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Topological quantization of current in quantum tunnel contacts

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It is shown that an account of the Berry phase (a topological θ-term) together with a dissipative term in the effective action $S[\phi]$ of the tunnel contacts induces a strong quantization of the tunnel current at low temperatures. This phenomenon like the Coulomb blockade reflects a discrete charge structure of the quantum shot noise and can ensure a quantization of the tunnel current without a capacitive charging energy E_C, when the latter is strongly suppressed by quantum fluctuations. Since a value of the θ-parameter is determined by the gate voltage, this effect allows to control a current through the contact. A possible physical application of this effect is proposed.

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1. Introduction. Last decade, in connection with the nanotechnologies, there is a great and permanently growing interest to various small (nanoscale) quantum systems such as quantum dots, tunnel contacts etc. One of the simplest example of such systems is a small normal metal tunnel contact – the single electron box (SEB) or the single electron transistor (SET) (see, for example, [1]). As is known, due to the discreteness of the electric charge of carriers (in this case electrons with a charge e) and a capacitive charging energy $E_c = e^2/2C$, the so called Coulomb blockade (CB) takes place in the SEB [2]. It means that the SEB can conduct only when the electrostatic energy E_C of the SEB is equal for states with n and (n+1) electrons. This takes place when the gate voltage V_g applied to the SEB is [2]

$$V_g = e(n + 1/2)/C_g, \qquad (1)$$

here C_g is a gate capacitance. This theoretical explanation of the Coulomb blockade is pure classical and is applicable only at moderately low temperatures $T < E_C$ and does not take into account quantum phase and/or charge fluctuations. The effective action of the SEB $S_e[\phi]$ in a phase representation and an imaginary time τ, accounting these fluctuations, was deduced in [3] (see also [4]) and has the form

$$S_e[\phi] = S_C[\phi] + S_D[\phi], \quad S_C[\phi] = \int d\tau \frac{\dot{\phi}^2}{4E_C},$$

$$S_D[\phi] = \iint d\tau d\tau' \alpha(\tau - \tau') \sin^2(\frac{\phi(\tau) - \phi(\tau')}{2}), \quad (2)$$

$$\alpha(\tau) = g\frac{T^2}{\sin^2(\pi\tau T)}, \quad g = g_t/g_q = R_q/R_t,$$

$$0 \le \tau \le 1/T, \quad \phi(1/T) = \phi(0)\ (mod 2\pi).$$

Here g is a dimensionless tunnel conductance, $R_q = 1/g_q = h/e^2$ is the quantum resistance and $R_t = 1/g_t$ is a tunnel resistance. The action S_D describes a phase correlation and, in some sense, is universal for scale invariant (in τ) systems.

The action $S_e[\phi]$ is a quite general and with an obvious change of the value of the charge e can be also applied for a description of the phase fluctuations in superconducting tunnel contacts (where it was firstly obtained)[3], in various tunnel contacts [4] and granular metals (see, for example, a recent review [5]). It is worth also to note that the CB effect can exist also in the other quantum contacts with different types of scattering [6, 7].

The full partition function, describing the CB effect in the SEB and containing also an external gate voltage V_g, has an additional, the Berry phase like, term (the so called θ-term) [4],

$$Z_\theta = \int D\phi \exp(-S[\phi]), \quad S[\phi] = S_e[\phi] + iS_\theta[\phi],$$

$$S_\theta[\phi] = \theta Q, \quad Q = \frac{1}{2\pi}\int d\tau \dot{\phi}, \quad \theta = C_g V_g 2\pi/e, \quad (3)$$

where $Q \in \mathbb{Z}$ is a topological charge of mappings $S^1 \to$ $\to S^1$. The θ-term $S_\theta[\phi]$ has a pure topological structure. All physical quantities of the model must be periodic in θ with a period 2π.

The SEB model effective action S_e has many properties similar to those of two-dimensional nonlinear sigma-models (NSM's). It is connected with two facts:

1) at low T (or at large τ) the main role plays S_D, a dissipative part of S_e, while the capacitive part S_C is irrelevant [8] in the renorm-group (RG) sence and is strongly suppressed [4];

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2) on a classical level S_D is invariant under transformations of a small conformal (linear fractional) group [9].

