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# Longitudinal resonance and identification of the order parameter of the $A$-like phase of superfluid $^3$He in aerogel

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Interpretation of the recent experiments of Dmitriev et al., on longitudinal resonance in the $A$-like phase, based on specific properties of "robust" order parameter is proposed.

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In the recent experiments of Dmitriev et al. [1] the longitudinal NMR in the superfluid $^3$He in aerogel was first observed and investigated. The results obtained for the $A$-like phase provide new input for identification of this phase. In these experiments in parallel with the longitudinal NMR the transverse line was registered. For the ensuing discussion the following results of Ref. [1] are of particular importance:

1) when the A-like phase is obtained by cooling without specially applied external perturbations there exist well defined longitudinal resonance;

2) transverse NMR signal at the same conditions is positively shifted with respect to the Larmor frequency. The line has two-maximum shape. The shift of one of the two maxima (referred as $f$-line in the Ref. [1]) is approximately 4 times greater then of the other ($c$-line);

3) after application of a series of 180° pulses on cooling through $T_c$ both the longitudinal resonance and the $f$-line in the transverse signal disappear. The intensity of the $c$-line increases.

In what follows it is shown that these qualitative features, in particular the disappearance of the longitudinal resonance, can be related to specific properties of the previously proposed "robust" order parameter [2, 3]:

$$A^R_{\mu j} = \Delta\frac{1}{\sqrt{3}}e^{i\psi}[\hat{d}_\mu(\hat{m}_j + i\hat{n}_j) + \hat{e}_\mu\hat{l}_j]. \qquad (1)$$

Here $l_j, m_j, n_j$ are mutually orthogonal unit vectors in momentum space and $d_\mu, e_\mu$ – mutually orthogonal unit vectors in spin space.

NMR probes the form of the order parameter via the dipole energy and via the anisotropy of magnetic susceptibility. For the order parameter Eq. (1) the dipole energy has the following form:

$$U_D = \frac{\chi_n}{8g^2}\Omega^2\left[(\hat{\mathbf{d}}\cdot\hat{\mathbf{m}} + \hat{\mathbf{e}}\cdot\hat{\mathbf{l}})^2 + (\hat{\mathbf{d}}\cdot\hat{\mathbf{n}})^2 + \hat{\mathbf{f}}\cdot\hat{\mathbf{n}}\right]. \qquad (2)$$

Here the spin vectors $d_\mu, e_\mu$ are complemented up to a triad by introduction of a unit vector $\hat{\mathbf{f}} = \hat{\mathbf{d}}\times\hat{\mathbf{e}}$.

Dependence of $U_D$ on a relative orientation of $\hat{\mathbf{f}}$ and $\hat{\mathbf{n}}$ reflects the lack of symmetry of the order parameter with respect to the interchange of $\hat{\mathbf{d}}$ and $\hat{\mathbf{e}}$. Minima of $U_D$ determine possible static orientations of the spin triad $\hat{\mathbf{d}}, \hat{\mathbf{e}}, \hat{\mathbf{f}}$ with respect to the orbital $\hat{\mathbf{l}}, \hat{\mathbf{m}}, \hat{\mathbf{n}}$. $U_D$ reaches its absolute minimum when

$$\hat{\mathbf{d}}\cdot\hat{\mathbf{m}} + \hat{\mathbf{e}}\cdot\hat{\mathbf{l}} = 0, \quad \hat{\mathbf{d}}\cdot\hat{\mathbf{n}} = 0, \qquad (3)$$

$$\hat{\mathbf{f}}\cdot\hat{\mathbf{n}} = -1, \qquad (4)$$

i.e. $\hat{\mathbf{f}}$ is antiparallel to $\hat{\mathbf{n}}$. With that restriction Eqs. (3) determine two mutually degenerate orientations of $\hat{\mathbf{d}}, \hat{\mathbf{e}}$ in a plane perpendicular to $\hat{\mathbf{n}}$. These are $\hat{\mathbf{d}} = \hat{\mathbf{l}}$, $\hat{\mathbf{e}} = -\hat{\mathbf{m}}$ and $\hat{\mathbf{d}} = -\hat{\mathbf{l}}$, $\hat{\mathbf{e}} = \hat{\mathbf{m}}$.

