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Next-to-leading order corrections to Bhabha scattering in renormalization group approach (I). Soft and virtual photonic contributions

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Soft and virtual loop photonic contributions to the second order next-to-leading QED radiative corrections to Bhabha scattering are calculated with help of the renormalization group approach. The results are in agreement with earlier calculations, where other methods were used. Scale dependence and the present theoretical accuracy of Bhabha cross section description are discussed.

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1. Introduction. Precision theoretical predictions for the differential cross section of Bhabha scattering are of ultimate importance for all experiments at electron-positron colliders. They are required for normalization purposes including luminosity determination, for several searches of new physics, and as a background contribution to many other processes studied at the colliders. To provide the required accuracy of the predictions, one should take into account radiative corrections in the first and higher order of perturbative QED. Certain effects of strong and weak interactions should be included as well.

It is natural to expand the QED part of radiative corrections to Bhabha scattering into a series in the fine structure constant α and in powers of the so-called large logarithm $L = \ln(M^2/m_e^2)$, where M is a large energy scale related to the beam energy, $M \gg m_e$. The terms enhanced by the large logs are known to give the bulk of the result in the kinematical regions of interest for experiments.

In this paper we present the derivation of the second order next-to-leading virtual and soft photonic radiative corrections to the cross section of Bhabha scattering. To get the corrections we use the renormalization group techniques borrowed from QCD. The result is found to be in agreement with earlier calculations [1, 2]. The advantage of our approach is its universality: in the same way one can get all the other remaining contributions to the radiative corrections in $\mathcal{O}(\alpha^2 L)$. For the case of small angle Bhabha scattering the complete result for the $\mathcal{O}(\alpha^2 L)$ is known [3], but for the general case of large angle scattering terms of this order are still not systematized, while there is a number of results for particular contributions scattered in the literature.

The paper is organized as follows. The notation is introduced in the next section. The application of the structure function approach is given in Sect. 3. The numerical results are presented in several plots followed by a short discussion and conclusions.

2. Preliminaries. Let us represent the differential Bhabha cross section as a series in α:

$$d\sigma = d\sigma^{\rm Born} + d\sigma^{(1)} + d\sigma^{(2)} + \mathcal{O}(\alpha^3), \qquad (1)$$

where $d\sigma^{\rm Born}$ represents the Born level cross section, and $d\sigma^{(1,2)}$ are the pure QED contributions of the first and second order corrections.

The first order contribution is usually decomposed into three parts:

$$d\sigma^{(1)} = d\sigma^{\rm V} + d\sigma^{\rm S} + d\sigma^{\rm H}, \qquad (2)$$

where superscripts V, S, and H are used to denote the virtual, soft, and hard photonic corrections, respectively. The small parameter Δ $(\Delta \ll 1)$ subdivides the kinematical domain of real photon emission into soft and hard parts with photon energy below and above $\Delta{\cdot}E_{\rm beam}$, where $E_{\rm beam}$ is the beam energy. The one-loop contributions are well known, see i.e. Refs. [4, 1, 5].

In the second order we construct a similar decomposition:

$$d\sigma^{(2)} = d\sigma^{\rm VV} + d\sigma^{\rm SV} + d\sigma^{\rm SS} + d\sigma^{\rm VH} + d\sigma^{\rm SH} + d\sigma^{\rm HH}, \qquad (3)$$

where the superscripts have the same meaning as in Eq. (2), so that for instance VH denotes the contribution due to emission of one hard photon accompanied by the effect of a single virtual loop.

In this paper we don't take into account the so-called pair contributions, related to emission of real or virtual

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pairs (e^+e^-, $\pi^+\pi^-$ etc.). Their numerical contribution to the observed cross section is typically small compared to the photonic correction, see Ref. [6, 7].

The second order contribution $d\sigma^{(2)}$ is decomposed into series in the powers of the large logarithm. There in particular contributions we meet terms of the orders $\mathcal{O}(\alpha^2 L^{4,3,2,1,0})$. The terms with the fourth and the third powers of L will cancel out in the sum of virtual and soft photon contributions.