Then, it was shown by the renorm-group method that this model is asymptotically free [8, 9] and, due to the nontrivial topology of the unit circle S_1, where a phase ϕ takes its values, has instanton solutions [9, 10].

The further intensive investigations of the CB properties of the SEB model, using various approximations and methods (from quasi-classical till numerical, see, for example, [11 – 14]), have given many interesting results (including the renormalized charging energy $\widetilde{E}_C$ and a possibility of a phase transition from a conducting to insulating state [4, 11]), though some of them differ from each other quantitatively (for $\widetilde{E}_C$) and even qualitatively (for a phase transition). Since the CB effect is connected with the charging energy E_C, the larger part of these studies was devoted to its renormalization and all of them confirm a strong suppression of E_C at low T (or large τ). But, a strong suppression of the charging energy E_C induces a question: how a discreteness of the charge can show up itself in this case? In particular, can one see the corresponding oscillations of the tunnel current at $T > \widetilde{E}_C$, when they must be washed out by thermal fluctuations?

In this letter, I propose a resolution of this dilemma basing on topological and symmetrical properties of S. It will be shown that at low T (or large time scales) a discreteness of the charge and current in the SEB can be realized *without the Coulomb blockade*, but due to a presence of the θ-term and the dissipative part S_D only.

2. Topology, symmetries and correlations in $D \leq 2$. A decisive role of a topology and symmetries in a determination of the dynamical and correlation properties of two-dimensional physical systems is known more than 30 years and can be seen in a framework of the effective action, containing in the simplest form the main ingredients connected with a topology and symmetries. In the most often cases, when only the lowest fluctuations are important, these effective actions are the actions of NSM's with the corresponding topological and symmetrical properties. Now the NSM's are widely used in different physical problems. The most known example is the NSM description of the integer quantum Hall effect (IQHE) in terms of the NSM on the Grassmanian manifold $G = U(2N)/[U(N) \times U(N)]$ (before taking a replica limit $N \to 0$)[15]. It incorporates two important ingredients, the weak localization theory (WLT) and an existence of instanton solutions, and can describe a phase transition. It was noted also in this theory that, in general, θ is not independent parameter, but can be renormalized by fluctuations (see, for example, [16, 17]). Moreover, it was more or less established for various 2D NSM's with θ-term that an account of S_θ can give the massless (power law) correlations at special values of the θ-parameter [18, 19]

$$\theta = \pm 2\pi(n + 1/2) \quad (n = 0, 1, 2, ...). \tag{4}$$

It means that at these values of θ the NSM is in other, massless, phase, while at other values of θ it has only massive correlations.

This result was firstly conjectured on the physical grounds alone [18], but later they were confirmed by different methods [19], including the exact solution ones [20]. However, one must note that an exact nature of the appearance of the massless phases at the points $\theta = \pm 2\pi(n + 1/2)$ still remains obscure. As it follows from (3), the partition function Z_θ has an alternating structure in this case

$$Z_{\theta=\pi} = \sum_{n=-\infty}^{\infty} (-1)^n Z_n, \tag{5}$$

where Z_n is the partition function for phase configurations with $Q = n$. For this reason, the appearance of the massless phases at these points can be connected with a some deep *topological* property of the functional space of the NSM's, admitting instantons, and possibly reflects a larger symmetry of the models at these points, which contains or reduces to a conformal symmetry group as a subgroup.

The action S from (3), though is effectively one-dimensional, also belongs to this kind of actions due to its two properties listed above. In our further consideration an account of the S_θ will play an important role. As it follows from (3) all physical quantities of the SEB model must satisfy the next symmetries:

1) they must be periodic in θ with a period equal to 2π (thus, one can confine oneself by a range $0 \leq \theta \leq$ $\leq 2\pi$);

2) they can be divided into even and odd parts relative θ.

Recently, it was shown under some assumptions that S_e is exactly solvable and a series of higher order conserved currents were constructed [21]. Moreover, it was shown also that the point $\theta = \pi$ has some special properties, corresponding to a massless phase. Thus, one can see that our one-dimensional model also has a massless fixed points at $\theta = \pi$. This allows us to analyze our model (3) basing on its topological and above listed properties.