Except for these two minima the dipole energy has also a meta-stable minimum

$$\hat{\mathbf{f}}\cdot\hat{\mathbf{n}} = 1. \qquad (5)$$

In this case Eqs. (3) are met for arbitrary orientation of $\hat{\mathbf{d}}, \hat{\mathbf{e}}$ in a plane perpendicular to $\hat{\mathbf{n}}$, i.e. the minimum is continuously degenerate. Two configurations: Eq. (4) and Eq. (5) have different NMR "signatures". For the stable minimum (Ref. [4]) the longitudinal resonance frequency is:

$$\omega_\parallel^{(s)} = \Omega \qquad (6)$$

and transverse resonance frequency, in a principal order on $\Omega/\omega_L \ll 1$ is:

$$\omega_\perp^{(s)} = \omega_L + \Omega^2/4\omega_L. \qquad (7)$$

For the meta-stable orientation because of its continuous degeneracy

$$\omega_\parallel^{(m)} = 0. \qquad (8)$$

There is no shift of the transverse resonance frequency from the Larmor frequency in the order $(\Omega/\omega_L)^2$:

$$\omega_\perp^{(m)} = \omega_L + O\left[(\Omega/\omega_L)^4\right], \qquad (9)$$

i.e. the shift is much smaller then for the stable minimum.

Comparison of these signatures with the results of Ref. [1] suggests the following qualitative interpretation. In the experimental cell regions with the stable configuration $\widehat{\mathbf{f}}\cdot\widehat{\mathbf{n}}=-1$ coexist with the regions where meta-stable configuration $\widehat{\mathbf{f}}\cdot\widehat{\mathbf{n}}=1$ is realized. The line $f$ in the transverse NMR signal originates from regions with the stable configuration, while the line $c$, which has much smaller shift – from the meta-stable. The longitudinal signal comes from regions with the stable configuration only. Application of 180° pulses at cooling favors formation of the meta-stable configuration. This results in suppression of the $f$-line and in disappearance of the longitudinal resonance.

The reason why the pulses suppress the stable configuration is not clear. One can see that application of a $180^o$ pulse removes a barrier for conversion of the stable configuration in the meta-stable. This conversion requires reversal of the direction of spin $\mathbf{S}$ with respect to the triad $\widehat{\mathbf{d}},\widehat{\mathbf{e}},\widehat{\mathbf{f}}$. Motion of the triad $\widehat{\mathbf{d}},\widehat{\mathbf{e}},\widehat{\mathbf{f}}$ can be parametrized by Euler angles $\alpha,\beta,\gamma$ (cf. [5]), so that $\alpha$ has a meaning of the phase of precession of $\mathbf{S}$, $\gamma$ – the phase of rotation of the triad $\widehat{\mathbf{d}},\widehat{\mathbf{e}},\widehat{\mathbf{f}}$ around $\mathbf{S}$ and $\beta$ – of the tipping angle. Averaging of the dipole energy over a period of precession results in:

$$\bar{U}_D=\frac{\chi_n}{8g^2}\Omega^2\left[(1+\cos\beta)^2\sin^2(\alpha+\gamma)+ +\frac{1}{2}\sin^2\beta-\cos\beta\right]. \tag{10}$$

The sum $\alpha+\gamma$ is a “slow variable”, it keeps relative phase of precession and rotation close to the minimum of the $\bar{U}_D$ for a given tipping angle $\beta$: $\alpha+\gamma=n\pi$. As a result when $\beta$ is not close to $\pi$ a relative orientation of $\mathbf{S}$ with respect to the triad $\widehat{\mathbf{d}},\widehat{\mathbf{e}},\widehat{\mathbf{f}}$ does not change. At $\beta=\pi$ the coefficient in front of $\sin^2(\alpha+\gamma)$ turns to zero, angles $\alpha$ and $\gamma$ are decoupled and during relaxation of $\mathbf{S}$ to the equilibrium its absolute value and orientation with respect to the triad $\widehat{\mathbf{d}},\widehat{\mathbf{e}},\widehat{\mathbf{f}}$ can change. The average dipole energy for the meta-stable configuration does not depend on $\alpha+\gamma$ for all $\beta$. There is no symmetry between the stable and meta-stable configurations with respect to their reaction to 180° pulses, but it is not clear why the opposite conversion is not going on after tipping for 180°. So we have to take as experimental fact that when 180° pulses are applied at cooling the $c$-state is formed predominatingly, which within the suggested identification scheme corresponds to the meta-stable configuration.

