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Critical behavior of transport and magnetotransport in 2D electron system in Si in the vicinity of the metal-insulator transition

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We report on studies of the magnetoresistance in strongly correlated 2D electron system in Si in the critical regime, in the close vicinity of the 2D metal-insulator transition. We performed self-consistent comparison of our data with solutions of two equations of the cross-over renormalization group (CRG) theory, which describes temperature evolutions of the resistivity and interaction parameters for 2D electron system. We found a good agreement between the $\rho(T, B_{\parallel})$ data and the RG theory in a wide range of the in-plane fields, 0–2.1 T. This agreement supports the interpretation of the observed 2D MIT as the true quantum phase transition.

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Low-temperature transport in many high mobility 2D electron systems was found to manifest a critical behavior [1 – 4]. This phenomenon takes place at low carrier densities, therefore the electron-electron correlations play a crucial role. In transport studies, as temperature decreases, the overall picture of the temperature dependence of resistivity ρ shows clearly distinct behavior in two different domains on the density scale. At low densities, $n < n_c$, resistivity exponentially increases with cooling, whereas at high densities, $n > n_c$, resistivity significantly decreases (here n_c is a sample-dependent critical density, $\sim 10^{11}\,\mathrm{cm}^{-2}$ for high mobility Si samples). In the former domain, transport is consistent with a conventional picture of hopping conduction, that is typical for an insulator [5]. In the latter domain, far from the critical density (and at $\rho \ll h/e^2$), temperature dependence of conductivity was experimentally shown [6] to be explained by the Fermi-liquid effects that were calculated within the framework both of the quantum interaction corrections [7], and temperature dependent screening [8]. It was so far unclear whether the metallic type transport persists to $T = 0$ and whether the cross-over from the metallic to the insulating behavior (that is observed at low though finite temperatures) signifies a true quantum phase transition.

The successful comparison with the theory of interaction corrections in the high density regime, $n \gg n_c$, encouraged us to extend the comparison to the critical regime of lower densities $n \approx n_c$ and higher resistivities $\rho \sim h/e^2$. The method that now commonly in use for this regime and in the diffusive interaction limit $T\tau \ll 1$ is a generalization of the nonlinear σ-model theory, which has been developed by Finkelstein [9]. The RG equations [9, 10] describe length scale (temperature) evolutions of the resistivity and interaction parameters for 2D electron system in the first order in $\rho/(\pi h/e^2)$ and in all orders in interaction.

Earlier [11], only experimental $\rho(T)$-data [12] in zero field has been compared with one of the RG-equations, while temperature dependence of the interaction parameter $\gamma_2(T)$ [9] was not tested. In this paper we have extracted, for the first time, $\gamma_2(T)$ from the low-field magnetoresistance measurements, using cross-over RG (CRG) equations, proposed in Ref. [13] for in-plane magnetic fields, varying from vanishingly low to large. We have compared $\gamma_2(T)$ with theoretical dependence calculated from the RG-theory [11] in zero field. Also, we have compared our $\rho(T, B_{\parallel})$ data with the solutions of the CRG equations. For these purposes, we measured $\rho(T, B_{\parallel})$ in a wide range of the in-plane fields $B_{\parallel} = 0 - 2.5\,\mathrm{T}$. We have found a good agreement of the measured $\rho(T, B_{\parallel})$ and $\gamma_2(T)$ with the RG-theory.

Measurements were performed with a Si-MOS sample (peak mobility $3\,\mathrm{m^2/Vs}$ at $T = 1.3\,\mathrm{K}$) of the rectangular geometry $5\times 0.8\,\mathrm{mm}^2$. We used four-terminal ac-technique at 5 Hz frequency; source-drain current was chosen low enough, $I = 10\,\mathrm{nA}$, in order to avoid electron overheating. The sample was located at a precise rotation stage that enabled to align its plane parallel to the magnetic field directions with accuracy of $\sim 1'$. The alignment was controlled by observation of the suppression of the weak localization (WL) peak in ρ_{xx} in the magnetic field B. For studies we have chosen the temperature range $(1.3–4.2)\,\mathrm{K}$, because for the studied high-mobility Si-MOS sample in the critical regime $n \approx n_c$ the resistivity exhibits a well-pronounced maximum at about $T_{\max} \approx 2-3$ K with relatively low resistivity $\rho_{\max} \sim h/e^2$. For lower densities, the $\rho(T)$ maximum shifts to lower temperatures and higher resistivities; as a result, $\rho_{\max}(e^2/\pi h)$ is no longer a small parameter as needed for comparison with the one-loop RG theory.

