Pis'ma v ZhETF, vol. 83, iss. 12, pp. 675 – 680

Intervortex quasiparticle tunneling and electronic structure of multi-vortex configurations in type-II superconductors

A. S. Mel'nikov[1)], *M. A. Silaev*

Institute for Physics of Microstructures RAS, 603950 Nizhny Novgorod, GSP-105, Russia

Submitted 22 May 2006

The electronic spectrum of multi-vortex configurations in type-II superconductors is studied taking account of the effect of quasiparticle tunneling between the vortex cores. The tunneling is responsible for the formation of strongly coupled quasiparticle states for intervortex distances $a < a_c$, where the critical distance a_c is of the order of several coherence lengths ξ. Analyzing the resulting spectra of vortex clusters bonded by the quasiparticle tunneling we find a transition from a set of degenerate Caroli – de Gennes – Matricon branches to anomalous branches similar to the ones in multi-quanta giant vortices. This spectrum transformation results in the oscillatory behavior of the density of states at the Fermi level as a function of a and could be observed in mesoscopic superconductors and disordered flux line arrays in the bulk systems.

PACS: 74.25.–q, 74.78.Na

Theoretical investigations of the quantum mechanics of low energy quasiparticle (QP) excitations in isolated vortices and vortex arrays are of crucial importance for understanding the basic thermodynamic and transport phenomena in the mixed state of type-II superconductors at low temperatures. This issue has been studied for several decades starting from the pioneering work by Caroli, de Gennes and Matricon (CdGM) [1]. For an isolated singly-quantized vortex there appears a so-called anomalous branch of the subgap spectrum which corresponds to the QP states bound to the vortex core because of the Andreev reflections from the gap potential profile $\Delta = |\Delta(r)|e^{i\theta}$, where (r, θ, z) is a cylindrical coordinate system, and z-axis is chosen parallel to the vortex line. The low energy spectrum of these localized states can be written as follows: $E = -\mu\omega \simeq -\mu\Delta_0/(k_\perp\xi)$, where Δ_0 is the gap value far from the vortex core, ξ is the superconducting coherence length, $k_\perp = \sqrt{k_F^2 - k_z^2}$ is the Fermi momentum projection on the plane perpendicular to the vortex axis, and the angular momentum quantum number μ is half an odd integer. The overlapping of QP wave functions of neighboring vortices should perturb the CdGM spectrum and result, e.g., in the band structure effects in periodic vortex lattices [2]. With the increase in the vortex concentration at high magnetic fields close to the upper critical field H_{c2} there occurs a crossover to the Landau-type quantization for QPs precessing along the whole Fermi surface (see [3] and references therein). A scenario of such crossover from discrete CdGM levels to the Landau-type spectrum is an appealing problem which would allow to understand the nature of de Haas – van Alphen oscillations observed experimentally [4] even for magnetic fields $H \sim (0.3 - 0.4)H_{c2}$ when the vortices are well-separated. The transformation of the QP spectrum in the latter case should be controlled by the effect of QP tunneling between the vortex cores. This tunneling phenomenon is expected to play an essential role also for exotic vortex configurations formed in mesoscopic superconductors of the size of several coherence lengths. In such systems the balance of competitive forces acting on vortices due to the screening current flowing at the sample boundary and the intervortex repulsion, leads to the formation of small size multi-vortex configurations (vortex molecules) and multi-quanta (giant) vortices [5]. These vortex states can transform into each other via magnetic field-driven first or second order phase transitions. According to the general theory [6] the number of anomalous branches (per spin) crossing the Fermi level at certain impact parameters $\mu_F/k_\perp \lesssim \xi$ equals to the winding number M of a multi-quanta vortex. The anomalous branch with $\mu_F = 0$ responsible for the peak in the density of states (DOS) at the vortex center exists only for vortices with an odd winding number. Generally the spatial distribution of the DOS has the shape of rings with radii of the order of ξ [7, 8]. The splitting of a multi-quanta vortex into $M-$ vortex molecule with the intervortex distance a results in the transformation of these rings in the DOS profile into a set of peaks at the centers of individual vortices. The initial stage of such DOS transformation has been studied in [8] by the perturbation method valid for $a \ll \xi$.