3. RG phase diagram and charge quantization. Now I will consider the RG flow phase diagram in the g, θ-plane using above mentioned facts. In particular, a renormalized physical conductance g and its β-function $\beta_g(g,\theta)$, determining its change under RG flow, are an even functions of θ, while the corresponding β-function of θ, $\beta_\theta(g,\theta)$, must be odd in θ. Since they must also be periodic in θ, one can write the following functional representation for them

$$d_l g = \beta_g(g,\theta), \quad \beta_g(g,\theta) = a_0(g) + \sum_1^\infty a_n(g)\cos(n\theta), \tag{6}$$

$$d_l\theta = \beta_\theta(g,\theta),\ \beta_\theta(g,\theta) = \sum_1^\infty b_n(g)\sin(n\theta),\ l = \ln\tau/\tau_0. \tag{6'}$$

Here $\tau = \tau_0 e^l$ is a running scale with τ_0 fixing an initial scale, which is in our case a scale of the SEB. The periodic parts appear due to the large phase fluctuations, the instantons (or phase slips) contributions, in an analogy with the two-dimensional IQHE case [17]. The function $a_0(g)$ is now known up to two-loop level [14]

$$a_0(g) = -(1+1/g). \tag{7}$$

One can write, for instance, the one-instanton contribution. It has a form

$$a_1(g) = -Dg^2\exp(-g) = b_1(g), \tag{8}$$

where the constant $D > 0$ is connected with the fluctuations over one instanton solution. Note that one instanton contribution in β_g enhances a decreasing of g at $\theta = 0$ and slows down it at $\theta = \pi$. The renormalization of θ is absent in the RG approach at small $1/g$ and contains only the instanton contributions. This coincides with the IQHE case, where a_0 has a similar form before taking a replica limit $N \to 0$.

As follows from (6′) the lines $\theta = \pm n\pi$ ($n = 0,\ 1,\ 2,\ \ldots$) are integral lines, since they correspond to zero's of β_θ. On the line $\theta = 0$ (and on other $\theta = 2n\pi$ lines) one has usual asymptotic freedom behaviour with a trivial infra-red (IR) stable fixed point $g = 0$. This means that on this line at low T the initial tunnel conductance g_0 is firstly reduced logarithmically by the small quantum phase fluctuations (like in the WLT) till $\tau_c \sim \tau_0\exp(g)$, a correlation imaginary time (or a characteristic temperature $T_c \sim 1/\tau_c$). The correlation time τ_c is similar simultaneously to the dephasing time τ_ϕ and a correlation length L_c of the WLT, since in the model (3) there exists only one effective dimension, τ. Further, at $\tau > \tau_c$, a renormalized conductance $g \to 0$ exponentially $g \sim e^{-\tau/\tau_c}$ due to the large phase fluctuations - instantons (or phase slips). Analogously, $g \sim e^{-T_c/T}$ at $T_c > T$. In the limit $\tau \to \infty$ or $T \to 0$ the SEB does not conduct on this line at all.

On the line $\theta = \pi$ a behaviour of g differs from that for $\theta = 0$. There must be the finite fixed point (FP) g^*, corresponding to the massless phase, which is IR-stable for initial values of g on this line and IR-unstable for initial values of g out of this line. This FP ensures a charge transfer in the SEB at $\theta = \pi$. Such behaviour of g means that the SEB conducts on this line at arbitrary initial values g_0.

Unfortunately, the exact value of g^* is now not known. Here is some difficulty. $a_n(g)$ ($n > 0$), entering in β_g, contain the constants like D, which are not defined unambiguously: they depend on scheme of renormalization [18]. At the same time the condition of the existence of only one nontrivial fixed point is very restrictive. To see this let us consider one instanton contribution. Then one obtains the equation

$$\beta_g(g^*){=}a_0 - a_1{=}0 \Longrightarrow (1+1/g^*){=}Dg^{*s}\exp(-g^*). \tag{9}$$

