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Cross-over behavior of disordered interacting two-dimensional electron systems in a parallel magnetic field

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We present analysis of the cross-over behavior of disordered interacting two-dimensional electron systems in the parallel magnetic field. Using the so-called cross-over one-loop renormalization group equations for the resistance and electron-electron interaction amplitudes we qualitatively explain experimentally observed transformation of the temperature dependence of the resistance from a reentrant (nonmonotonic) behavior in relatively weak fields into an insulating-type behavior in stronger fields.

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Disordered two-dimensional electron systems (2DES) have been in the focus of experimental and theoretical research for several decades [1]. The interest to 2DES has been renewed because of the discovery of the metal-insulator transition (MIT) in a high mobility silicon metal-oxide-semiconductor field-effect transistor (Si-MOSFET) [2]. During last decade an interesting behavior of resistance and spin susceptibility has been found experimentally not only in Si-MOSFET but in other 2D electron systems [3]. Recently, a major step toward the theoretical proof for the MIT existence in 2DES has been made in Ref. [4].

If an electron density is higher than the critical one (metallic phase) at low temperatures $T \ll \tau^{-1}$ [τ stands for the transport mean-free path time] the increase of the resistance with decreasing temperature is replaced by the drop as T becomes lower than some $T_{\max}$ (see Fig.1) [3]. This nonmonotonic behavior of the resistance has been predicted from the renormalization group (RG) analysis of the interplay between disorder and electron-electron interaction in 2DES [5, 6]. As a weak magnetic field $B_{\|}$ is applied parallel to 2DES the decrease of the resistance is stopped at some temperature and the resistance increases again [7]. Further increase of $B_{\|}$ leads to the monotonic growth of the resistance as temperature is lowered.

In the Letter we present the theoretical explanation for this striking behavior of the resistance in parallel magnetic field. We demonstrate that it can be explained with the help of the RG analysis of disorder and electron-electron interaction in 2DES in the presence of the Zeeman splitting.

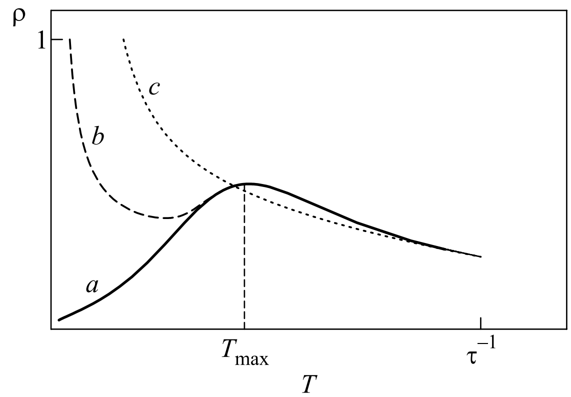

Fig.1. The sketch of the temperature dependence of the resistance in the presence of $B_{\|}$. The curve a corresponds to the case $B_{\|} = 0$, the curve b to the case $g_L \mu_B B_{\|} \ll T_{\max}$ and the curve c to the case $g_L \mu_B B_{\|} \gg T_{\max}$

The presence of the parallel magnetic field introduces a new energy scale $g_L \mu_B B_{\|}$ into the problem. Here g_L and μ_B stand for the Landé factor and the Bohr magneton respectively. The Zeeman splitting $g_L \mu_B B_{\|}$ sets the cut-off for a pole in the diffusive modes (diffusons) with opposite electron spin projections [5]. In the temperature range $g_L \mu_B B_{\|} \ll T \ll \tau^{-1}$ this cut-off is irrelevant and 2DEG behaves as if no parallel magnetic field is applied. However, at lower temperatures $T \ll g_L \mu_B B_{\|}$ the diffusive modes with opposite electron spin projections do not contribute and 2DES behaves as in the presence of the strong parallel magnetic field $B_{\|}^{\infty} \sim (g_L \mu_B \tau)^{-1}$. It is well known [5] that in this case the resistance monotonically grows as T is lowered. Therefore, if the parallel magnetic field is weak $g_L \mu_B B_{\|} \ll T_{\max}$, then the T-dependence of the resistance manifests the reentrant behavior with maximum and minimum. At higher parallel magnetic fields $g_L \mu_B B_{\|} \gg T_{\max}$ the insulating-type

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behavior of the resistance restores such that it grows monotonically with decreasing T (see Fig.1).

