Pis'ma v ZhETF, vol. 85, iss. 1, pp. 3 – 8

No realistic wormholes from ghost-free scalar-tensor phantom dark energy

K. A. Bronnikov, A. A. Starobinsky[+]

Centre of Gravitation and Fundamental Metrology, VNIIMS, 117361 Moscow, Russia;

Institute of Gravitation and Cosmology, PFUR, 117198 Moscow, Russia

[+]*Landau Institute for Theoretical Physics RAS, 119334 Moscow, Russia*

Submitted 7 December 2006

It is proved that no wormholes can be formed in viable scalar-tensor models of dark energy admitting its phantom-like ($w < -1$) behaviour in cosmology, even in the presence of electric or magnetic fields, if the non-minimal coupling function $f(\Phi)$ is everywhere positive and the scalar field Φ itself is not a ghost. Some special static, spherically symmetric wormhole solutions may exist if $f(\Phi)$ is allowed to reach zero or to become negative, so that the effective gravitational constant becomes negative in some region making the graviton a ghost. If f remains non-negative, such solutions require severe fine tuning and a very peculiar kind of model. If $f < 0$ is allowed, it is argued (and confirmed by previous investigations) that such solutions are generically unstable under non-static perturbations, the instability appearing right near transition surfaces to negative f.

PACS: 04.50.+h, 04.70.-s, 95.36.+x

1. Wormholes are hypothetic objects described by nonsingular solutions of gravitational field equations with two large or infinite regions of space-time connected by a throat (a tunnel). These two regions may lie in the same universe or even in different universes. The existence of stable static or stationary (traversable, Lorentzian) wormholes can lead to remarkable astrophysical effects as was recently emphasized in [1], as well as to the possibility of realizing hyperspace jumps ('null-transportation') or time machines [2]. That is why it is so important to investigate if really existing matter can produce and support such objects (it is well known that vacuum Einstein equations do not admit static wormhole solutions).

It is not difficult to construct a static wormhole without worrying about a matter source, as was done, e.g., by Morris and Thorne [3]. Problems arise when trying to find an explicit, internally self-consistent matter model whose energy-momentum tensor (EMT) has a structure required for wormhole existence and stability. Really, it is known that such matter should be rather exotic in the sense that its effective EMT should violate the weak or/and null energy conditions

$$T_{\mu\nu}u^\mu u^\nu \geq 0, \qquad (1)$$

where u^μ is a time-like or null 4-vector ($u_\mu u^\mu \geq 0$)[1)]. Nevertheless, beginning with the 1970s [4, 5], a number of static wormhole solutions supported by classical scalar fields in the Einstein and scalar-tensor theories of gravity have been constructed, see, e.g., [1, 2, 6] for references to later work.

However, in all these solutions, to violate the above energy conditions, such models always contain at least one ghost[2)], i.e., a classical field with a negative kinetic term, whose energy density may become arbitrarily negative for high frequency oscillations. From the quantum field theory point of view, this property is bad: it leads to the dramatic possibility of generating an unlimited amount of positive energy in the form of equal amounts of all known particles and antiparticles in *laboratory*, accompanied by production of equal negative energy of the ghost field in the form of particles and antiparticles of that field, too (see [7] for the most recent consideration). Note that this process requires gravitational creation of four particles from vacuum. That is why this instability is not seen in the behaviour of linear quantum perturbations in a classical background. Since nothing of this kind is observed, it seems that nature somehow avoids ghosts. Moreover, serious problems with ghosts appear even at the classical level. First, different in-

[1)]Our conventions are: the metric signature $(+ - - -)$; the curvature tensor $R^\sigma{}_{\mu\rho\nu} = \partial_\nu \Gamma^\sigma_{\mu\rho} - \ldots$, $R_{\mu\nu} = R^\sigma{}_{\mu\sigma\nu}$, so that the Ricci scalar $R > 0$ for de Sitter space-time and the matter-dominated cosmological epoch; the system of units $8\pi G = c = 1$.

[2)]The words "ghost" and "phantom" are often used on equal footing in papers on gravitation and cosmology; however, for clarity, we here distinguish between "phantom" dark energy as matter with $w < -1$ (see below) and "ghosts" as fields with negative kinetic energy.

stabilities arise at boundary surfaces dividing ghost and normal field behaviour which generically transform these surfaces into singular ones [8 – 11]. Second, as was recently argued in [12], cosmological models with a ghost field cannot explain the observed large-scale homogeneity and isotropy of the Universe. Thus, even though there exist some counter-arguments in favour of ghost fields (see, e.g., [13]), it is reasonable to try to avoid such fields in modelling real or hypothetic phenomena.