Here, for a generality, we consider a possibility, when the power of g in a_1 is some integer $s = 1, 2, 3, \ldots$. The left side of (9) is a smooth function convex down, while the right side is convex up with a maximum at $g = s$. Then, neglecting a smooth dependence of the left side of (9) (its derivative $\sim 1/g^2$), we see that one solution exists only for a special value of D:

$$g^* = s, \quad D_s = \frac{g^*+1}{(g^*)^{s+1}}e^{g^*}. \tag{10}$$

It is very intriguing that in this approximation g^* is an integer. If the exact value of $g^* = s$, then it means that $g_t^* = sg_q = sh/e^2$. An account of the left side derivative gives

$$g^* = \frac{s-1+\sqrt{(s+1)^2+4}}{2}. \tag{10'}$$

For larger values of D one obtains two solutions, while for smaller values there is no any solution. A case of two solutions, taken formally, has some difficulties, because two nearest fixed points cannot be both IR-stable on line. Then there are two possibilities: 1) the smaller FP g_1^* is IR-stable and the larger FP g_2^* is IR-unstable or 2) an opposite case. In the first case the renormalized g for initial values $0 < g_0 < g_2^*$ tends at $\tau \to \infty$ (or $T \to 0$) to the FP g_1^*, which will define the effective conductance of the SEB for these initial values. For $g_1^* < g_0 < \infty$ it tends to ∞, what would mean that for these initial values the effective conductance of the SEB is ∞. In the second case, when g_1^* is IR-unstable one obtains the effective conductance $g = g_2^*$ for all $g_0 > g_1^*$ and $g = 0$ for $g_0 < g_1^*$. Both these results are unacceptable since they contradict to the exact result [21] about one massless phase at $\theta = \pi$. In order the two FP on a line be

IR-stable one needs one IR-unstable fixed point between them, what is impossible in our case in the region near a maximum of a_1. At the same time an existence of two FP could be interpreted as a mathematical signal that physically in the model there is a narrow band of the massless states with a width equal to a distance between these solutions (this hypothesis has been proposed in 80ies during an investigation of the IQHE).

An account of higher order terms does not improve a situation (in 2D NSM's as well as in our 1D model), since a presence of the alternating factor $(-1)^n$ in the corresponding equation

$$\beta_g(g^*) = a_0 + \sum_1^\infty (-1)^n a_n = 0. \qquad (11)$$

only complicates a situation, because they can partially cancel each other. For this reason, in the RG approach one needs to know all terms to make unambiguous conclusion about finite nontrivial fixed point. This can be done only with an usage of some exact results, as it takes place in [6] and in the exact solution approach of [21].

Nevertheless, basing on the abovementioned exact properties, one can conjecture a following global RG flow phase diagram of the SEB model at $T \to 0$ (Fig.1). Here it is supposed that there are no any other fixed

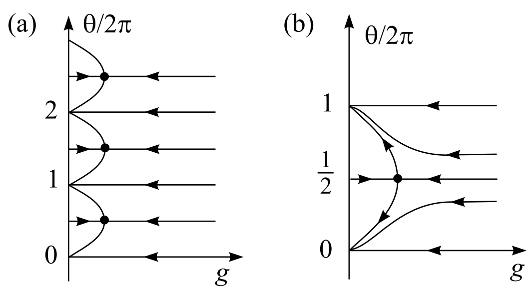

Fig.1. (a) A schematic picture of the SEB $g-\theta$-plane phase diagram at $T = 0$. The arrows on the horizontal lines $\theta/2\pi = 0, \pm 1/2, \pm 1, \pm 3/2, ...$ indicate the RG flow. (b) The same zoomed picture for one period part. Here the RG flow shows that θ is also renormalized at small g and $\theta/2\pi \neq \pm(n+1/2)$ $(n = 0, 1, 2, ...)$

points. The RG flow near the fixed points and separatrices, connecting these fixed points, shows a strong renormalization of the parameter θ. If the parameter θ were unrenormalized, all RG flow lines would be straight lines. Then a separatrice, connecting the trivial fixed point $g = 0 = \theta$ with $g = g^*, \theta = \pi$ would be a continuous line of the fixed points. This would mean that the SEB model is conducting at all θ except $\theta = 0$ and critical correlations have power law form with the exponents depending on θ. This is not a case in the 2D NSM's and contradicts to the results [21].