Below we shall illustrate this simple physical explanation of non-monotonic behavior of the resistance by the so-called cross-over RG equations (cf. Eqs. (17)-(21)) for the resistance and the electron-electron interaction amplitudes that smoothly interpolate between the well-known cases of $B_{\|}=0$ and strong field.

We consider two-dimensional interacting electrons with n_v valleys in the presence of the quenched disorder and the parallel magnetic field at low temperatures $T \ll \tau^{-1}$. We assume that a magnetic field $B_\perp \gtrsim T/(De)$ where e and D stand for the electron charge and diffusion coefficient respectively is applied perpendicular to 2DES in order to suppress the Cooper channel. We suppose that both the inverse intervalley scattering time and the valley splitting are much less than the temperature.

Following Finkelstein [5], the effective quantum theory of disordered interacting two-dimensional electrons is given in terms of the generalized non-linear σ-model involving unitary matrix field variables $Q_{mn}^{\alpha_1\alpha_2;\zeta_1\zeta_2}(\mathbf{r})$ which obey the constraint $Q^2(\mathbf{r})=1$. Here the integers $\alpha_i=1,2,\dots,N_r$ denote the replica indices, m,n correpond to the discrete set of the Matsubara frequencies $\omega_n=\pi T(2n+1)$. The integers $\zeta_i=\pm1,\pm2,\dots,\pm n_v$ are the combined spin and valley indices. The effective action is given as

$$S=\int d^2\mathbf{r}\left(\mathcal{L}_\sigma+\mathcal{L}_F+\mathcal{L}_{B_\|}+\mathcal{L}_h\right). \qquad (1)$$

Here $\mathcal{L}_\sigma$ is the free electron part [8]

$$\mathcal{L}_\sigma=\frac{\sigma}{16n_v}\mathrm{Tr}\,(\nabla Q)^2, \qquad (2)$$

where σ denotes the mean-field conductance in units e^2/h with h being the Plank constant and symbol Tr is the trace over Matsubara, replica, spin and valley indices. The $\mathcal{L}_F$ involves the electron-electron interaction amplitudes which describe the scattering on small (Γ) and large (Γ_2) angles and the quantity z (originally introduced by Finkelstein [5]) which is responsible for the specific heat renormalization [9],

$$\mathcal{L}_F=4\pi Tz\mathrm{Tr}\,\eta(\Lambda-Q)-\pi T\Gamma\sum_{\alpha n}\mathrm{Tr}\,I_n^\alpha Q\mathrm{Tr}\,I_{-n}^\alpha Q$$
$$+\pi T\Gamma_2\sum_{\alpha n}(\mathrm{Tr}\,I_n^\alpha Q)\otimes(\mathrm{Tr}\,I_{-n}^\alpha Q)+2\pi Tz\mathrm{Tr}\,\eta\Lambda. \qquad (3)$$

Here $\mathrm{Tr}\,A\otimes\mathrm{Tr}\,B=A_{nn}^{\alpha\alpha;\zeta_1\zeta_2}B_{mm}^{\beta\beta;\zeta_2\zeta_1}$ and the matrices Λ, η and I_k^γ are given as

$$\Lambda_{nm}^{\alpha\beta;\zeta_1\zeta_2}=\mathrm{sign}\,(\omega_n)\delta_{nm}\delta^{\alpha\beta}\delta^{\zeta_1\zeta_2},$$
$$\eta_{nm}^{\alpha\beta;\zeta_1\zeta_2}=n\delta_{nm}\delta^{\alpha\beta}\delta^{\zeta_1\zeta_2}, \qquad (4)$$
$$(I_k^\gamma)_{nm}^{\alpha\beta;\zeta_1\zeta_2}=\delta_{n-m,k}\delta^{\alpha\gamma}\delta^{\beta\gamma}\delta^{\zeta_1\zeta_2}.$$

In the presence of the parallel magnetic field $B_\|$ the Zeeman splitting should be taken into account [5]

$$\mathcal{L}_{B_\|}=-iz_2g_L\mu_BB_\|\mathrm{Tr}\,\tau_zQ+\frac{n_vg_L^2\mu_B^2z_2}{2\pi T}N_rB_\|^2. \qquad (5)$$