In this Letter we address the limit $a > \xi$ of well-separated vortices in mesoscopic superconductors and

[1)]e-mail: melnikov@ipm.sci-nnov.ru

disordered vortex arrays in the bulk systems well below H_{c2}. The goal of the present work is to study a scenario of the QP spectrum transformation caused by the formation of vortex clusters bonded by the QP tunneling between the Andreev wells in the cores. Note that hereafter we neglect all the possible normal scattering effects at impurities [9] or mesoscopic sample boundaries [10] which also can affect the low energy spectrum.

Let us start with a qualitative analysis of the intervortex tunneling effect and consider a set of vortex lines parallel to the z-axis. In the plane (xy) the vortex centers defined as zeros of the superconducting order parameter are positioned at certain points $\mathbf{r}_i$. For QPs propagating along the classical trajectories parallel to $\mathbf{k}_\perp = k_\perp(\cos\theta_p, \sin\theta_p)$ we introduce the angular momenta $\mu = [\mathbf{r}, \mathbf{k}_\perp]\cdot\mathbf{z_0} = k_\perp r\sin(\theta_p - \theta)$ and $\tilde{\mu}_i = \mu - [\mathbf{r}_i, \mathbf{k}_\perp]\cdot\mathbf{z}_0$ defined with respect to the z-axis passing through the origin and with respect to the i-th vortex axis passing through the point $\mathbf{r}_i$, correspondingly. Neglecting the QP tunneling between the vortex cores we get degenerate CdGM energy branches: $E_i = -\omega\tilde{\mu}_i$. For a fixed energy E we can define a set of crossing branches on the plane (μ, θ_p): $\mu_i(\theta_p) = -E/\omega + [\mathbf{r}_i, \mathbf{k}_\perp]\cdot\mathbf{z_0}$. These branches are shown in Fig.1 for two vortices with

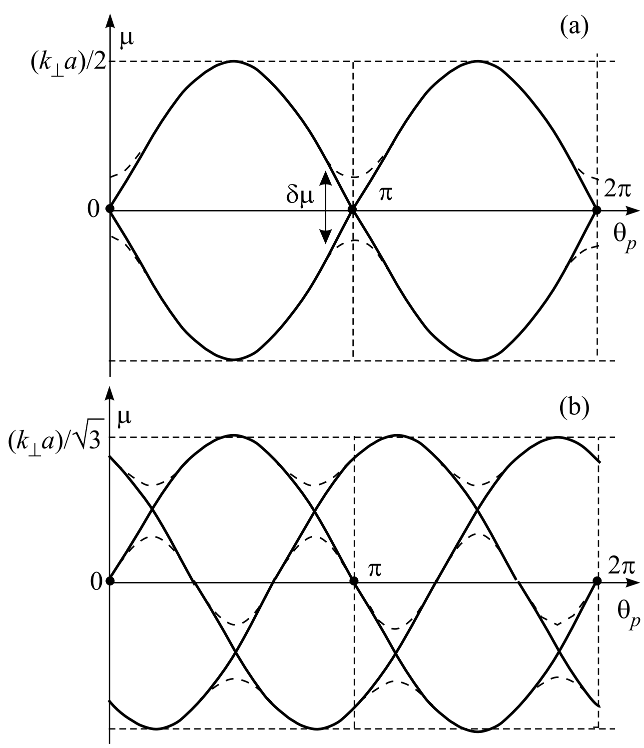

Fig.1. Angular momentum μ as a function of θ_p at $E = 0$ for (a) two and (b) three vortices

$\mathbf{r}_1 = (-a/2, 0)$ and $\mathbf{r}_2 = (a/2, 0)$ and three vortices at the apexes of the equilateral triangle with the center at the origin: $\mathbf{r}_1 = (0, a/\sqrt{3})$, $\mathbf{r}_2 = (a/2, -a/(2\sqrt{3}))$ and $\mathbf{r}_3 = (-a/2, -a/(2\sqrt{3}))$. Each crossing point of branches $\mu_i(\theta_p)$ and $\mu_j(\theta_p)$ corresponds to the trajectories passing through the cores of i-th and j-th vortices. It is natural to expect that the degeneracy at these points will be removed if we take account of a finite probability of QP tunneling between the cores. The tunneling is determined by the exponentially small overlapping of wave functions localized near the cores and results in the splitting of energy levels: $\delta E \sim \Delta_0 \exp(-k_F a_{ij}/(k_\perp\xi))$, where $a_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$. The estimate for the branch splitting $\delta\mu$ shown in Fig.1 by dash lines reads:

$$\delta\mu(a_{ij}) \sim \frac{\delta E}{\omega} = k_\perp\xi \exp\left(-\frac{k_F a_{ij}}{k_\perp\xi}\right). \qquad (1)$$