Using the factorization properties of soft photon radiation, we can write immediately the Soft-Soft (SS) and Soft-Virtual (SV) contributions [8]:

$$d\sigma^{\rm SS} = \frac{1}{2!}(\delta^{\rm S})^2 d\sigma^{\rm Born}, \quad d\sigma^{\rm SV} = \delta^{\rm S}\delta^{\rm V} d\sigma^{\rm Born}, \qquad (4)$$

where $\delta^{\rm S,V} = d\sigma^{\rm S,V}/d\sigma^{\rm Born}$. Note that for the contribution of double soft photon emission, $d\sigma^{\rm SS}$, in the formula above we apply an upper limit on the energy of each of the photons independently. The Virtual-Virtual (VV) contribution can't be received in such a simple manner. Below we will show how to reconstruct the logarithmically enhanced part of it using the renormalization group techniques or in other words the electron structure function approach.

3. Structure Function Approach. The structure (fragmentation) function approach, widely used in QCD can be applied to QED problems [9–12]. With help of it we can analytically find the most important contributions reinforced by the large logarithm L, since they can be treated as electron mass singularities.

We are going to drop the pair contributions, so we need here the pure photonic part of the non-singlet structure (fragmentation) functions for the initial (final) state corrections. These functions describe the probability to find a massless (massive) electron with energy fraction z in the given massive (massless) electron. In our case with the next-to-leading accuracy we have

$$\begin{aligned} \mathcal{D}_{ee}^{\rm str,frg}(z) &= \delta(1-z) + \\ &+ \frac{\alpha}{2\pi}\Big(d_1(z,\mu_0,m_e) + LP^{(0)}(z)\Big) + \left(\frac{\alpha}{2\pi}\right)^2 \times \\ &\times \bigg(\frac{1}{2}L^2 P^{(0)}\otimes P^{(0)}(z) + LP^{(0)}\otimes d_1(z,\mu_0,m_e) + \\ &+ LP_{ee}^{(1,\gamma)\rm str,frg}(z)\bigg) + \mathcal{O}(\alpha^2L^0,\alpha^3), \end{aligned} \qquad (5)$$

where the superscripts "str" and "frg" are used to mark the structure and fragmentation functions, respectively. The difference between the functions appear only due to the difference in the next-to-order splitting functions $P^{(1,\gamma)}$. The modified minimal subtraction scheme $\overline{\rm MS}$ is used. We have chosen the factorizations scale equal to M, and the renormalization scale μ_0 will be taken equal to m_e. More details on application of the approach to calculation of second order next-to-leading QED corrections can be found in Refs. [10, 12].

The function d_1 corresponds to next-to-leading order initial condition in the $\overline{\rm MS}$ scheme for iterative solutions of the QED DGLAP equation. It reads

$$d_1(z,\mu_0,m_e) = \left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu_0^2}{m_e^2} - 2\ln(1-z) - 1\right)\right]_+, \qquad (6)$$

where the usual plus prescription is exploited to regularize the singularity at $z\to 1$. The relevant lowest order splitting function is

$$P^{(0)}(z) = \left[\frac{1+z^2}{1-z}\right]_+. \qquad (7)$$

The next-to-leading order functions $P_{ee}^{(1,\gamma)}$ is provided by the set of Feynman diagrams with pure photonic corrections. Since at the present moment we are interested only in the soft and virtual part of the corrections, we need only the so-called Δ-parts of the functions, which coincide:

$$\left(P_{ee}^{(1,\gamma)\rm frg,str}\right)_\Delta = \frac{3}{8} - 3\zeta(2) + 6\zeta(3). \qquad (8)$$

So in what follows, we can drop the superscripts distinguishing the structure and fragmentation functions.