Here $z_2=z+\Gamma_2$ and the Pauli matrix τ_z is defined as

$$(\tau_z)_{nm}^{\alpha_1\alpha_2;\zeta_1\zeta_2}=\mathrm{sign}\,(\zeta_1)\delta_{nm}\delta^{\alpha\beta}\delta^{\zeta_1\zeta_2}. \qquad (6)$$

We mention that the last term in $\mathcal{L}_{B_\|}$ corresponds to the Fermi-liquid spin susceptibility. Finally, the term

$$\mathcal{L}_h=-\frac{\sigma h^2}{4n_v}\mathrm{Tr}\,\Lambda Q \qquad (7)$$

is not a part of the theory but we shall use it later on as a convenient infrared regulator of the theory.

The action (1) involves the matrices which are formally defined in the infinite Matsubara frequency space. In order to operate with them we have to introduce a cut-off for the Matsubara frequencies. Then the set of rules which is called $\mathcal{F}$-algebra can be established [10]. At the end of all calculations we tend the cut-off to infinity.

The theory (1) should be supplemented by the important constraint that the combination $z+\Gamma_2-2n_v\Gamma$ remains constant in the course of the RG flow. Physically, it corresponds to the conservation of the number of particles [5]. In the special case of the Coulomb interaction which is of the main interest for us in the paper the relation $z+\Gamma_2-2n_v\Gamma=0$ holds and the action (1) with $B_\|=0$ is invariant under a global rotation of the matrix Q ($\mathcal{F}$-invariance) [10].

The most significant physical quantities in the theory comprising the complete information on its low-energy dynamics are the physical observables σ', z_2' and z' associated with the mean-field parameters σ, z_2 and z of the action (1). The quantity σ' is the conductance of 2DES as one can obtain from a linear response to electromagnetic field, $n_vg_L^2\mu_B^2z_2'/\pi$ is the spin susceptibility of 2DES, and z' is related with the specific heat of 2DES [10]. Extremely important to remind that the observable parameters σ', z_2' and z' are precisely the same as those defined by the background field procedure [11].

The conductance σ' is expressed in terms of the current-current correlations as [10]

$$\sigma' = -\frac{\sigma}{8n_v n}\left\langle \mathrm{Tr}\,[I_n^\alpha, Q][I_{-n}^\alpha, Q]\right\rangle + \tag{8}$$
$$+\frac{\sigma^2}{16n_v^2 nd}\int d^d\mathbf{r}'\langle\langle \mathrm{Tr}\, I_n^\alpha Q(\mathbf{r})\nabla Q(\mathbf{r})\mathrm{Tr}\, I_{-n}^\alpha Q(\mathbf{r}')\nabla Q(\mathbf{r}')\rangle\rangle,$$

where the limit $n \to 0$ is assumed and d denotes the dimension. Here and from now onwards the expectations are defined with respect to the theory (1). The observable z_2' is given by [5]

$$z_2' = \frac{\pi}{n_v(g_L\mu_B)^2 N_r}\frac{\partial^2\Omega}{\partial B_\|^2}, \tag{9}$$

where Ω denotes the thermodynamic potential of the unit volume. A natural definition of z' is obtained through the derivative of Ω with respect to T [10],

$$z' = \frac{1}{2\pi \mathrm{Tr}\,\eta\Lambda}\frac{\partial}{\partial T}\frac{\Omega}{T}. \tag{10}$$

To define a theory for perturbative expansions we use the "square-root" parameterization

$$Q = W + \Lambda\sqrt{1-W^2}, \qquad W = \begin{pmatrix} 0 & w \\ w^\dagger & 0\end{pmatrix}. \tag{11}$$

The action (1) can be written as an infinite series in the independent fields $w_{n_1 n_2}^{\alpha_1\alpha_2,\zeta_1\zeta_2}$ and $w_{n_4 n_3}^{\dagger\alpha_1\alpha_2,\zeta_1\zeta_2}$. We use the convention that Matsubara indices with odd subscripts $n_1, n_3, \ldots$ run over non-negative integers whereas those with even subscripts $n_2, n_4, \ldots$ run over negative integers.