As a result, we get the branches $\mu_i^*(\theta_p)$ with a qualitatively new behavior: each of these branches consists of parts corresponding to the classical QP trajectories passing through the cores of different vortices. Keeping in mind general conditions of the semiclassical approach validity it is natural to expect that a reasonable criterion on the intervortex tunneling efficiency can be obtained if we compare the splitting $\delta\mu(a_{ij})$ with the quantum mechanical uncertainty $\Delta\mu$ of the angular momentum. The latter value can be estimated from the uncertainty principle $\Delta\mu\Delta\theta_p \sim 1$, where the expression for the angle uncertainty near the branch crossing points reads: $\Delta\theta_p \sim \Delta\mu/k_\perp a_{ij}$. Provided the branch splitting is rather small, i.e. $\delta\mu(a_{ij}) \ll \Delta\mu \sim \sqrt{k_\perp a_{ij}}$ for all crossing points, the branches $\mu_i(\theta_p)$ are almost independent. In the opposite limit $\delta\mu(a_{ij}) \gg \sqrt{k_\perp a_{ij}}$ the QP states of the i-th and j-th vortices appear to be strongly coupled by tunneling: near the branch crossing point the QP trajectory performing a precession in the core of the i-th vortex experiences a transition to the core of the j-th vortex. According to the above condition on $\delta\mu(a_{ij})$ the tunneling is most efficient for $k_\perp = k_F$ and $a_{ij} < a_c$, where $a_c \simeq (\xi/2)\ln(k_F\xi)$ is a critical intervortex distance. Using the percolation theory language we can consider the vortices to be bonded if $a_{ij} < a_c$ and define a cluster in a disordered flux line system as a set of M vortices bonded either directly or via other vortices. The number M grows with the increase in the vortex concentration and becomes infinite at the percolation threshold or for a periodic vortex lattice. Certainly in mesoscopic superconductors the cluster dimensions L_v can not exceed the sample size R. Our further consideration of the bonded QP states is restricted to the limit of finite clusters with $L_v \ll r_L$, where $r_L \sim k_\perp\phi_0/H$ is the cyclotron orbit radius, and $\phi_0 = \pi\hbar c/|e|$ is the flux quantum. The cluster is characterized by a set of hybridized QP states: with a change in the $\mathbf{k}_\perp$ direction the wave function experiences a number of subsequent transitions between the cores of neighboring vortices. Taking, e.g., the upper branch in Fig.1a we obtain the wave function concentrated near the cores of the right and left vortices for the angular intervals $0 < \theta_p < \pi$ and $\pi < \theta_p < 2\pi$, respectively. Further decrease in the intervortex distance results in the increase in the tunneling probability and, thus, the increase in $\delta\mu(a_{ij})$. Finally, for $a_{ij} \to 0$ we get a set of M lines $\mu = \text{const}$ parallel to the θ_p axis, i.e. M anomalous branches crossing zero energy at angular independent impact parameters and corresponding to the $M-$ quanta vortex. Certainly this limit can be realized only in mesoscopic samples.

Following [11] one can obtain the discrete energy levels applying the Bohr-Sommerfeld quantization rule for canonically conjugate variables μ and θ_p:

$$\int_0^{2\pi n_\theta} \mu(\theta_p) d\theta_p = 2\pi(n+\beta), \tag{2}$$

where n and $1 \le n_\theta \le M$ are integers, $2\pi n_\theta$ is the period of the $\mu(\theta_p)$ function, and $\beta = 1/2$ ($\beta = 0$) for odd (even) n_θ values. Depending on the ratio $\delta\mu(a_{ij})/\Delta\mu$ one should apply this quantization rule either to the branches $\mu_i(\theta_p)$ or to the branches $\mu_i^*(\theta_p)$. In the momentum region

$$k_F\sqrt{1-(\min(a_{ij})/a_c)^2} \ll |k_z| < k_F \tag{3}$$

one can neglect the branch splitting $\delta\mu(a_{ij}) \ll \Delta\mu$ at all and Eq.(2) written for the branches $\mu_i(\theta_p)$ gives us the CdGM spectrum with a minigap $\omega_0/2 = \omega(k_z = 0)/2$. For $\min(a_{ij}) > a_c$ the CdGM expression holds for the entire momentum range. For vortices forming a cluster the QP states bonded by intervortex tunneling appear in a finite momentum interval