The problem of phantom matter has got a new twist after the recent discovery of the Universe late-time acceleration [14]. A new form of matter dubbed dark energy (DE) is needed to support this acceleration if the Einsteinian form of gravitational field equations is assumed (see [15, 16] for reviews). Moreover, it appeared that though DE is well described by a cosmological constant in the first approximation, some observational data, in particular, the 'Gold' supernova sample [17], slightly favour a phantom behaviour of DE. Namely, the DE equation of state $w \equiv p_{\rm DE}/\rho_{\rm DE}$ may be less than -1 for small redshifts $z < 0.3$ along with crossing of the phantom divide $w = -1$ at larger z [18] (see [16] for a list of further references). Other supernova samples like the SNLS one [19] as well as the WMAP3 data [20] have a cosmological constant as the best fit but still do not exclude recent phantom DE behaviour, see [21].

Does it mean that if the transient phantom behaviour of DE will be confirmed by future, more precise observational data, we have to introduce ghosts? No, not at all. It is known that there do exist models without ghosts admitting a phantom DE behaviour and even a super-accelerated expansion of the present Universe, $\dot H > 0$, where $H(t)$ is the Hubble parameter, $H \equiv d \ln a(t)/dt$, $a(t)$ is the scale factor of a Friedmann-Robertson-Walker cosmological model. The simplest of them is generic scalar-tensor gravity generalizing the original Brans-Dicke theory to the case of a non-zero scalar potential, see Eq. (2) below. It was explicitly shown in [22, 12] (see also [23]) that this DE model has sufficient freedom to describe all possible observational data on the luminosity distance and the inhomogeneity growth factor including the possible present phantom behaviour and smooth crossing of the phantom divide in the recent past. More complicated models of phantom DE without ghosts can exist, too, but they are less investigated.

Now, it becomes very important to investigate if this physically reasonable and potentially existing kind of exotic matter may support static and stable wormholes. This problem is solved in this paper. Following and extending our previous considerations in [24, 25], we will also add an electromagnetic field to the scalar-tensor DE since this might be important both for stabilizing a wormhole and for potential astrophysical applications [1]. Thus, the Lagrangian density of a general scalar-tensor theory (STT) in a Jordan-frame manifold $\mathbb{M}_{\rm J}$ with the metric $g_{\mu\nu}$ is taken as

$$2L = f(\Phi)R + h(\Phi)g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu} - 2U(\Phi) - F^{\mu\nu}F_{\mu\nu}, \qquad (2)$$

where R is the Ricci scalar, $F_{\mu\nu}$ is the electromagnetic field tensor, f, h and U are arbitrary functions. It is assumed that a fermion matter Lagrangian is not coupled to Φ, so that the Jordan frame is the physical one (in particular, fermion masses are constant and atomic clocks measure the proper time t in it). We will still use the Einstein frame, defined as a manifold $\mathbb{M}_{\rm E}$ with the metric

$$\overline{g}_{\mu\nu} = |f(\Phi)| g_{\mu\nu}, \qquad (3)$$

as a convenient tool for studying the properties of $g_{\mu\nu}$, employing results obtainable from the corresponding Lagrangian

$$2L_E = (\operatorname{sign} f)\left[\overline{R} + (\operatorname{sign} l)\overline{g}^{\mu\nu}\phi_{\mu}\phi_{,\nu}\right] - \\ - 2V(\phi) - F^{\mu\nu}F_{\mu\nu}, \qquad (4)$$

where bars mark quantities obtained from or with $\overline{g}_{\mu\nu}$, indices are raised and lowered with $\overline{g}_{\mu\nu}$ and the following relations hold:

$$l(\Phi) := fh + \frac{3}{2}\left(\frac{df}{d\Phi}\right)^2, \quad \frac{d\phi}{d\Phi} = \frac{\sqrt{|l(\Phi)|}}{f(\Phi)}, \\ V(\phi) = |f|^{-2}U(\Phi). \qquad (5)$$

The conditions of (quantum) stability and absence of ghosts in the theory (2) are $f(\Phi) > 0$ (the graviton is not a ghost) and $l(\Phi) > 0$ (the Φ field is not a ghost).