Figure 1 shows typical temperature dependences of the resistivity in the critical regime for various $B_{\parallel}$ fields in the range from 0 to 2.5 T. For the studied sample, the metal-insulator transition in zero field takes place at the critical density $n_c = 0.86 \cdot 10^{11}\,\mathrm{cm}^{-2}$. Data shown in Fig.1 were taken for the carrier density $n =$

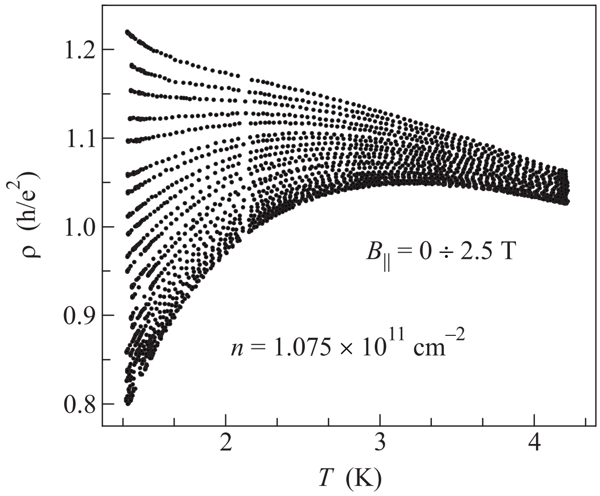

Fig.1. $\rho(T)$ temperature dependences for various in-plane magnetic fields varied in steps of 0.1 T in the range (0 − 2.5) T (from bottom to top)

$= 1.075 \cdot 10^{11}\,\mathrm{cm}^{-2}$ that is very close to n_c. There are several evidences for the presented data belong to the critical regime: (i) the proximity of n to n_c, (ii) the non-monotonic $\rho(T)$ temperature dependence with a clearly pronounced maximum and with a sharp drop in $\rho(T)$ at temperatures below the maximum, and (iii) the high value of the low-temperature resistivity $\rho \approx h/e^2$, that is close to the critical value $\sim 2h/e^2$ for this sample.

Rough estimate of the transport time τ based on the Drude formula shows that $T\tau$ is ≤ 0.3 for all "metallic-like" curves (i.e. for the curves with $\rho(T)$ maximum in Fig.1), over the whole studied temperature range. This indicates that the data shown in Fig.1 belong to the diffusive interaction regime.

In zero magnetic field, there is a well-pronounced maximum in $\rho(T)$ at $T_{\max} \approx 3$ K. Application of the in-plane magnetic field gradually drives the system to the insulating state. As $B_{\parallel}$ field increases, the $\rho(T)$ maximum becomes shallow and shifts progressively to lower temperatures (see Fig.1). At $B_{\parallel} = 2.3$ T the maximum completely vanishes; for higher fields resistivity monotonically increases with cooling that signals the onset of the insulating state. The cross-over from non-monotonic to monotonic temperature dependence of the magnetoresistance occurs in fields $g\mu_B B_{\parallel} \sim kT_{\max}$ in agreement with the predictions of the CRG theory [13].

Figure 2 shows the magnetoconductivity data $\Delta\sigma = \sigma(B_{\parallel}) - \sigma(0)$, replotted from Fig.1 for various fixed temperatures and for $B_{\parallel}$ varying from 0 to 1.1 T. For low fields, $b = g\mu_B B_{\parallel}/kT \ll 1$, the data, as expected, are linear in $B_{\parallel}^2$. When $\Delta\sigma(B)$ is plotted as a function of b^2, the slopes of the linear $|\Delta\sigma(B)|$ curves *increase as temperature decreases* (compare the data with the guiding parallel dashed lines in Fig.2). This proves that

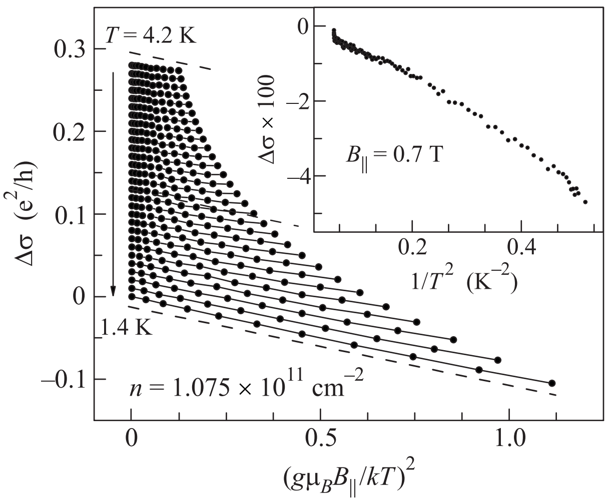

Fig.2. Magnetoconductivity in the critical regime versus square of the ratio of the Zeeman to thermal energy. Different curves correspond to different temperatures, varying, from top to bottom, in the range 4.2 to 1.4 K. Dashed parallel lines are guides to the eye. Inset shows magnetoconductivity at $B_{\parallel} = 0.7$ T versus $1/T^2$