$$|k_z| \ll k_F\sqrt{1-(\min(a_{ij})/a_c)^2} < k_F. \tag{4}$$

In this limit the QP tunneling between the cores results in the qualitative modification of spectrum which can be obtained by substituting $\mu_i^*(\theta_p)$ into Eq.(2):

$$E_{ni}(k_z) \approx \frac{\Delta_0}{\xi}\left(\frac{n+\beta}{k_\perp} + b_i(\mathbf{r}_1,\ldots\mathbf{r}_M)\right), \tag{5}$$

where $i = 1\ldots M$. The spectrum (5) is similar to the one of a multi-quanta vortex [6–8] which recovers in the limit $a_{ij} \to 0$ when $|b_i| \lesssim \xi$. The multi-vortex cluster geometry and its dimensions L_v determine the effective impact parameters $b_i(\mathbf{r}_1,\ldots\mathbf{r}_M)$ which vary in the range $-L_v \lesssim b_i \lesssim L_v$. Taking a two- (three-) vortex molecule with $\xi < a < a_c$ as an example we get $b_{1,2} \sim \pm a$ ($b_{1,3} \sim \pm a$, $b_2 = 0$). Contrary to the CdGM case the spectrum branches (5) can cross the Fermi level as we decrease the characteristic intervortex distance a and the minigap is suppressed. The DOS consists of M sets of van Hove singularities corresponding to the extrema of $E_{ni}(k_z)$ branches. The energy interval between the peaks belonging to each set is ω_0. For a certain fixed energy the DOS as a function of a exhibits oscillations with the period of the order of the atomic length scale $\delta a \sim k_F^{-1}$. Experimentally the intervortex distance can be controlled by a varying magnetic field. For typical values $a \sim \sqrt{\phi_0/H}$ we get the following field scale of DOS oscillations: $\delta H/H \sim \sqrt{\hbar\omega_H/\varepsilon_F}$, where $\omega_H = |e|H/mc$ is the cyclotron frequency, m is the electron effective mass, and ε_F is the Fermi energy. The oscillatory behavior should affect both thermodynamic and transport properties at low temperatures though in real experimental conditions the DOS peak structure is certainly smeared due to the various mechanisms of level broadening, e.g., finite temperature, fluctuations in vortex positions, impurity scattering effects, etc. It should be noted that for typical values $k_F\xi = 10^2 - 10^3$ the critical distance $a_c/\xi \sim 2-3$ exceeds the core radius and the spectrum transformation starts at the fields $H \sim \phi_0/a_c^2 \sim H_{c2}[\ln(k_F\xi)]^{-2}$ when the vortices are indeed well-separated.

Now we proceed with a quantitative analysis of the intervortex tunneling on the basis of the Bogolubov – de Gennes theory:

$$\hat{H}_0\hat{\Psi} + \hat{\sigma}_x \operatorname{Re}\Delta\hat{\Psi} - \hat{\sigma}_y \operatorname{Im}\Delta\hat{\Psi} = E\hat{\Psi}, \tag{6}$$

where $\hat{H}_0 = \hat{\sigma}_z((\hat{\mathbf{p}} - e\hat{\sigma}_z\mathbf{A}/c)^2 - \hbar^2 k_\perp^2)/2m$, $\hat{\Psi} = (U, V)$, U and V are the particle- and hole-like parts of the QP wave function, $\hat{\sigma}_i$ are Pauli matrices, $\hat{\mathbf{p}} = -i\hbar\nabla$, $\Delta = \Delta(x,y)$ is an order parameter profile. For extreme type-II superconductors we can assume the magnetic field $\mathbf{H} = -H\mathbf{z}_0$ to be homogeneous on the spatial scale L_v and take the gauge $\mathbf{A} = [\mathbf{H},\mathbf{r}]/2$. Within the quasiclassical approach the wave function in the momentum representation can be taken in the form:

$$\hat{\Psi}(\mathbf{p}) = \frac{1}{k_\perp}\int_{-\infty}^{+\infty} ds e^{-i(|\mathbf{p}|-\hbar k_\perp)s/\hbar}\hat{\psi}(s,\theta_p) \ . \tag{7}$$

