {}^*\delta_x$. We have used the differential form notations on the lattice described in detail in the second paper in Ref. [4].

The operator (2) is clearly gauge invariant with respect to the $U(1)$ gauge transformations. One can also perform a formal duality transformation with respect to the quantum average of the operator (2) and show that in the dual model – which has form of the Abelian Higgs model – this operator is invariant under the (dual) gauge transformations [6]. Moreover, one can represent the partition function as a sum over closed monopole trajectories. In this representation the quantum average of the monopole creation operator $\Phi_{\mathrm{mon}}(x)$, Eq. (2), is given by a sum over all closed monopole trajectories plus one open trajectory which begins at the point x, Refs. [18]. Thus, this operator creates a monopole at the point x.

Note that in this paper we are using the "old" definition [18] of the monopole creation operator. The "new" definition [20] takes into account charged matter fields but it is very involved from the point of view of numerical calculations. Moreover, results of Ref. [21] clearly show that there is no qualitative difference between the old and the new definitions.

To get the monopole condensate we have to study the effective constraint potential for the monopole creation operator Φ_{mon},

$$V_{\mathrm{effc}}(\Phi) = -\ln\Bigl[\Bigl\langle\delta\Bigl(\Phi - \frac{1}{V}\sum_x \Phi_{\mathrm{mon}}(x)\Bigr)\Bigr\rangle\Bigr]. \qquad (3)$$

This potential selects the zero-momentum component of the creation operator. The minimum of this potential corresponds to the monopole condensate. However, a numerical calculation of the potential $V_{\rm effc}(\Phi)$ is time consuming, and in this paper we present results for the probability distribution

$$V(\Phi) = -\ln\Big[\Big\langle \delta\Big(\Phi - \Phi_{\rm mon}(x)\Big)\Big\rangle\Big], \qquad (4)$$

which has a meaning very similar to (3).

We perform our study in the Abelian Polyakov gauge and in the Abelian field strength gauge which are defined as we already discussed by the diagonalization of the (untraced) Polyakov loop, $P_x[U] = U_{x,4}U_{x+\hat{4},4}\dots U_{x-\hat{4},4}$, and of the $U_{x,12}$ plaquette, with respect to gauge transformations, $U_{x,\mu} \to \Omega_x U_{x,\mu}\Omega^\dagger_{x+\hat{\mu}}$.

We compare the potential in these gauges with the monopole potential obtained in the MA gauge in Ref. [6]. The MA gauge is defined by the maximization of the lattice functional

$$R_{MA}[U] = \sum_{s,\hat{\mu}} \mathrm{Tr}\Big(\sigma_3 U(s,\mu)\sigma_3 U^\dagger(s,\mu)\Big), \qquad (5)$$

with respect to gauge transformations. Here σ_3 is the Pauli matrix.

3. Numerical results. We simulate the $SU(2)$ gauge fields on the lattices $L_s^3\times 4$, $L_s = 12, 14, 16, 24$ with C-periodic boundary conditions in space directions [22]. The C-periodicity for non-Abelian fields corresponds to the anti-periodicity for the Abelian gauge fields. We need these conditions because one can not create the charged particle in a finite volume with periodic boundary conditions (the Gauss law) [6]. In the case of $SU(2)$ gauge group the C-periodic boundary conditions as defined by appropriate matching conditions at the space boundary: $U_{x,\mu} \to \Omega^+ U_{x,\mu}\Omega$, $\Omega = i\sigma_2$.

The effective potential in the AP and F_{12} gauges was calculated using 400 independent configurations of $SU(2)$ gauge fields for each value of the gauge coupling β at a fixed lattice volume. On each configuration the distribution of the monopole creation operator is evaluated in 20 points. The effective potential (4) is the logarithm (taken with "minus" sign) of the distribution function. The results for the MA gauge are taken from [6].

The statistical errors of the data for the potential is evaluated using the bootstrap method. First, we calculate the so-called "initial ensemble" of the values of the monopole creation operator. Second, we randomly take these values and construct $N_{\rm ens}$ additional ensembles (typically, $N_{\rm ens} = 500$). Note that any given element of the initial ensemble may enter the constructed ensembles more than one time. The number of entries in each of the additional ensembles is fixed to be the same as in the initial one. Third, for each ensemble we construct a histogram, the (minus) logarithm of which has a meaning of the monopole potential, $V(\Phi)$, according to Eq. (4). Therefore, for each value of the monopole field, Φ, we get $N_{\rm ens}$ values of the potential, V, distributed as a Gaussian. The central value of this distribution gives us the value of the potential at given lattice volume $L_s^2 \times L_t$, β and Φ, $V = V(\Phi)$, while the width of the distribution provides us with the statistical error.