the data are taken in the diffusive interaction regime [7]. Indeed, in the ballistic interaction regime, where the magnetoconductance data scale as (B^2/T) [7], the slope would behave in the opposite way, i.e. decrease linearly with cooling. The diffusive interaction behavior can also be seen from the inset to Fig.2, where the same data for a fixed $B_{\parallel} = 0.7$ T are plotted versus $(1/T)^2$. A positive curvature of the data demonstrates that the T-dependence is even steeper than $1/T^2$.

For the diffusive interaction regime, the first order interaction corrections (which are applicable for low resistivity/high density regime) result in the magnetoconductance as follows [14, 15]:

$$\sigma(T,B) = \sigma(T,0) - c n_v^2 \gamma_2(\gamma_2+1)\frac{(g\mu_B B_{\parallel})^2}{(kT)^2}, \qquad (1)$$

where $c \approx 0.029$, $\gamma_2 = -F_0^\sigma/(1+F_0^\sigma)$, F_0^σ – Fermi-liquid interaction constant, and n_v^2 takes into account correct number of triplet terms for the multivalley system [11]. Thus, within the framework of the interaction corrections, the slope of the magnetoconductance curves depends on the interaction parameter.

According to Eq. (1), the decrease in slopes of the $\Delta\sigma(b)$ curves with temperature in Fig.2 indicates a temperature dependence of the interaction parameter, which is one of the goals of our studies. However, Eq. (1) is valid only for $\rho \ll h/e^2$, i.e. for high density "metallic" regime, where temperature dependence of the interaction constant should be logarithmically weak [15]. On the other hand, our main interest is the critical regime of low densities and high resistivities $\rho \sim h/e^2$.

The two-parameter RG-theory has been developed to describe the behavior of the 2D system in the critical regime for the zero magnetic field case [9, 11]. It predicts a strong (power-low) temperature dependence of $\gamma_2(T)$. This theory was already shown [11] to describe qualitatively the nonmonotonic $\rho(T)$ experimental data in the critical regime. In RG theory, the $\rho(T)$ maximum signifies a turnover from the "localizing" to "delocalizing" behavior. The $(\rho_{\max}, T_{\max})$ coordinates of this point, therefore, are convenient for selecting the proper boundary conditions (integration constants). We have calculated numerically the $\gamma_2(T)$ dependence by solving the system of RG-equations [11] in the one-loop approximation for the two-valley case [11]. This zero field result is plotted in Fig.3 as a solid curve.

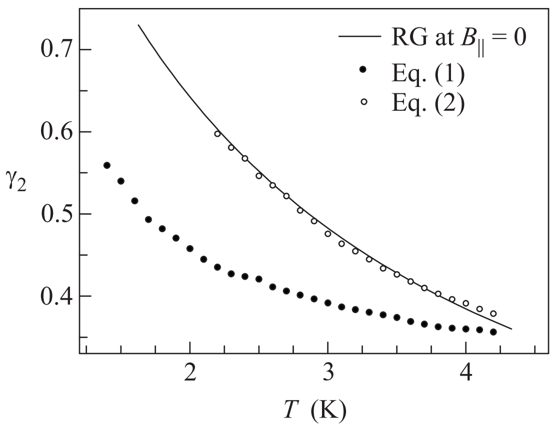

Fig.3. Temperature dependence of γ_2. Solid line – theoretical curve calculated from the RG-equations [11]. The symbols denote the empirical γ_2 obtained from our data using Eq. (1) (filled circles), and using Eq. (2) (empty circles)

The experimental observation of such strong $\gamma_2(T)$ dependence would be a crucial test of the RG-theory. In order to extract the $\gamma_2(T)$ dependence from magnetoconductance data, a parallel magnetic field is evidently required. If magnetic field is weak in all experimentally studied temperature range ($g\mu_B B_\parallel \ll kT$) the dependence of the normalized resistance $\rho(T, B_\parallel)/\rho(B_\parallel)_{\max}$ on $n_v \rho(B_\parallel)_{\max} \ln T/T(B_\parallel)_{\max}$ is given by the same universal curve as that for $B_\parallel = 0$ [9, 13]. Here $T(B_\parallel)_{\max}$ and $\rho(B_\parallel)_{\max}$ denote the temperature and the resistivity maximum values at a given $B_\parallel$ field. Expanding $T(B_\parallel)_{\max}$ and $\rho(B_\parallel)_{\max}$ to the second order in $B_\parallel$ one can obtain a novel expression for the magnetoresistivity:

$$\frac{\rho(T, B_\parallel)}{\rho(B_\parallel)_{\max}} = \frac{\rho(T,0)}{\rho_{\max}} + v(\gamma_2)\frac{(g\mu_B B_\parallel)^2}{(kT)^2}. \qquad (2)$$