The equation for $\hat{\psi}(s,\theta_p)$ reads: $\hat{H}\hat{\psi} = E\hat{\psi}$, where

$$\hat{H} = -i\hbar V_\perp \hat{\sigma}_z \frac{\partial}{\partial s} + \hat{\sigma}_x \operatorname{Re}\Delta - \hat{\sigma}_y \operatorname{Im}\Delta + \frac{\hbar\omega_H}{2}\hat{\mu}, \tag{8}$$

$mV_\perp = \hbar k_\perp$, $\Delta = \Delta(\hat{x},\hat{y})$, $\hat{\mu} = -i\partial/\partial\theta_p$, and the terms quadratic in H are neglected assuming $L_v \ll r_L$. The coordinate operator takes the form:

$$\hat{\mathbf{r}} = s\mathbf{k}_\perp/k_\perp + \{[\mathbf{k}_\perp,\mathbf{z}_0],\hat{\mu}\}/2k_\perp^2, \tag{9}$$

where $\{A, B\} = AB + BA$ is an anticommutator. Let us emphasize that the Hamiltonian (8) takes account of noncommutability of $\hat{\mu}$ and θ_p and, thus, the above description involves the angular momentum quantization. Replacing $\hat{\mu}$ by a classical variable we get Andreev equations along straight trajectories. For an isolated vortex positioned at $\mathbf{r} = 0$ and described by the gap function $\Delta_1(\mathbf{r}) = \Delta_v(r)e^{i\theta}$ a standard solution of the above equations corresponding to low energy CdGM levels reads: $\hat{\psi} = \exp(i\mu\theta_p)\hat{\psi}_0(s,\theta_p)$, where

$$\hat{\psi}_0(s,\theta_p) = \exp\left(\frac{i}{2}\hat{\sigma}_z\theta_p\right)\sqrt{\frac{\Delta_0}{2\hbar V_\perp \Lambda}}\begin{pmatrix} 1 \\ -i \end{pmatrix} e^{-K(s)},$$

$$K(s) = \frac{1}{\hbar V_\perp}\int\limits_0^s \frac{t\Delta_v(t)}{|t|}dt \;,\; \Lambda = \frac{2\Delta_0}{\hbar V_\perp}\int\limits_0^{+\infty} e^{-2K(t)}dt.$$

The physical picture of the intervortex QP tunneling effect can be illustrated by consideration of the simplest generic problem: two neighboring vortices positioned at $\mathbf{r}_\pm = (\pm a/2, 0)$. The distances from this vortex pair to other vortices or to a mesoscopic sample boundary are assumed to be larger than the critical distance a_c so that their influence can be neglected. To describe the case of well-separated vortices $a > \xi$ it is natural to use the tight binding approximation for the QP wave function:

$$\hat{\psi} = c_+(\theta_p)\widehat{T}_+\hat{\psi}_0(s,\theta_p) + c_-(\theta_p)\widehat{T}_-\hat{\psi}_0(s,\theta_p), \qquad (10)$$

where the operator

$$\widehat{T}_\pm = \exp\left(\frac{i\hat{\sigma}_z\varphi_\pm}{2} \mp \frac{a\cos\theta_p}{2}\left(ik_\perp + \frac{\partial}{\partial s}\right) \pm \frac{ik_\perp a}{2}\right)$$

transforms the CdGM states of a single vortex at the origin into the eigenstates for isolated vortices at $\mathbf{r}_\pm = (\pm a/2, 0)$, $\varphi_\pm$ is the order parameter phase induced at the center of right (left) vortex due to the presence of neighboring vortices or boundaries. The coefficients $c_\pm$ satisfy the condition $c_\pm(\theta_p + 2\pi) = -c_\pm(\theta_p)$, so that the total wave function is single valued. In the angular intervals $|\sin\theta_p| > \xi/a$ the QP trajectories can not pass through both vortex cores and we have two solutions with either $c_+ = 0$ or $c_- = 0$, i.e. the states corresponding to one of the vortices. The only effect of neighboring vortices or boundaries on the QP energy for such trajectories could be associated with the Doppler shift ϵ_d caused by the superfluid velocity field induced by the external sources in a given vortex core. However this superflow would produce the Lorentz force acting on a vortex and, thus, ϵ_d should vanish for a static vortex configuration. This condition gives us the relation $a = a(H)$ which depends on the vortex arrangement and sample geometry (see, e.g., [12]).