The effective monopole potentials in the MA, AP and F_{12} gauges on $16^3 \times 4$ lattice were previously investigated in Ref. [9]. It was clearly observed that in the AP and F_{12} gauges the global minimum $\Phi_{\rm min}$ of the effective potentials – which corresponds to the value of the monopole condensate – is the same for these gauges within numerical errors. Moreover, the potential in the MA gauge is different from AP and F_{12} potentials. In the strong coupling regime the condensates in the three gauges are the same, $\Phi_{\rm min} \approx 1$. In the deep deconfinement phase the monopole condensate tend to vanish in MA gauge while in AP and F_{12} gauges the condensate is still non-zero. Thus, there is a clear evidence of the failure of the universality hypothesis.

In order to make the qualitative conclusion of Ref. [9] strict, one needs to perform an extrapolation of the results to the thermodynamical limit. In fact, in the $SU(2)$ gauge theory the phase transition is of the second order, so that the finite volume effects may be essential for the determination of the monopole condensate. In an unfortunate case the results of Ref. [9] may be spoiled by the strong volume dependence at the transition temperature.

We calculate the condensate on the lattices with $L_s = 12, 14, 16, 24$ spatial extensions and perform the extrapolation $L_s \to \infty$ using the formula:

$$\Phi_c = \Phi_c^{\rm inf} + C/L. \qquad (6)$$

The examples of the fits for the AP and the F_{12} gauges are shown in Fig.1. The values of $\chi^2/d.o.f.$ are in the range $0.2 \sim 1$.

The monopole condensates in the thermodynamic limit ($L_s \to \infty$) for all three Abelian projections are shown in Fig.2 as functions of β. One can clearly see that the monopole condensate in the MA projection vanishes at a certain critical $\beta = \beta_c$ which is very close to the phase transition point, $\beta_c \approx 2.3$. Contrary to the MA gauge the monopole condensates obtained in the AP and the F_{12} gauges do not vanish at $\beta = \beta_c$. This result is in contradiction with observations of Ref. [13].

The dependence of the monopole condensate on β can be fitted by the following function:

$$\Phi_c^{\rm inf}(\beta) = 1 - (\beta/\beta_c)^\gamma, \qquad (7)$$

with $\chi^2/d.o.f. \approx 0.3$. It occurs that $\gamma = 1.2(5)$ and $\beta_c = 2.31(3)$. The value of β_c coincides within error

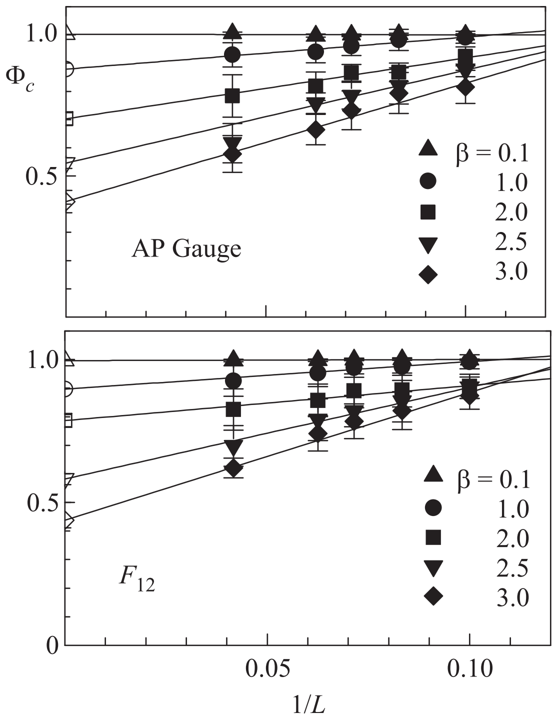

Fig.1. The extrapolation (6) of the monopole condensate to the thermodynamic limit in (top) the AP and (bottom) the F_{12} gauges for various values of the gauge coupling β and $L_s = 12, 14, 16, 24$

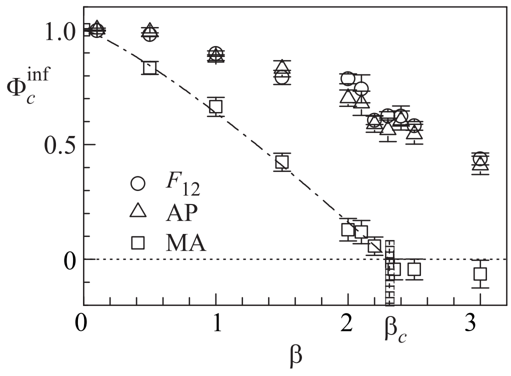

Fig.2. The monopole condensate in the thermodynamic limit in the MA, the AP and the F_{12} gauges. The dash–dotted line is the fit of the monopole condensate in the MA gauge by Eq. (7). The critical value of the gauge coupling (along with the numerical error) is denoted by the vertical dashed line

bars with the known critical value [23] on $L_s^3 \times 4$ lattices.

4. Conclusion. We have presented a clear evidence of non-universality of the dual superconductor mechanism in the thermodynamic limit. Our results confirm the conclusion of Ref. [9] made in a finite-volume case. We show that the value of the monopole condensate depends on the choice of the Abelian projection. The independence was found only in the unphysical strong coupling region. We studied three the Polyakov, the field strength and the Maximal Abelian gauges, and the proper behavior of the condensate was observed only in the MA gauge. Our conclusion contradicts the results of Ref. [13] where condensate was found to be projection–independent.