Here the function $v(\gamma_2)$ depends not only on γ_2 but on $T(B_\parallel)_{\max}$, $\rho(B_\parallel)_{\max}$, as well as derivatives $dT(B_\parallel)_{\max}/db^2$ and $d\rho(B_\parallel)_{\max}/db^2$ at $b = 0$. In notations of Ref. [13]

$$v(\gamma_2) = a_0(\gamma_2) e^{2F(\gamma_2) - 2F(\gamma_2^{\max})}\Big[-\frac{d\ln\rho_{\max}}{db^2} \times$$
$$\times \int_{\gamma_2^{\max}}^{\gamma_2} \frac{du}{b_0(u)} e^{-F(u)+F(\gamma_2^{\max})} + \frac{\rho_{\max}}{2}\frac{d\ln T_{\max}}{db^2}\Big]\Big|_{b=0}. \quad (3)$$

To compare our data in the low field limit $b \ll 1$ with Eq. (2), for each temperature we have determined the difference $\delta\rho = \rho(T, B_\parallel)/\rho(B_\parallel)_{\max} - \rho(T,0)/\rho_{\max}$ and plotted it as function of b^2. The $v(\gamma_2)$ function was calculated numerically, by solving the CRG equations (17) and (18) from Ref. [13] in the b^2 approximation. The slopes of the resulting $\delta\rho(b^2)$ curves enables us to extract the experimental temperature dependence of γ_2 by using Eq. (2). The result is shown in Fig.3 by empty circles. One can see that our data agrees with the theoretical $\gamma_2(T)$ dependence [11] with no adjustable parameters in the wide range of temperatures. We show in Fig.3 the data only down to $T = 2.2$ K at which the universal curve given by the one-loop RG theory starts to overestimate $\rho(T,0)/\rho_{\max}$. For lower temperatures, it is no longer possible to extract $\gamma_2(T)$ from $v(\gamma_2)$ since the latter becomes non-monotonic. Presumably, it indicates the significance of higher-order (in ρ) terms in the RG theory at such low temperatures.

For comparison, we also present in Fig.3 the $\gamma_2(T)$ dependence (filled circles) determined from our experimental data by using Eq. (1), i.e. the first-order interaction corrections. There is only a qualitative similarity between thus determined $\gamma_2(T)$ and the theoretical RG-result (solid line). The disagreement is not surprising because the first-order (in ρ) interaction quantum corrections are inapplicable for the $\rho \sim h/e^2$ case.

It should be noted that the observed strong growth in $\gamma_2(T)$ (see Fig.3) at first sight is inconsistent with almost temperature independent F_0^σ, determined from Shubnikov-de Haas (ShdH) measurements [16, 17]. The are several possible reasons for this apparent in consistency: (i) a finite perpendicular and parallel fields, such as needed for the observation of beats in ShdH measurements, affects strongly the solution of the RG equations by reducing significantly $\gamma_2(T)$, (ii) ShdH measurements of F_0^σ miqht be made away of the critical regime of electron densities, and (iii) ShdH measurements at low densities were made at low temperatures $T < 1\,\mathrm{K}$, whereas the critical $\gamma_2(T)$ behavior in Fig.3 is established reliably only for higher temperatures $T > 2.2\,\mathrm{K}$.

In Ref. [13] cross-over RG equations have been suggested to describe the transition from weak ($b \ll 1$) to strong ($b \gg 1$) parallel magnetic fields. In order to accomplish the comprehensive comparison of our data with the theory, we have directly compared the measured $\rho(T, B_\parallel)$ dependences with solutions of the cross-over RG equations [13] for various fixed magnetic fields. For this purpose, we normalized the $\rho(T)$ data by its maximum value $\rho(B)_{\max}$ for each magnetic field. The comparison is presented in Fig.4. One can see the $\rho(T)$