The angular intervals $|\sin\theta_p| < \xi/a$ should be considered separately since the corresponding QP trajectories pass through both vortex cores. Substituting Eq.(10) into the quasiclassical equations with the Hamiltonian (8), multiplying them by the functions $\left(\widehat{T}_\pm\hat{\psi}_0\right)^*$, and integrating over s, one obtains:

$$-\omega\hat{\mu}\hat{c} - t\exp\{i\hat{\sigma}_z k_\perp a(\cos\theta_p - 1)\}\hat{\sigma}_x\hat{c} = E\hat{c}, \qquad (11)$$

where $\hat{c} = (c_+, c_-)$. For the trajectories with θ_p close to 0 or π we find an approximate expression for the intervortex tunneling amplitude: $t \simeq \Delta_0 e^{-ak_F/\xi k_\perp}$. Note that here we omit small θ_p-dependent corrections to ω resulting from the angular dependent distortions of the gap profiles in the cores and neglect the renormalization of the ω and t values caused by the terms $\propto \hbar\omega_H \sim \omega_0(\xi/a)^2 \ll \omega_0$. Let us introduce the function $\hat{b} = \exp(iE\theta_p/\omega - ik_\perp a\hat{\sigma}_z(\cos\theta_p - 1)/2)\hat{c}$. For $|\theta_p| \ll \xi/a$ the Eq.(11) reads:

$$-\omega\hat{\mu}\hat{b} + \frac{k_\perp a\omega}{2}\theta_p\hat{\sigma}_z\hat{b} - t\hat{\sigma}_x\hat{b} = 0. \qquad (12)$$

This problem is equivalent to the one describing the interband tunneling [13] or the one-dimensional (1D) motion of a Dirac particle in a uniform electric field and the solution can be written in terms of the parabolic cylinder functions $D_{i\alpha}(\sigma)$:

$$\hat{b} = \begin{pmatrix} d_1 D_{i\alpha}(\sigma) + d_2 D_{i\alpha}(-\sigma) \\ \sqrt{-i\alpha}(d_1 D_{i\alpha-1}(\sigma) - d_2 D_{i\alpha-1}(-\sigma)) \end{pmatrix}, \qquad (13)$$

where $\alpha = t^2/(\omega^2 k_\perp a) \sim (\delta\mu(a)/\Delta\mu)^2$, and $\sigma = \theta_p\sqrt{k_\perp a/i}$. Taking the asymptotical expressions for $D_{i\alpha}(\sigma)$ in the interval $\max[\alpha, 1]/\sqrt{k_\perp a} \ll |\theta_p| < \xi/a$ we get: $\hat{b}(\theta_p > 0) = \hat{S}\hat{b}(\theta_p < 0)$, where

$$\hat{S} = e^{-\pi\alpha}\hat{I} + ie^{i\hat{\sigma}_z\gamma}\left(\hat{\sigma}_y \operatorname{Re}\tau + \hat{\sigma}_x \operatorname{Im}\tau\right)e^{-i\hat{\sigma}_z\gamma},$$

$$\tau = \tau(\alpha) = \frac{2\Gamma(1 - i\alpha)}{\sqrt{-2\pi i\alpha}}e^{-\pi\alpha/2}\sinh(\pi\alpha),$$

$\hat{I}$ is the unity matrix, $\gamma = k_\perp a\theta_p^2/4 + \alpha\ln(|\theta_p|\sqrt{k_\perp a})$, and $\Gamma(x)$ is the gamma function. For the angles θ_p close to π the solution can be obtained using a transformation: $\hat{b}(|\theta_p - \pi| \ll \xi/a) = \hat{\sigma}_x\hat{b}(|\theta_p| \ll \xi/a)$. Matching the wave function in various angular domains and using the condition $\hat{c}(\theta_p) = -\hat{c}(\theta_p + 2\pi)$ we obtain the spectrum:

$$\cos(\pi E/\omega) = \pm e^{-\pi\alpha/2}\sqrt{2\sinh(\pi\alpha)}\sin(\pi\chi), \qquad (14)$$

where $\pi\chi = k_\perp a + \alpha\ln(k_\perp\xi^2/a) + \arg(\Gamma(1 - i\alpha)) + \pi/4$. The generalization of such matching procedure for $M-$