The master formula describing the radiatively corrected Bhabha cross section in the structure function approach reads [5]:

$$\begin{aligned} d\sigma &= \int_{\bar z_1}^1 dz_1 \int_{\bar z_2}^1 dz_2 \mathcal{D}_{ee}^{\rm str}(z_1)\mathcal{D}_{ee}^{\rm str}(z_2) \times \\ &\times \Big(d\sigma^{\rm Born}(z_1,z_2) + d\bar\sigma^{(1)}(z_1,z_2) + \mathcal{O}(\alpha^2L^0)\Big) \times \\ &\times \int_{\bar y_1}^1 \frac{dy_1}{Y_1}\int_{\bar y_2}^1 \frac{dy_2}{Y_2}\mathcal{D}_{ee}^{\rm frg}\left(\frac{y_1}{Y_1}\right)\mathcal{D}_{ee}^{\rm frg}\left(\frac{y_2}{Y_2}\right), \end{aligned} \qquad (9)$$

where $d\bar\sigma^{(1)}$ is the $\mathcal{O}(\alpha)$ correction to the massless Bhabha scattering, calculated using the $\overline{\rm MS}$ scheme to subtract the lepton mass singularities. Energy fractions of incoming partons are $z_{1,2}$, and $Y_{1,2}$ are the energy fractions of the outcoming electron and positron.

Here we are interested in the contributions due to virtual and soft photons, so all the four integrals will have the same lower limit being equal to $1-\Delta$. First we can perform convolution of the four structure functions entering Eq. (9) with each other:

$$D^{\otimes 4}(z) = \delta(1-z) + 4\frac{\alpha}{2\pi}\left(d_1(z) + LP^{(0)}(z)\right) + \\ + 4\left(\frac{\alpha}{2\pi}\right)^2\left(2L^2(P^{(0)}(z))^{\otimes 2} + LP^{(1)}(z) + \\ + 4L\, d_1(z)\otimes P^{(0)}(z)\right) + \mathcal{O}(\alpha^2 L^0, \alpha^3), \qquad (10)$$

with a short notation for multiple convolution operation: $(A)^{\otimes n} \equiv \underbrace{A\otimes A\otimes A\cdots A}_{n}$. If $z = 1-\Delta$ and $\Delta \ll 1$, this function gives the probability to find such a situation where one looses in total due to photon emission ΔE_{beam} from the total energy of the process under consideration (the center-of-mass reference frame is used throughout the paper).

Let us fix now the factorization scale $M = \sqrt{s}$ and define $L_s \equiv \ln(s/m_e^2)$, $s = 4E_{\text{beam}}^2$. Later on we will consider another choice of the scale.

Convolution of the function found above with the Born part of the kernel cross section gives us the corresponding part to the cross section (with the upper limit on the energy lost):

$$\int_{1-\Delta}^{1} \mathcal{D}^{\otimes 4}(z) d\sigma^{\text{Born}}(z) dz = \\ = d\sigma^{\text{Born}}\Bigg\{1 + \frac{\alpha}{2\pi}\left[4L_s\left(2\ln\Delta + \frac{3}{2}\right) + \mathcal{O}(L_s^0)\right] + \\ + \left(\frac{\alpha}{2\pi}\right)^2\left[8L_s^2\left(P^{(0)}\right)_\Delta^{\otimes 2} + 16L_s(P^0\otimes d_1)_\Delta + \\ + 4L_s P_\Delta^{(1)}\right] + \mathcal{O}(\alpha^2 L_s^0, \alpha^3)\Bigg\}, \qquad (11)$$

where we used subscript Δ to specify the so-called Δ-part of the corresponding function (see i.e. Refs. [11]).

Convolution with the $d\bar{\sigma}^{(1)}$ is more complicated since the latter is a non-trivial function of z. But we restricted ourselves to consider only the terms reinforced by the large logarithm. Therefore we need to compute only the following part:

$$4\frac{\alpha}{2\pi}L_s\int_{1-\Delta}^{1} dz[P^{(0)}\otimes d\bar{\sigma}^{(1)}](z). \qquad (12)$$