The propagators can be written in the following form

$$\langle w_{n_1 n_2}^{\alpha_1\alpha_2;\zeta_1,\zeta_2}(p) w_{n_4 n_3}^{\dagger\alpha_4\alpha_3;\zeta_4\zeta_3}(-p)\rangle = \frac{8n_v}{\sigma}\delta^{\alpha_1\alpha_3}\delta^{\alpha_2\alpha_4}\times \tag{12}$$
$$\times\delta_{n_{12},n_{34}}\Big[\delta_{n_1,n_3}\delta^{\zeta_1\zeta_3}\delta^{\zeta_2\zeta_4}D_p(\omega_{12}, i\omega_{12}^B) -$$
$$-\frac{16\pi n_v T z\gamma_2}{\sigma}\delta^{\alpha_1\alpha_2}\delta^{\zeta_1\zeta_3}\delta^{\zeta_2\zeta_4}D(\omega_{12}, i\omega_{12}^B)D_p^t(\omega_{12}, i\omega_{12}^B)$$
$$+\frac{8\pi T z(1-\alpha+\gamma_2)}{\sigma}\delta^{\alpha_1\alpha_2}\delta^{\zeta_1\zeta_2}\delta^{\zeta_3\zeta_4}D^s(\omega_{12})D_p^t(\omega_{12})\Big],$$

where we introduce the notations $z\alpha = z + \Gamma_2 - 2n_v\Gamma$, $\gamma_2 = 1 + \Gamma_2/z$, $\omega_{12} = 16n_v\pi T z(n_1 - n_2)/\sigma$, $\omega_{12}^B = \omega_B(\mathrm{sign}\,\zeta_1 - \mathrm{sign}\,\zeta_2)/2$ with $\omega_B = 8n_v z_2 g_L\mu_B B_\|/\sigma$ and

$$D_p(\omega, x) = [p^2 + h^2 + \omega + x]^{-1},$$
$$D_p^s(\omega) = [p^2 + h^2 + \alpha\omega]^{-1},$$
$$D_p^t(\omega, x) = [p^2 + h^2 + (1+\gamma_2)\omega + x]^{-1},$$
$$D_p(\omega) \equiv D_p(\omega, 0), \qquad D_p^t(\omega) \equiv D_p^t(\omega, 0). \tag{13}$$

The standard one-loop analysis for the physical observables σ', z_2' and z' performed at $T = 0$ yields

$$\sigma' = \sigma + \frac{8}{d}\int\frac{d^d p\, p^2}{(2\pi)^d}\int_0^\infty d\omega\big[(2n_v^2 - 1)\gamma_2 D_p^2(\omega)D_p^t(\omega) +$$
$$+2n_v^2\gamma_2\,\mathrm{Re}\,D_p^2(\omega, i\omega_B)D_p^t(\omega, i\omega_B) - (1-\alpha)D_p^2(\omega)D_p^s(\omega)\big],$$

$$z_2' = z_2 + \frac{8n_v^2}{\sigma}z_2(1+\gamma_2)\,\mathrm{Re}\int\frac{d^d p}{(2\pi)^d}\int_0^\infty d\omega\,\times$$
$$\times\Big[D_p^2(\omega, i\omega_B) - D_p^{t2}(\omega, i\omega_B)\Big],$$
$$z' = z + \frac{2z}{\sigma}\int\frac{d^d p}{(2\pi)^d}\Big[\alpha D_p^s(0) - 2n_v^2\,\mathrm{Re}\,D_p(0, i\omega_B) -$$
$$-2n_v^2 D_p(0) + (2n_v^2 - 1)(1+\gamma_2)D_p^t(0) +$$
$$+2n_v^2(1+\gamma_2)\,\mathrm{Re}\,D_p^t(0, i\omega_B)\Big]. \tag{14}$$

In what follows we shall employ the dimensional regularization scheme with $d = 2 + 2\epsilon$. Evaluating the momentum and frequency integrals in Eqs. (14) for the case of the Coulomb interaction ($\alpha = 0$) we obtain for the physical observables $\rho' = 4\Gamma(1-\epsilon)(4\pi)^{-d/2}/\sigma'$, z_2' and z'

$$\frac{1}{\rho'} = \frac{1}{\rho} + \frac{h^{2\epsilon}}{\epsilon}\Big[1 + g_t(\gamma_2) - 2n_v^2 g_t(\gamma_2)(1 + f_\epsilon(\omega_B/h^2))\Big],$$
$$z_2' = z_2\left[1 - \frac{2n_v^2\gamma_2\rho\, h^{2\epsilon}}{\epsilon}f_\epsilon(\omega_B/h^2)\right], \tag{15}$$
$$z' = z\Big[1 + \frac{\rho\, h^{2\epsilon}}{2\epsilon}\Big(1 - \gamma_2(2n_v^2 - 1) + 2n_v^2\gamma_2 f_\epsilon(\omega_B/h^2)\Big)\Big],$$

where $\rho = 4\Gamma(1-\epsilon)(4\pi)^{-d/2}/\sigma$, $f_y(x) = \mathrm{Re}(1+ix)^y$ and

$$g_t(\gamma_2) = \frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2) - 1. \tag{16}$$