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1. V. Singh, D. A. Browne, and R. W. Haymaker, Phys. Lett. B **306**, 115 (1993); G. S. Bali, C. Schlichter, and K. Schilling, Phys. Rev. D **51**, 5165 (1995); G. S. Bali, C. Schlichter, and K. Schilling, Prog. Theor. Phys. Suppl. **131**, 645 (1998); F. V. Gubarev, E. M. Ilgenfritz, M. I. Polikarpov, and T. Suzuki, Phys. Lett. B **468**, 134 (1999); Y. Koma, M. Koma, E. M. Ilgenfritz et al., Phys. Rev. D **68**, 094018 (2003); Y. Koma, M. Koma, E. M. Ilgenfritz, and T. Suzuki, Phys. Rev. D **68**, 114504 (2003).
2. G. 't Hooft, in *High Energy Physics*, Ed. A. Zichichi, EPS Intern. Conf., Palermo, 1975; S. Mandelstam, Phys. Rep. **23**, 245 (1976).
3. G. 't Hooft, Nucl. Phys. B **190**, 455 (1981).
4. T. Suzuki, Nucl. Phys. Proc. Suppl. **30**, 176 (1993); M. N. Chernodub and M. I. Polikarpov, *Abelian projections and monopoles*, in *Confinement, duality, and non-perturbative aspects of QCD*, Ed. P. van Baal, Plenum Press, (1998) p. 387; hep-th/9710205; R. W. Haymaker, Phys. Rept. **315**, 153 (1999).
5. T. Suzuki and I. Yotsuyanagi, Phys. Rev. D **42**, R4257 (1990); H. Shiba and T. Suzuki, Phys. Lett. B **333**, 461 (1994); G. S. Bali, V. Bornyakov, M. Müller-Preussker, and K. Schilling, Phys. Rev. D **54**, 2863 (1996).
6. M. N. Chernodub, M. I. Polikarpov, and A. I. Veselov, Phys. Lett. B **399**, 267 (1997); Nucl. Phys. Proc. Suppl. **49**, 307 (1996).
7. A. Di Giacomo and G. Paffuti, Phys. Rev. D **56**, 6816 (1997).
8. A. S. Kronfeld, M. L. Laursen, G. Schierholz, and U. J. Wiese, Phys. Lett. B **198**, 516 (1987); A. S. Kronfeld, G. Schierholz, and U. J. Wiese, Nucl. Phys. B **293**, 461 (1987).
9. V. A. Belavin, M. N. Chernodub, and M. I. Polikarpov, JETP Lett. **79**, 245 (2004).
10. M. N. Chernodub, Phys. Rev. D **69**, 094504 (2004).
11. J. D. Stack, S. D. Neiman, and R. J. Wensley, Phys. Rev. D **50**, 3399 (1994); Y. Matsubara, S. Ilyar, T. Okude et al., Nucl. Phys. Proc. Suppl. **42**, 529 (1995); S. Ejiri, S. I. Kitahara, Y. Matsubara et al., Nucl. Phys. Proc. Suppl. **47**, 322 (1996); A. J. van der Sijs, Nucl. Phys.

Proc. Suppl. **73**, 548 (1999); S. Fujimoto, S. Kato, T. Suzuki, and T. Tsunemi, Prog. Theor. Phys. Suppl. **138**, 36 (2000).
12. P. Cea and L. Cosmai, Phys. Lett. B **349**, 343 (1995).
13. A. Di Giacomo, B. Lucini, L. Montesi, and G. Paffuti, Phys. Rev. D **61**, 034503, 034504 (2000).
14. A. I. Veselov and M. A. Zubkov, unpublished (1998).
15. K. Bernstein, G. Di Cecio, and R. W. Haymaker, Phys. Rev. D **55**, 6730 (1997).
16. R. M. Woloshyn, Phys. Rev. D **51**, 6411 (1995).
17. F. X. Lee, R. M. Woloshyn, and H. D. Trottier, Phys. Rev. D **53**, 1532 (1996).
18. J. Fröhlich and P. A. Marchetti, Commun. Math. Phys. **112**, 343 (1987).
19. M. N. Chernodub, M. I. Polikarpov, and A. I. Veselov, Phys. Lett. B **342**, 303 (1995).
20. J. Frohlich and P. A. Marchetti, Phys. Rev. D **64**, 014505 (2001).
21. V. A. Belavin, M. N. Chernodub, and M. I. Polikarpov, Phys. Lett. B **554**, 146 (2003); JETP Lett. **75**, 217 (2002).
22. U. J. Wiese, Nucl. Phys. B **375**, 45 (1992).
23. J. Engels, J. Fingberg, and M. Weber, Nucl. Phys. B **332**, 737 (1990).