Using the techniques of dealing with the singular functions regularized by introduction of the Δ and Θ parts [11, 13] we cast it into the following form:

$$4\frac{\alpha}{2\pi}\int_{1-\Delta}^{1} dy \int_0^1 \frac{dz}{z}\, L_s P^{(0)}\left(\frac{y}{z}\right) d\bar{\sigma}^{(1)}(z) = \\ = 4\frac{\alpha}{2\pi}\, d\bar{\sigma}_\Delta^{(1)} L_s P_\Delta^{(0)} + 4\frac{\alpha}{2\pi}\, L_s \delta(\bar{\sigma}^{(1)}), \qquad (13)$$

where

$$d\bar{\sigma}_\Delta^{(1)} = \int_{1-\Delta}^{1} d\bar{\sigma}^{(1)}(z)dz = d\sigma^{\text{V}} + d\sigma^{\text{S}} - d\sigma^{\text{Born}}\frac{\alpha}{2\pi}4L_s P_\Delta^{(0)}, \\ \delta(\bar{\sigma}^{(1)}) = \int_{1-\Delta}^{1} d\bar{\sigma}^{(1)}(z)\; 2\,\ln\left(1 - \frac{1-z}{\Delta}\right)dz. \qquad (14)$$

Let us note that the last non-trivial term the function under the integral above is not singular for $z \to 1$. So the virtual loop contribution to the function $d\bar{\sigma}^{(1)}$ doesn't contribute to the result of the integral. And we can put the upper limit of the integral over z to be equal to $1-\Delta_1$, $\Delta_1 \ll \Delta \ll 1$. Therefore we need to consider only the soft photon part of the function:

$$\frac{d\bar{\sigma}^{(1,\text{S})}(z)}{dz} = d\sigma^{\text{Born}}\Bigg\{-\frac{\alpha}{4\pi^2}\frac{d|\mathbf{k}|}{dz}\frac{|\mathbf{k}|^2}{\sqrt{|\mathbf{k}|^2+\lambda^2}} \times \\ \times\left[\int_0^{2\pi} d\phi \int_{-1}^{1} dc\left(\frac{p_{e^+}}{p_{e^+}k} - \frac{p_e}{p_e k} + \frac{p'_e}{p'_e k} - \frac{p'_{e^+}}{p'_{e^+}k}\right)^2\right] - \\ - \frac{\alpha}{2\pi}4\left[d_1(z) + L_s P^{(0)}(z)\right]\Bigg\}, \qquad (15)$$

where $1-z = |\mathbf{k}|/E_{\text{beam}}$; λ is a fictitious photon mass, $\lambda \ll m_e$; p_e (p_{e^+}) is momentum of incoming electron (positron), p'_e, p'_{e^+}, and k are the momenta of the outgoing electron, positron, and the photon. Using the standard techniques of calculations of soft photon contributions we get

$$\frac{\delta(\bar{\sigma}^{(1)})}{d\sigma^{\text{Born}}} = 4\frac{\alpha}{2\pi}\left[4\zeta(2)\ln\frac{1+c}{1-c} + 8\Big(\zeta(3) - \zeta(2)\ln\Delta\Big)\right],$$

where c is the cosine of the electron scattering angle, $c = \cos\widehat{\mathbf{p}_e\mathbf{p}'_e}$.

Summing up the contributions in Eq. (13) and then summing up Eqs. (4), (11, (13) we receive the leading and next-to-leading second order contributions to quasi-elastic Bhabha cross section, where the total energy loss (due to soft photon emission) is limited by ΔE_{beam}. It is useful to describe also the case when the energies of the soft photons (if they are two) are limited independently. The transition between the two cases was derived in Ref. [8]. Applying it we get the final result:

$$d\sigma^{\text{VV}} + d\sigma^{\text{SV}} + d\sigma^{\text{SS}} = \left(\frac{\alpha}{2\pi}\right)^2 d\sigma^{\text{Born}} \times \\ \times\Bigg\{L_s^2\left[32\ln^2\Delta + 48\ln\Delta + 18\right] + \\ + L_s\left[64\left(\ln\left(\frac{x}{1-x}\right) - 1\right)\ln^2\Delta + \right.$$

$$+ 16\Bigg(2\left(\mathrm{Li}_2\left(1-x\right)-\mathrm{Li}_2\left(x\right)\right)+3\ln\left(\frac{x}{1-x}\right)+ \\ + f(x) - 7 - \frac{2\pi^2}{3}\Bigg)\ln\Delta + 24\mathrm{Li}_2\left(1-x\right) - \\ - 24\mathrm{Li}_2\left(x\right)+12f(x)+24\zeta(3)-\frac{93}{2}-10\pi^2\Bigg] + \\ + 4\delta_0^{(2)} + \mathcal{O}\left(\frac{m_e^2}{s}\right)\Bigg\}, \qquad x \equiv \frac{1-c}{2}, \qquad (16)$$

function $f(x)$ can be found in Ref. [1]. In the formula above besides the logarithmically enhanced terms derived here, we included also the known contribution without the large logs, $\delta_0^{(2)}$, which is given by Eq. (3) from Ref. [2].