2. Let us first assume that $f(\Phi)$ and $l(\phi)$ are smooth and positive everywhere, including limiting points, or, equivalently, that f and l are bounded above and below by some positive constants. It was shown in [26] that a static wormhole throat (defined as a minimal 2-surface in a 3-manifold) necessarily implies violation of the null energy condition (NEC) by matter sources of the Einstein equations. Meanwhile, the matter sources in (1) always satisfy the NEC, hence wormholes (and even wormhole throats) cannot exist in Einstein's frame. Further, under the assumptions made, the conformal mapping $\overline{g}_{\mu\nu} = f(\Phi)g_{\mu\nu}$ always transfers a flat spatial infinity in one frame to a flat spatial infinity in another (though, the corresponding Schwarzschild masses may be different due to scalar field effects). If we suppose that there is an asymptotically flat wormhole in $\mathbb{M}_{\rm J}$, then its each flat infinity has a counterpart in $\mathbb{M}_E$, the whole manifold is smooth, and we obtain a wormhole in $\mathbb{M}_E$, in contradiction to the above-said. Thus static and asymptotically flat wormholes are absent in the Jordan frame as well.

This simple reasoning does not even require any spatial symmetry assumption and means that any static wormholes are ruled out in the theory (2), if everywhere $f > 0$ and $l > 0$. Moreover, no positivity condition or any other restriction on $U(\Phi)$ has been assumed (apart from that needed for the existence of asymptotic flatness).

It should be stressed that throats are not ruled out in the Jordan frame since the energy conditions may be violated locally, in full analogy with phantom DE behaviour in cosmology. Even though the NEC holds for the fields ϕ and $F_{\mu\nu}$ in $\mathbb{M}_E$, it can be violated for Φ in $\mathbb{M}_J$. Conformal mappings like (3) preserve the timelike or null character of the vectors u^μ in (1), but $T_{\mu\nu}$ can change drastically. The EMT $T_{\mu\nu}[\phi]$ in $\mathbb{M}_E$ has its usual form; however, if we write the gravitational field equations in $\mathbb{M}_J$ in the Einstein form, the EMT $T_{\mu\nu}[\Phi]$ will contain second-order derivatives. Nevertheless, as we see, wormholes as global entities cannot appear in $\mathbb{M}_J$.

This no-wormhole statement can be further strengthened in several respects. First, the asymptotic flatness requirement may be omitted: it is sufficient to require the existence of two spatial infinities (which may be defined in terms of sequences of closed 2-surfaces with intinitely growing areas in the spatial sections of the space-time manifold), so that spatial sections of both $\mathbb{M}_E$ and $\mathbb{M}_J$ have the topology of a 3-cylinder $\mathbb{R} \times \mathbb{S}^2$. (Our reasoning does not cover other possible wormhole geometries: that of a "dumbbell", where a throat connects two large but finite universes and the 3-topology is $\mathbb{S}^3$, and that of a "hanging drop", where one of the universes is finite and the 3-topology is $\mathbb{R}^3$.)

Second, the restriction to the static case is not necessary. Indeed, Hochberg and Visser [27] extended their previous result [26] on necessary NEC violation to dynamic wormhole throats (which required a more general definition of a throat in terms of anti-trapped surfaces). In other words, even a dynamic throat cannot exist in $\mathbb{M}_E$ without NEC violation. We therefore can assert that even dynamic wormholes cannot exist in any $\mathbb{M}_J$ connected with such $\mathbb{M}_E$ by well-behaved (though possibly time-dependent) conformal factors f.

One reservation should be made here: the above reasoning employs the fact that a regular conformal mapping transfers a spatial infinity to a spatial infinity and does not change the spatial topology. This is true under the strong conditions that we have imposed on $f(\Phi)$. It is known, however, that spatial topology of the same space-time manifold may be different in different reference frames (a well-known example is the appearance of de Sitter space in closed and open FRW forms). Speaking of dynamic wormholes, we should understand that their properties can be drastically different in different reference frames, but we consider conformal mappings connecting, in a sense, similar reference frames in different manifolds.

Third, the no-wormhole statement is valid not only for STT but for any metric theory of gravity (e.g., high-order theories with Lagrangians containing $f(R)$ or $f(R, \Phi)$) whose physical manifold $\mathbb{M}_J$ is conformally related to some other manifold $\mathbb{M}_E$ in which the Einstein equations hold with a matter source respecting the NEC, provided the conformal factor is everywhere smooth and positive.

3. Returning to STT, let us slightly weaken our assumptions and allow the function $f(\Phi)$ to become zero at some $\Phi = \Phi_0$. Note that the DE description in cosmology does not require this, but let us conjecture that in local configurations the scalar field Φ may reach values not attainable in a cosmological setting. The situation then becomes more complex since now the mapping (3) is able to transfer a point or surface of finite area to spatial infinity, i.e., a limiting surface of infinite area.