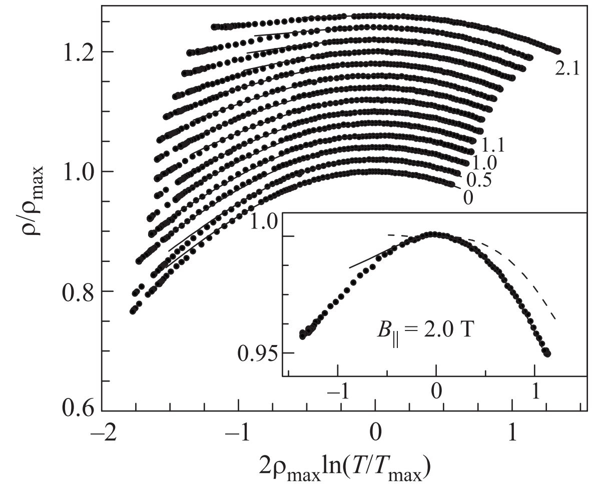

Fig.4. Comparison of the measured $\rho(T)$ data (symbols) with the theoretical dependences (lines) calculated from RG-equations [13] for various fixed magnetic fields (indicated next to each curve, in Tesla). Data and curves are offset shifted vertically by 0.02, relative to each other, starting from the zero field curve. Inset shows example of the comparison between the $\rho(T)$ data for $B_\parallel = 2\,\mathrm{T}$ and the CRG equations [13] with $A = 1/2\pi$ (dashed curve) and $A = 0.71/\pi$ (solid curve)

data agree with the RG theory not only in zero field (as was demonstrated earlier [11]), but also in the wide range of $B_\parallel$ fields.

In low fields $B_\parallel < 1.5\,\mathrm{T}$ there is a quantitative agreement between the experiment and the RG theory. In fields $< 0.7\,\mathrm{T}$, as discussed above, the $\rho(T)$ data collapse onto the universal curve for $B_\parallel = 0$ with no adjustable parameters. For higher magnetic field, the data collapse deteriorates because the system enters the non-universal cross-over regime. In theory [13], magnetic field enters the cross-over RG equations as

$$\mathcal{B} = A[1 + \gamma_2(T_{\max})]\frac{g\mu_B B_\parallel}{kT_{\max}}, \tag{4}$$

where the numerical prefactor $A = 1/2\pi$.

In the cross-over regime for $B_\parallel > 1.5\,\mathrm{T}$, there is only qualitative agreement between the theory [13] and the data as shown in the inset to Fig.4. However, by changing the numerical factor A, we could bring the theoretical curves into a good agreement with the data. For example, for $B_\parallel = 2\,\mathrm{T}$ the calculated curve agrees with the data when A equals $0.71/\pi$ (see Fig.4). In even higher fields $B > 2.3\,\mathrm{T}$, the $\rho(T)$ maximum vanishes and comparison with the theory is no longer possible. We mention that in high fields the comparison between the data and the RG theory is limited to not too low temperatures due to the divergence of $\rho(T)$ (in theory) at finite temperature in the one-loop CRG equations [13].

Finally, we wish to note that earlier [11] only one of the RG-equations, for $\rho(T)$ in zero field, has been tested by comparing with experimental data [12]. In this comparison the temperature dependence of $\gamma_2(T)$ was taken solely from theory and other RG equations were not tested therefore. Recently [18], an attempt was undertaken to test both variables, $\rho(T)$ and $\gamma_2(T)$, where the zero field $\rho(T)$-data was compared with solution of the RG-equation, and γ_2 was obtained from the magnetoconductance data using Eq. (1). As discussed above, Eq. (1) describes interaction quantum corrections and, strictly speaking, is inapplicable in the critical regime of $\rho \sim h/e^2$. Indeed, Fig.3 demonstrates that the $\gamma_2(T)$ dependence, obtained in this way, is only qualitatively similar to the exact solution of the RG-equations [11].

In contrast, in this paper, we compared self-consistently both variables $\rho(T)$ and $\gamma_2(T)$ with solutions of two RG-equations [11] and found a good consistency between the data and the theory. Moreover, we have found a good agreement of the $\rho(T, B_\parallel)$ data with the solutions of the CRG-equations [13] for various $B_\parallel$ fields.

In conclusion, we performed a comprehensive comparison of the transport and magnetotransport critical behavior in a wide temperature range, with the cross-over RG-equations which take magnetic field into account. Specifically, we compared with the theory temperature dependences of both, the interaction constant γ_2 and resistivity ρ. We have found that (i) the experimental $\gamma_2(T)$ determined from the low-field magnetoresistance grows rapidly as temperature decreases, in agreement with calculated $\gamma_2(T)$ [11], and (ii) the calculated temperature dependence of the magnetoresistance is in a good agreement with the experimental $\rho(T, B_{\|})$ data in a wide magnetic field range. This agreement strongly supports the theoretical interpretation of the observed 2D MIT as the true quantum phase transition.

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