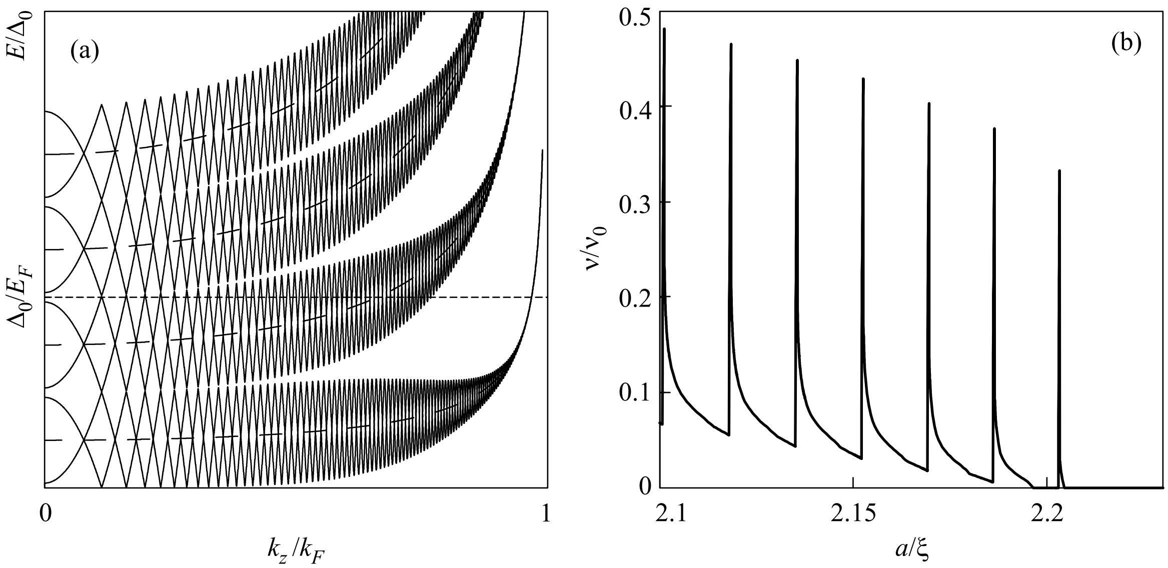

Fig.2. Two-vortex molecule ($k_F\xi = 200$): (a) energy spectrum for $a = 2.1\xi$; (b) the DOS per spin projection averaged over the interval $\delta E = 0.05\omega_0$ around $E = 0$ as a function of the intervortex distance a. The CdGM spectrum is shown by the dash lines. $\nu_0 = k_F/4\omega_0$ is the averaged CdGM DOS

vortex clusters is straightforward. The spectrum calculated using the Eq.(14) is shown in Fig.2a for a typical parameter $k_F\xi = 200$ and for a rather small intervortex distance $a < a_c$. One can see that the transformation of the spectrum $E(k_z)$ occurs according to the scenario suggested above: as we decrease the distance a below a_c the crossover to the double quanta vortex spectrum starts in the region of small k_z values defined by the condition $\alpha > 1$. For $\alpha \ll 1$ we get the CdGM spectrum with a small oscillatory correction:

$$E - \omega(n + 1/2) \simeq \mp(-1)^n \omega\sqrt{2\alpha/\pi}\sin(k_\perp a + \pi/4). \quad (15)$$

The effective minigap $E_{\min} = \omega_0(1/2 - \sqrt{2\alpha/\pi})$ vanishes for $a \sim a_c$. For intermediate values $\sqrt{k_\perp\xi^2/a} \gg \alpha \gg 1$ the spectrum takes the form (5) if we put $b_{1,2} = \pm\chi/k_\perp$. This expression can be matched with the one obtained using a standard WKB expansion for Eq.(11) in the limit $\alpha \gg 1$ if we put

$$\frac{\pi\chi}{k_\perp a} = 1 + \int_0^{\xi/a} \left(\sqrt{\theta_p^2 + \frac{4\alpha}{k_\perp a}} - \theta_p\right) d\theta_p. \quad (16)$$

In agreement with general arguments presented above the DOS $\nu(E)$ in the limit $a < a_c$ consists of two sets of peaks shifted by the value $\omega_0(2\chi - [2\chi])$, where [...] denotes the integer part. The oscillatory behavior of the DOS at the Fermi level as a function of a is shown in Fig.2b.