Let us restrict ourselves to static spherical symmetry, considering the theory (1) in a space-time with the metric

$$ds_E^2 = A(\rho)dt^2 - \frac{d\rho^2}{A(\rho)} - r^2(\rho)(d\theta^2 + \sin^2\theta\, d\varphi^2) \quad (6)$$

and assuming $\phi = \phi(\rho)$. The Maxwell fields compatible with spherical symmetry are radial electric fields ($F_{01}F^{10} = q_e^2/r^4$) and radial magnetic fields ($F_{23}F^{23} = q_m^2/r^4$) where the constants q_e and q_m are the electric and magnetic charges, respectively.

The scalar field equation and three independent combinations of the Einstein equations read

$$(Ar^2\phi')' = r^2 dV/d\phi, \quad (7)$$

$$(A'r^2)' = -2r^2V + 2q^2/r^2; \quad (8)$$

$$2r''/r = -\phi'^2; \quad (9)$$

$$A(r^2)'' - r^2A'' = 2 - 4q^2/r^2, \quad (10)$$

where the prime denotes $d/d\rho$ and $q^2 = q_e^2 + q_m^2$. Eq. (7) follows from (8)–(10), which, given a potential $V(\phi)$, form a determined set of equations for the unknowns $r(\rho)$, $A(\rho)$, $\phi(\rho)$.

Now, let us inquire whether or not an asymptotically flat geometry of $\mathbb{M}_E$ described by a solution to Eqs. (8)–(10) can be conformally mapped according to Eq. (3)

(where now $f \geq 0$ is simply some function of ρ) to a twice asymptotically flat wormhole geometry in a manifold $\mathbb{M}_J$. One flat infinity in $\mathbb{M}_E$ is assumed at $\rho = \infty$, and it maps to a flat infinity in $\mathbb{M}_J$ provided f has a finite limit as $\rho \to \infty$. Another flat infinity in $\mathbb{M}_J$ may be obtained either from a centre $r = 0$ (where $A/r^2 \to 0$ as $r \to 0$) or from a horizon (where $A = 0$ at finite r) in $\mathbb{M}_E$. A reason is that the ratio A/r^2 does not change at conformal mappings and preserves its geometric meaning in $\mathbb{M}_J$, where $g_{tt} = A/f$ and $-g_{\theta\theta} = r_J^2 = r^2/f$, and a flat infinity implies $g_{tt} \to 1$ while $r_J^2 \to \infty$ (recall that $g_{\mu\nu}$ is the metric in $\mathbb{M}_J$). Thus we must have $A/r^2 = 0$ at a preimage of a flat infinity in $\mathbb{M}_J$.

Assuming that a centre in $\mathbb{M}_E$ is located (without loss of generality) at $\rho = 0$ and $r(\rho) \sim \rho^a$, $a = \text{const} > 0$ at small ρ, we can evaluate the possible behaviour of $A(\rho)$ at small ρ from Eq. (10) and then check whether a conformal factor f may be chosen in such a way that $\rho = 0$ is a flat infinity in $\mathbb{M}_J$. An inspection, performed separately for $q = 0$ and $q \neq 0$, shows that such a choice is impossible.

Horizons are not excluded in solutions to (8)–(10) (though the potential $V(\phi)$ must be then at least partly negative to conform with the well-known no-hair theorems). Moreover, in the scalar-vacuum case $q = 0$, considering asymptotic flatness at large ρ, only one simple horizon ($A \sim \rho - \rho_h$ near the horizon $\rho = \rho_h$) may appear as shown in [28]. If $q \neq 0$, both simple and double ($A \sim (\rho - \rho_h)^2$) horizons may appear.

A simple horizon in $\mathbb{M}_E$ could map into a flat asymptotic in $\mathbb{M}_J$ if $f \sim \rho - \rho_h$, then $r_J \sim (\rho - \rho_h)^{-1/2} \to \infty$ as $\rho \to \rho_h$. However, the requirement of the proper circumference to radius ratio at flat infinity,

$$|g^{\rho\rho}|(r_J')^2 \to 1, \tag{11}$$

is violated in this case: the expression in question behaves as $(\rho - \rho_h)^{-1} \to \infty$ instead of tending to unity.

We conclude that *static, spherically symmetric scalar-vacuum configurations in $\mathbb{M}_E$ cannot be conformally mapped with conformal factors $f(\rho) \geq 0$ into twice asymptotically flat wormholes.*

The situation is different for double horizons. Indeed, if $A \sim (\rho - \rho_h)^2$, the requirement $g_{tt} = A/f \to 1$ leads to $f \sim (\rho - \rho_h)^2$, hence $r_J \sim (\rho - \rho_h)^{-1} \to \infty$, and it is straightforward to see that the expression in the l.h.s. of Eq. (11) tends to a finite limit. Additional fine tuning (besides the special choice of parameters leading to a double horizon) is then required to bring this expression to precisely 1, otherwise there is a spatial infinity with solid angle excess or deficit as, e.g., in global monopole models.