Quantitative comparison reveals certain discrepancies. First, the shift of the $c$-line in the transverse resonance data, being much smaller then the shift of the $f$-line, is still much greater then one would expect from Eq. (9). This discrepancy could be ascribed to a deviation of the $A$-like order parameter realized in particular experiment from the limiting “robust” form. As an example of possible effect of such deviation on NMR signal consider the class of axi-planar order parameters, which are represented in a form:

$$A_{\mu j}=\Delta e^{i\phi}[\widehat{d}_\mu(a_y\widehat{m}_j+ia_z\widehat{n}_j)+a_x\widehat{e}_\mu\widehat{l}_j], \tag{11}$$

where $a_x,a_y,a_z$ are real numbers restricted by the normalization condition: $a_x^2+a_y^2+a_z^2=1$. This class includes the ABM phase ($a_y^2=a_z^2=1/2, a_x=0$) and the “robust” phase ($a_x^2=a_y^2=a_z^2=1/3$). Guided by the experiment [1] consider only such deviations from the ”robust” form, which preserve the degeneracy of the meta-stable minimum with respect to rotations of $\widehat{\mathbf{d}},\widehat{\mathbf{e}}$ in the plane perpendicular to $\widehat{\mathbf{n}}$. This condition is satisfied if $a_x=a_y\equiv u, a_z\equiv v$, i.e.

$$A_{\mu j}=\Delta e^{i\phi}[\widehat{d}_\mu(u\widehat{m}_j+iv\widehat{n}_j)+u\widehat{e}_\mu\widehat{l}_j]. \tag{12}$$

The dipole energy for this form of the order parameter is

$$U_D=\frac{\chi_n}{8g^2}\Omega^2\left[(\widehat{\mathbf{d}}\cdot\widehat{\mathbf{m}}+\widehat{\mathbf{e}}\cdot\widehat{\mathbf{l}})^2+w^2(\widehat{\mathbf{d}}\cdot\widehat{\mathbf{n}})^2+\widehat{\mathbf{f}}\cdot\widehat{\mathbf{n}}\right], \tag{13}$$

where $w=v/u$. The normalization constant $\Omega^2$ is defined so that $\omega_\parallel^{(s)}=\Omega$. Standard calculation renders:

$$\omega_\parallel^{(m)}=0 \tag{14}$$

and

$$\omega_\perp^{(m)}=\omega_L+(w^2-1)\frac{\Omega^2}{8\omega_L}. \tag{15}$$

For the robust state $w^2=1$ and $\omega_\perp^{(m)}=\omega_L$, but if $w^2>1$ the shift is finite and positive. The shift of the stable configuration also changes:

$$\omega_\perp^{(s)}=\omega_L+(w^2+1)\frac{\Omega^2}{8\omega_L}. \tag{16}$$

Experimentally measured ratio of the shift of the stable line to that of the meta-stable can be used for extraction of $w^2$ from the data:

$$\frac{\omega_\perp^{(m)}-\omega_L}{\omega_\perp^{(s)}-\omega_L}=\frac{w^2-1}{w^2+1}. \tag{17}$$

Assuming the r.h.s. is about $1/4$ we arrive at $w^2\approx 5/3$. With that value the order parameter, suitable for description of the experimental data can be written in a form Eq. (10) with $u^2 = 3/11$ and $v^2 = 5/11$ instead of 1/3 for the "robust" state.

Additional support for the proposed identification comes from the pulsed NMR data [1]. Dependence of the transverse shift on the tipping angle $\beta$ for the metastable configuration is given by:

$$\omega_{\perp}^{(m)} - \omega_L = (w^2 - 1)\frac{\Omega^2}{8\omega_L}\cos\beta. \tag{18}$$

The $\cos\beta$ dependence agrees with the data obtained in the experiment [1]. For the stable configuration the tipping angle dependence is given by:

$$\omega_{\perp}^{(s)} - \omega_L = \frac{\Omega^2}{8\omega_L}(1 + w^2\cos\beta). \tag{19}$$

When $w^2 = 1$ this formula coincides with that, obtained in Ref. [4]. The $1 + \cos\beta$ dependence was confirmed in the experiments with the 97% aerogel [6]. Dependence, following from Eq. (19) with $w^2 = 5/3$ also does not contradict to the data of [6].