In conclusion we note that the bending of vortex lines in a cluster can strongly affect the behavior of the wave functions along the magnetic field direction. Let us take, e.g., a two-vortex molecule and assume the function $a(z)$ to change slowly on a scale $\sim k_F^{-1}$. One can replace the momentum k_z in Eq.(5) by the operator $\hat{k}_z = -i\partial/\partial z$ and consider the spectrum branches as a set of effective Hamiltonians $E_{ni}(\hat{k}_z)$ describing the 1D quantum mechanics of QPs. Such approach is analogous to the one suggested in [14] for QPs in superconducting/normal metal layered structures. Taking a simple model profile $a(z) = \bar{a} - \tilde{a}z^2/L^2$ and choosing $n \simeq \mp k_F\bar{a}/\pi$ such 1D problem for energies close to the $E_{ni}(k_z)$ extremum can be reduced to the one of a quantum mechanical harmonic oscillator with the effective mass $m^* \sim mk_F\xi^2/\bar{a}$ and interlevel spacing $\Omega \sim \omega_0\sqrt{\bar{a}\tilde{a}}/L$. The corresponding wave functions are localized at a length scale $\sim L(k_F^2\tilde{a}\xi)^{-1/4}$. Such localization effect should result, in particular, in the suppression of the heat transport in the $z-$ direction.

We thank D. Ryzhov, V. Pozdnyakova, and I. Shereshevskii for help with computer codes, and N. Kopnin and G. Volovik for stimulating discussions. This work was supported, in part, by Russian Foundation for Basic Research, by Program "Quantum Macrophysics" of RAS, and by Russian Science Support and "Dynasty" Foundations. ASM acknowledges the support by the Academy of Finland.

1. C. Caroli and P. G. de Gennes, J. Matricon, Phys. Lett. **9**, 307 (1964).
2. E. Canel, Phys. Lett. **16**, 101 (1965); U. Klein, Phys. Rev. B **40**, 6601 (1989); B. Pöttinger and U. Klein,

Phys. Rev. Lett. **70**, 2806 (1993); M. Ichioka, N. Hayashi, and K. Machida, Phys. Rev. B **55**, 6565 (1997).

3. S. Dukan and Z. Tesanović, Phys. Rev. Lett. **74**, 2311 (1995); L. P. Gor'kov and J. R. Schrieffer, Phys. Rev. Lett. **80**, 3360 (1998); N. B. Kopnin and V. M. Vinokur, Phys. Rev. B **62**, 9770 (2000); T. Maniv et al., Rev. Mod. Phys. **73**, 867 (2001).
4. T. Terashima et al., Phys. Rev. B **56**, 5120 (1997).
5. G. Boato et al., Solid St. Commun. **3**, 173 (1965); D. S. McLachlan, Solid St. Commun. **8**, 1589 (1970); V. A. Schweigert et al., Phys. Rev. Lett. **81**, 2783 (1998); A. K. Geim et al., Phys. Rev. Lett. **85**, 1528 (2000); L. F. Chibotaru et al., Nature **408**, 833 (2000).
6. G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **57**, 233 (1993) [JETP Lett. **57**, 244 (1993)].
7. Y. Tanaka, A. Hasegawa, and H. Takayanagi, Solid St. Commun. **85**, 321 (1993); D. Rainer, J. A. Sauls, and D. Waxman, Phys. Rev. B **54**, 10094 (1996); S. M. M. Virtanen and M. M. Salomaa, Phys. Rev. B **60**, 14581 (1999); K. Tanaka, I. Robel, and B. Janko, PNAS **99**, 5233 (2002).
8. A. S. Mel'nikov and V. M. Vinokur, Nature **415**, 60 (2002); Phys. Rev. B **65**, 224514 (2002).
9. A. I. Larkin and Yu. N. Ovchinnikov, Phys. Rev. B **57**, 5457 (1998).
10. N. B. Kopnin et al., Phys. Rev. Lett. **95**, 197002 (2005).
11. N. B. Kopnin and G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **64**, 641 (1996) [JETP Lett. **64**, 690 (1996)]; N. B. Kopnin, Phys. Rev. B **57**, 11775 (1998).
12. A. I. Buzdin and J. P. Brison, Phys. Lett. **196**, 267 (1994).
13. E. O. Kane and E. I. Blount, in *Tunneling Phenomena in Solids*, Eds. E. Burstein and S. Lundqvist, New York, Plenum Press, 1969.
14. A. V. Shytov, P. A. Lee, and L. S. Levitov, Usp. Fiz. Nauk **168**, 222 (1998) [Physics–Uspekhi **41**, 207 (1998)].