4. Numerical Results and Conclusions. In this way we received the photonic part of the second order next-to-leading logarithmic contribution to Bhabha cross section. Some more details of our calculations and numerical results can be found in the preprint [14]. The result agree with earlier calculations by means of different methods. Our approach allows to get all the next-to-leading contributions systematically. It can be applied to any kind of a process, where one has to look for the radiative corrections enhanced by large logarithms. In particular, we applied the same approach to the description of the contribution of real photon emission to Bhabha scattering [to be described elsewhere].

Let us compare the numerical values of the leading, next-to-leading, and next-to-next-to-leading corrections for two choices of the factorization scale: $M = \sqrt{s}$, which has been used in Refs. [1, 2], and $M = \sqrt{-t}$, which has been advocated in Ref. [3]. Since we have the complete answer (16), we can easily choose any other the factorization scale by changing the argument of the large logarithm, while the total sum is kept unchanged. For this purpose we use the relations

$$t = -xs, \quad L_s = L_t - \ln x, \quad L_t \equiv \ln\frac{-t}{m_e^2}. \qquad (17)$$

In Figures 1,2 we show the values of the second order soft and virtual photonic radiative corrections in different approximations with respect to the power of the large logarithm. Values of the corrections are given in terms of $10^{-3}\cdot d\sigma^{\mathrm{Born}}$. In particular, $r^{(2)}_{2,1,0}$ represent the leading $\mathcal{O}(\alpha^2L^2)$, the next-to-leading $\mathcal{O}(\alpha^2L^1)$, and the next-to-next-to-leading $\mathcal{O}(\alpha^2L^0)$ relative contributions to the cross section, respectively. Since the dependence on the parameter Δ should disappear in the sum of the virtual and soft corrections with the remaining three contributions (see Eq. (3)), we put $\Delta = 1$. In this way we receive only an estimate of the magnitude and the relative size of the corrections in different approximations. Nevertheless this evaluation helps us to get an idea about the size of the unknown second order contributions and to estimate the theoretical uncertainties.

Fig.1 shows the size of the corrections under consideration relevant for the studies at electron-positron colliders of moderate energy (VEPP-2M, BEPS etc.).