Deviation of the order parameter from the robust form switches on disorienting effect of aerogel, so that the glass-like state could form. This state is based on the nearly robust order parameter (12) and not on the ABM, as suggested in Ref. [7]. For the nearly robust order parameter (12) disorienting effect of aerogel is described by a term proportional to $(v^2 - u^2)\eta_{jl}(\mathbf{r})n_j n_l$, where $\eta_{jl}(\mathbf{r})$ is a real symmetric traceless random tensor. The disorienting effect is weakened by a small factor $(v^2 - u^2)$. Characteristic length $L$ of the disordered state is inversely proportional to a square of this combination, so it must be much greater then in the ABM phase favoring a situation when $L$ is much greater then the dipole length and orientation of spin triad $\widehat{\mathbf{d}}, \widehat{\mathbf{e}}, \widehat{\mathbf{f}}$ with respect to the orbital $\widehat{\mathbf{l}}, \widehat{\mathbf{m}}, \widehat{\mathbf{n}}$ follows a minimum of the dipole energy. This property is crucial for the very existence of the longitudinal resonance. It means also that in a magnetic field with $\omega_L \gg \Omega$ the orbital vector $\mathbf{n}$ in equilibrium is aligned or counter aligned with the field. Since $\mathbf{n}$ is fixed the disorienting effect of the random anisotropy is suppressed and disordered state is not formed.

There remains a discrepancy between the value of the dipole frequency $\Omega$ extracted from the magnitude of the shift of $f$-line and of the directly measured longitudinal resonance frequency. According to the data [1] for 29.3 bar at $T = 0.835T_{ca}$ the measured longitudinal resonance frequency $\omega_{\parallel}$ is only 0.7 of the value of $\Omega$ obtained from the shift of the $f$-line for the same conditions via Eq.(16) with $w^2 = 5/3$. This discrepancy could partly originate from a difference in definitions of the resonance frequency in Eq. (6) and in the measurement procedure. Usually the resonance frequency $\omega_r$ is registered as a maximum of the imaginary part of magnetic susceptibility $\chi''(\omega, T)$ with respect to $\omega$ at constant $T$. Formally $\omega_r = \omega_r(T)$ is a root of equation:

$$\left(\frac{\partial\chi''(\omega, T)}{\partial\omega}\right)_T = 0. \tag{20}$$

In the experiment [1] $\chi''(\omega, T)$ was measured as a function of $T$ at constant $\omega = \omega_0$. Resonance temperature $T_r$ was determined as a maximum of $\chi''(\omega_0, T)$ with respect to $T$:

$$\left(\frac{\partial\chi''(\omega, T)}{\partial T}\right)_{\omega=\omega_0} = 0 \tag{21}$$

and $\omega_0$ was taken as a resonance frequency at $T = T_r$. Two definitions need not give the same frequency: $\omega_r(T = T_r) \neq \omega_0$. When the difference $\nu = \omega_0 - \omega_r$ is small it can be expressed in terms of derivatives of $\chi''(\omega, T)$, taken at $\omega = \omega_r(T)$:

$$\nu = \frac{(\partial\chi''/\partial T)}{(d\omega_r/dT)(\partial^2\chi''/\partial\omega^2)}. \tag{22}$$

For the present situation $\nu$ is negative i.e. the resonance frequency $\omega_r$ and hence $\omega_{\parallel}^{(s)}$ is greater then its apparent value $\omega_0$. Crude estimation of the derivatives renders $\nu \sim \Gamma^2/\omega_r$, where $\Gamma$ is a width of resonance. When $\Gamma \ll \omega_r$ this correction is negligible, but it is not the case for the longitudinal NMR line in the $A$-like phase. Relative correction $\nu/\omega_r$ for the line, measured in Ref. [1] could be of the order of 0.1, which is a substantial decrease of the discrepancy.

In conclusion, it should be pointed out that the proposed identification of the observed NMR lines follows naturally from the assumption of the robust form for the order parameter of the $A$-like phase. It gives consistent qualitative description of the data. Quantitative discrepancies are partly removed by account of possible deviations of the order parameter from the robust form and by introduction of the discussed correction in the analysis of the data. Further NMR experiments, as suggested in Ref. [1], will help to make definitive conclusion about the form of the order parameter of the $A$-like phase.

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