The standard minimal subtraction scheme is the ω_B-independent and, therefore, is unable to treat the limits of vanishing ($\omega_B = 0$) and strong ($\omega_B \to \infty$) magnetic fields simultaneously. In order to avoid this problem and obtain the renormalization that correctly describes the two limiting cases of $\omega_B = 0$ and $\omega_B \to \infty$ we shall use the so-called cross-over renormalization scheme [12].

The parameter h naturally sets the momentum scale at which the bare parameters ρ, z_2 and z of the action (1) are defined. The physical observables ρ', z_2' and z' correspond to the momentum scale h' [13, 14] which is determined as $\sigma' h' \mathrm{Tr}\, 1 = \sigma h \mathrm{Tr}\,\Lambda\langle Q\rangle$. By using the relation $h' = h[1 + O(\rho)]$ we can substitute h' for h in Eqs. (15). Since the bare parameters ρ, z_2, and z are independent of h', we obtain from Eqs. (15) the following one-loop cross-over RG equations in two dimensions ($\epsilon = 0$)

The function $F(\gamma_2)$ for the cases of vanishing and strong magnetic field. Symbol $\mathrm{li}_n(x) = \sum_{k=1}^{\infty} x^k/k^n$ denotes the polylogarithmic function

f_c	$F(\gamma_2)$
0	$\frac{8n_v^2\gamma_2}{1+\gamma_2} + (4n_v^2-1)\ln^2(1+\gamma_2) + 2(4n_v^2-1)\mathrm{li}_2(-\gamma_2)$
1	$\begin{cases} 2g_t\left(\frac{-1}{2n_v^2}\right)\ln[1-(2n_v^2-1)\gamma_2] + 2(2n_v^2-1)\left[\mathrm{li}_2(-\gamma_2) - \mathrm{li}_2\left(\frac{1-(2n_v^2-1)\gamma_2}{2n_v^2}\right)\right] + 2\ln(1+\gamma_2), & \gamma_2 < \gamma_2^\star \\ 2g_t\left(\frac{-1}{2n_v^2}\right)\ln\left[1-\frac{1}{(2n_v^2-1)\gamma_2}\right] + 2(2n_v^2-1)\left[\mathrm{li}_2\left(1-\frac{1}{(2n_v^2-1)\gamma_2}\right) - \mathrm{li}_2\left(\frac{(2n_v^2-1)\gamma_2-1}{2n_v^2\gamma_2}\right)\right] + 2\ln\frac{1+\gamma_2}{\gamma_2}, & \gamma_2 \geqslant \gamma_2^\star \end{cases}$

$$\frac{d\rho'}{d\eta} = [a_0(\gamma_2') + f_c a_1(\gamma_2')]\rho'^2, \tag{17}$$

$$\frac{d\gamma_2'}{d\eta} = -\rho'\Big[b_0(\gamma_2') + f_c b_1(\gamma_2')\Big], \tag{18}$$

$$\frac{d\ln z'}{d\eta} = -\rho'\Big[c_0(\gamma_2') + f_c c_1(\gamma_2')\Big], \tag{19}$$

$$\frac{df_c}{d\eta} = -4f_c(1-f_c), \quad f_c(0) = \mathcal{B}^2/(1+\mathcal{B}^2). \tag{20}$$

Here we have introduced $\gamma_2' = z_2'/z' - 1$ and variable $\eta = \ln lh'$ where $l \propto h^{-1}$ corresponds to a mean-free path length at which, physically, the bare parameters of the action (1) are defined and

$$a_0(\gamma_2') = -2\left[1-(4n_v^2-1)g_t(\gamma_2')\right], \; b_0(\gamma_2') = (1+\gamma_2')^2,$$
$$c_0(\gamma_2') = (4n_v^2-1)\gamma_2' - 1, \quad a_1(\gamma_2') = -4n_v^2 g_t(\gamma_2'),$$
$$b_1(\gamma_2') = -2n_v^2\gamma_2'(1+\gamma_2'), \quad c_1(\gamma_2') = -2n_v^2\gamma_2'. \tag{21}$$