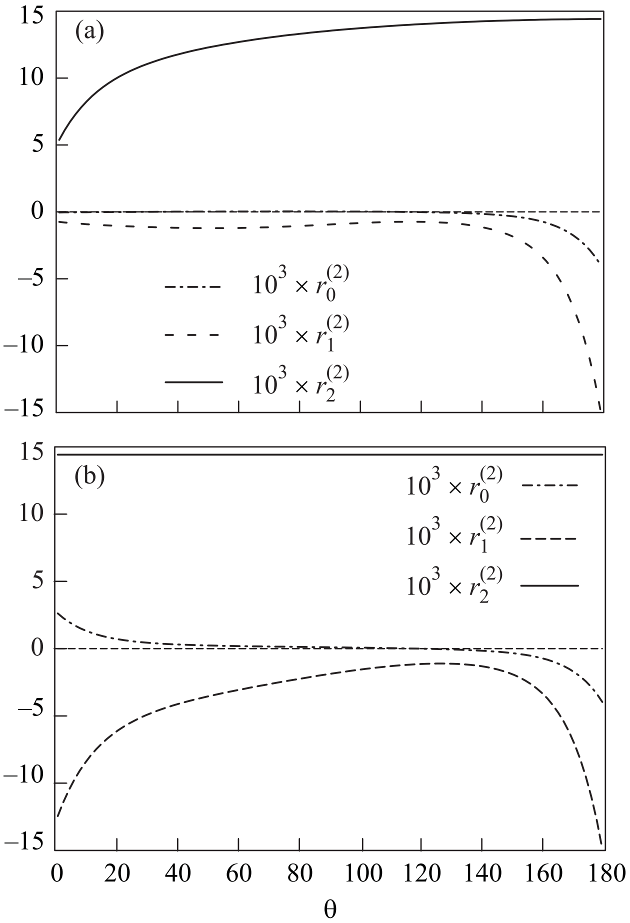

Fig.1. Soft and virtual second order photonic radiative corrections versus the scattering angle in degrees for $\Delta = 1$, $\sqrt{s} = 1\,\mathrm{GeV}$. (a) $M = \sqrt{s}$; (b) $M = \sqrt{-t}$

Fig.2 gives us results for Bhabha scattering at LEP/SLC. We checked that for $\sqrt{s} = 200\,\mathrm{GeV}$ the plots are very close the the ones shown for $\sqrt{s} = 100\,\mathrm{GeV}$.

Looking at the plots representing the contributions of different powers in L, we conclude that with the proper choice of the factorization scale $M = \sqrt{-t}$, the magnitude of the non-logarithmic corrections is below $1\cdot 10^{-2}\%$, everywhere except the region of very large scattering angles ($\theta \gtrsim 160°$). That region requires a special treatment, and it doesn't seem to be of interest for the experiments. Note that the estimate of the size of the non-logarithmic second order corrections agree with the one made earlier in Ref. [3]. At the same moment it

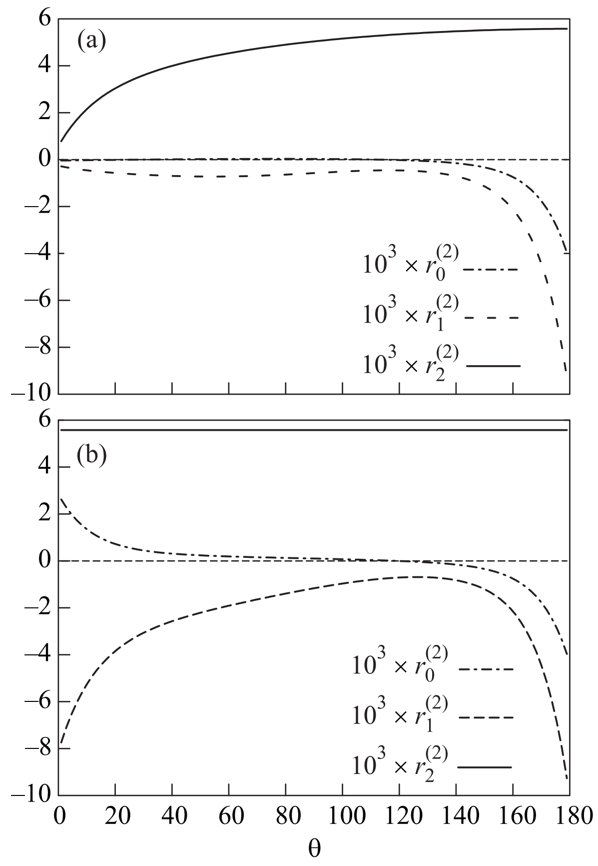

Fig.2. Soft and virtual second order photonic radiative corrections versus the scattering angle in degrees for $\Delta = 1$, $\sqrt{s} = 100\,\mathrm{GeV}$. (a) $M = \sqrt{s}$; (b) $M = \sqrt{-t}$

is clear that to reach the $1 \cdot 10^{-2}\%$ level in the precision of theoretical description of Bhabha scattering we should take into account the complete $\mathcal{O}(\alpha^2 L^0)$ calculations including the effects of virtual and real corrections due to pairs and photons.

Taking the result of the present study and the $\mathcal{O}(\alpha^2 L)$ results of papers [6], where the pair corrections were evaluated, and of Refs. [15], where real photon radiation was taken into account, we arrive at the complete result for the second order next-to-leading radiative corrections to Bhabha scattering. The results are valid both for the small and large angle scattering.

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