The presented RG flow diagram is very similar to that in the theory of the IQHE [17, 18], since they both are a consequence of general symmetries, topology and an existence of one finite fixed point only. The renormalized $\tilde{\theta}$ has the jumps at $\theta = \pi$, like the sign function (Fig.2a), while a behavior of the conductance g is similar

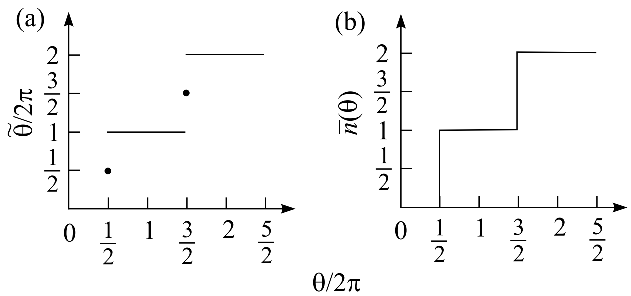

Fig.2. A schematic picture of (a) $\tilde{\theta}(\theta)$ and (b) $\bar{n}(\theta)$ at $T \to 0$

to a behavior of σ_{xx}. The average number of electrons in the SEB grain $\bar{n}$ has a sharp, stepwise, dependence on θ like the Hall conductivity σ_{xy} on magnetic field (Fig.2b). Using the results of [15], I have supposed also that the RG flow has a linear form near trivial fixed points $g = 0$, $\theta = 2n\pi$. When a perturbation with a some characteristic scale τ_p is introduced or T is small, but finite (or τ is large, but finite), the renormalization must be stopped at scale $\max[T, 1/\tau_p]$ on an energy scale or at $\min[1/T, \tau_p]$ on a τ scale. This gives a smeared behaviour near the lines $\theta = 2\pi(n + 1/2)$. For instance, the steps on Fig.2 will be more rounded and smooth.

Finally, one can see that the obtained topological charge quantization does not depend on E_C and works even if the renormalized $\widetilde{E}_C < T$, when the CB is already impossible: it will be washed out by the thermal fluctuations. For this reason one obtains at $\widetilde{E}_C < T$ a quantized (or oscillating) behaviour, smeared by finite T, which takes place completely due to the topological quantization. Since $\widetilde{E}_C \ll E_C$, there is a wide temperature interval $\widetilde{E}_C \ll T \ll E_C$, where a discreteness of the charge can demonstrate itself through a topological quantization. The existence of this temperature interval allows to verify the proposed theory of the charge quantization and the corresponding oscillations. At the same time the CB can still show up itself, since one can treat $\widetilde{E}_C$ as one of $1/\tau_p$. Then at low T one must stop renormalization in an energy scale at $\max[T, \widetilde{E}_C]$, and for $\widetilde{E}_C \geq T$ (if this region exists) one can see the smeared CB oscillations.

4. Conclusions and discussions. Thus, using topological and symmetrical properties of the SEB model, it is shown that the quantization of charge and current in tunnel contacts is possible even when the Coulomb charging energy E_C is strongly suppressed by quantum fluctuations to $\widetilde{E}_C \leq T$. It means that the charge quantization and the corresponding oscillation behaviour of various characteristics of the SEB model will survive at low $T \ll E_C$, but a smearing of these functions will be determined by $\max[T, \widetilde{E}_C]$.

It is interesting that a qualitatively similar phase diagram picture was obtained for the quantum point contact connecting quantum dot with external lead [6]. In this case the corresponding model Hamiltonian describes from very beginning the fermions and the stable fixed points at $\theta/2\pi = n+1/2$ are identified with the Toulouse fixed point in the theory of the 2-channel Kondo model. One can hope that the obtained results can be expand also on quantum contacts with other types of scattering [7].

Though the θ-parameter depends on the gate voltage V_g, it turns out that really the physics of the SEB model at low T weakly depends on its value, since effectively θ is strongly renormalized to $0 \pmod{2\pi}$ for all θ except $\theta = 2\pi(n+1/2)$.

At the end of this letter a few words about a possible physical application of the obtained results. An useful quantum device connected with the SEB model is the SET. Then the obtained topological quantization of the charge in the SEB model at $\widetilde{E}_C \leq T$ can be observed as a topological quantization of the tunnel current in the SET.

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