The cross-over parameter f_c is expressed via the quantity $\mathcal{B} = \omega_B l^2$. Eqs. (17)–(19) smoothly interpolate between the known results for the cases $f_c = 0$ and $f_c = 1$ [5, 6]. They were derived under assumption that $\bar{\rho}' \ll 1$. From here onwards we omit the 'prime' sign for convenience.

In general, the function $f_c(\eta)$, i.e., the right hand side of Eq. (20), is not universal. It depends on a method employed to derive the cross-over RG equations. The only universal properties of Eq. (20) are the existence of two fixed points $f_c = 0$ and $f_c = 1$. Fortunately, the qualitative features of the RG flow for Eqs. (17)–(21) are independent of the choice of $f_c(\eta)$ provided it smoothly interpolates between $f_c = 0$ at $\mathcal{B}e^{-2\eta} \ll 1$ and $f_c = 1$ at $\mathcal{B}e^{-2\eta} \gg 1$.

Let us start the analysis of Eqs. (17)–(19) from the case $f_c = 0$. Then we find $\rho(\eta) \propto \exp F(\gamma_2(\eta))$ where the function $F(\gamma_2)$ is given in Table. For all values of n_v γ_2 increases monotonically with decreasing η and diverges at $\eta_c = \Upsilon(\infty; \bar{\rho}, \bar{\gamma}_2)$ where

$$\Upsilon(\gamma_2; \bar{\rho}, \bar{\gamma}_2) = -\frac{1}{\bar{\rho}} e^{F(\bar{\gamma}_2)} \int_{\bar{\gamma}_2}^{\gamma_2} \frac{du}{b(u)} e^{-F(u)}. \tag{22}$$

with $b(u) \equiv b_0(u)$, $\bar{\rho} = \rho(0)$ and $\bar{\gamma}_2 = \gamma_2(0)$. We present the RG flow diagram for $n_v = 1$ in Fig.2. The RG flow is qualitatively the same for all values of n_v. As γ_2 monotonous increases an initial growth of ρ changes

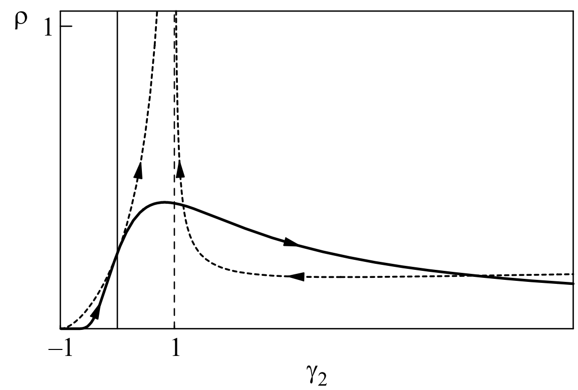

Fig.2. RG flow diagram in ρ versus γ_2 for $f_c = 0$ (solid line) and for $f_c = 1$ (dashed lines) with $n_v = 1$. The arrows indicate the direction towards the infrared $L \to \infty$ ($\eta \to -\infty$). See text

into a decline, i.e., 2DES remains in the metallic phase ($\rho \to 0$) at large lengthscales ($\eta \to -\infty$).

In the opposite case $f_c = 1$ the integration of Eqs. (17)–(19) yields $\rho(\eta) \propto \exp F(\gamma_2(\eta))$ with the function $F(\gamma_2)$ presented in Table. If $\bar{\gamma}_2 < \gamma_2^\star = (2n_v^2-1)^{-1}$ ($\bar{\gamma}_2 > \gamma_2^\star$) γ_2 increases (decreases) monotonous as η diminishes. It reaches finally the value $\gamma_2^\star$ at $\eta_c = \Upsilon(\gamma_2^\star; \bar{\rho}, \bar{\gamma}_2)$ where the function $\Upsilon(\gamma_2; \bar{\rho}, \bar{\gamma}_2)$ is given by Eq. (22) with $b(u) \equiv b_0(u) + b_1(u)$. We mention that simultaneously $\rho(\eta)$ diverges at η_c. We plot the RG flow diagram with $n_v = 1$ in Fig.2. Unlike the case $f_c = 0$, 2DES is in the insulating phase ($\rho \to \infty$) at large lengthscales for $f_c = 1$, γ_2 being quenched to $\gamma_2^\star$.

In the intermediate case $0 < f_c < 1$ the cross-over RG Eqs. (17)–(21) can be solved only numerically. If the parallel magnetic field is smaller than some $\mathcal{B}_X$ which is a function of $\bar{\rho}$ and $\bar{\gamma}_2$ the resistance has the reentrant behavior with the maximum and the minimum at some values of η (see Fig.1). At $\mathcal{B} > \mathcal{B}_X$ the insulator-type behavior of the $\rho(\eta)$ that one expects for $\mathcal{B} \to \infty$ is restored. For $\bar{\gamma}_2 < \gamma_2^\star$ the function $\gamma_2(\eta)$ has the maximum whereas $z_2(\eta) = z(\eta)(1+\gamma_2(\eta)$ (spin susceptibility) has the minimum. Their behavior does not change qualitatively when the parallel magnetic field passes through the value $\mathcal{B}_X$. For $\bar{\gamma}_2 > \gamma_2^\star$ the maximum in the function $\gamma_2(\eta)$ disappears for $\mathcal{B} > \mathcal{B}_X$ whereas $z_2(\eta)$ increases monotonous for all values of $\mathcal{B}$. As expected, the cross-over field $\mathcal{B}_X$ can be estimated as $\mathcal{B}_X \sim \exp(2\eta_{\max})$ where $\eta_{\max} = \Upsilon(\gamma_2^{\max}; \bar{\rho}, \bar{\gamma}_2)$ determines the position of the maximum on the curve $\rho(\eta)$ at $f_c = 0$. Here $\gamma_2^{\max}$ is given as the root of equation $a_0(\gamma_2) = 0$.

We have not considered above the contribution to the one-loop RG equations from the particle-particle (Cooper) channel. It can be shown [15] that the Zeeman splitting due to applied parallel magnetic field does not affect it in the one-loop approximation. Therefore, the Cooper-channel contribution to the RG equations can be taken into account by the substitution of $a_0(\gamma_2) - 2n_v$ for $a_0(\gamma_2)$ [6]. As one can check, the behavior of $\rho(\eta)$, $\gamma_2(\eta)$ and $z_2(\eta)$ in this case remains qualitatively the same as discussed above.

At zero temperature, h' plays the role of the inverse lengthscale L which is physically nothing else than a sample size of 2DES. If $L \gg \sqrt{D/T}$ the temperature behavior of the physical observables can be found from the cross-over RG Eqs. (17)–(19) stopped at the inelastic length L_{in}. Formally, it means that one should substitute $\eta_T = \frac{1}{2}\ln Tl^2/D$ for η with the help of the following equation $d\eta_T/d\eta = 1 - d\ln z/2d\eta$ [16]. The temperature dependence of the resistance in the presence of $B_{\|}$ obtained from $\rho(\eta)$ in this way agrees qualitatively with the experimental results of Ref. [7]. The detailed comparison between the theory and the experimental data on the resistance in the parallel magnetic field will be presented in the following Letter [17].

Also, we remind that the physical observables that we use throughout the paper are *ensemble averaged*. Even in a macroscopic sample of size $L \gg L_{\text{in}}$ they are different from the *measured* resistance $\rho(T)$ and spin susceptibility $z_2(T)$ due to statistically independent fluctuations of local conductance and electron-electron amplitudes in blocks of the size L_{in} [18].

Finally, we mention that the symmetry-breaking strain applied to 2DES with two-valleys [19] should affect the temperature dependence of the resistance in similar way as $B_{\|}$.

In summary, we have presented the analysis of the cross-over behavior of disordered interacting 2DES in the parallel magnetic field. The one-loop cross-over RG equations that smoothly interpolate between the two well-known limiting cases of vanishing and strong parallel magnetic field allow us to explain qualitatively the reentrant (nonmonotonic) behavior of the resistance of 2DES as a function of T in the presence of a relatively weak parallel magnetic field as well as its insulating behavior at stronger fields.

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