Thus, *some exceptional solutions to Eqs. (8)–(10) with $q \neq 0$ describe metrics that can be conformally mapped into twice asymptotically flat wormhole metrics in $\mathbb{M}_J$.*

Explicit examples of such solutions are yet to be found.

The next question is: which kind of STT admits such solutions? An answer is easily obtained with the aid of the relations (5). Let us use the Brans-Dicke parametrization of the general STT (2), namely, $f(\Phi) = \Phi$, $h(\Phi) = \omega(\Phi)/\Phi$ (the Brans-Dicke theory as such is the special case $\omega = \text{const}$). We also assume a generic behaviour of the ϕ field in $\mathbb{M}_E$ near $\rho = \rho_h$ putting $\phi'(\rho_h) = f_1 \neq 0$ and $A''(\rho_h) = A_2 > 0$. Then we find that

$$l(\Phi) = \omega(\Phi) + 3/2 \approx \Phi\phi_1^2/(2A_2) \to 0 \tag{12}$$

as $\Phi \to 0$. So, in this limit the STT approaches the boundary beyond which Φ would become a ghost.

4. Now let us further weaken our assumptions, allowing $f(\Phi)$ in (2) to become negative and cross zero at some point $\Phi = \Phi_0$. For static, spherically symmetric solutions of the theory (2), it has been shown [29] that, if $df/d\Phi \neq 0$ at $\Phi = \Phi_0$, there always exist such solutions that continue from positive to negative f. Moreover, such solutions (which are special with respect to the whole set of solutions) generically describe wormhole geometries [29]. There exist explicit examples of such continued solutions with both zero and nonzero charges q, obtained for massless conformal scalar fields in general relativity ($f(\Phi) = 1 - (1/6)\Phi^2$, $h(\Phi) \equiv 1$, $U(\Phi) \equiv 0$ in (2) [4] and for more general non-minimally coupled scalar fields, with $1/6$ replaced by an arbitrary coefficient $\xi > 0$ [30, 31]. However, the stability studies performed by now [31, 9, 25] show that such wormhole solutions are generically unstable under non-static monopole (spherically symmetric) perturbations, and the instability is related to a negative pole of the effective potential for perturbations situated precisely at the transition sphere at which $f = 0$. The existence of a generic space-like curvature singularity at $f \to 0$ whose structure was found in [8] suggests that a similar instability exists for non-spherically-symmetric perturbations, too. Therefore, one should expect that for the general STT (2) transitions to negative values of the effective gravitational constant ($\propto 1/f$) always (or at least generically) lead to instabilities.

5. Thus, we have proved a general theorem that *no wormholes (static or dynamic) connecting two spatial infinities can be formed in any ghost-free scalar-tensor theory of gravity, under the condition that the non-minimal* *coupling function $f(\Phi)$ in the Lagrangian (2) is everywhere positive, including possible limiting values.*

This result is valid in the presence of any matter whose EMT satisfies the null energy condition, e.g., the electromagnetic field. It is true, in particular, for the astrophysically relevant case of scalar-tensor models of dark energy admitting a phantom-like behaviour in cosmology ($w < -1$) [22, 12]. In other words, DE models of this class (as well as any other models conformally related to general relativity with everywhere positive conformal factors and without ghost fields) do not predict wormholes.

We have also tried to weaken the requirements and studied the possible behaviour of static, spherically symmetric vacuum and electro-vacuum configurations in scalar-tensor gravity (2), allowing $f(\Phi)$ to reach zero or even become negative. It has turned out that if f only reaches zero, twice asymptotically flat wormhole solutions in Jordan's frame can exist but only in exceptional cases: 1) the corresponding Einstein-frame solution must comprise an extreme black hole, whose double horizon is then mapped to the second spatial infinity in the Jordan frame, that is only possible with nonzero electric or magnetic fields; 2) additional fine tuning is necessary to avoid a solid angle deficit or excess at this second infinity, and 3) the theory itself should be very special: in the Brans-Dicke parametrization, it should hold $\omega(\Phi) + 3/2 \to 0$ as $\Phi \to 0$. So, at the second spatial infinity, the theory approaches a ghost boundary and, since $f \to 0$, the effective gravitational constant tends to infinity. Such solutions may be of certain theoretical interest but can hardly be called realistic.

Rather a wide (although still special) class of wormhole solutions exists in theories where a transition to $f < 0$ is allowed. However, previous studies have shown that such solutions are generically unstable under spherically symmetric perturbations, the instability appearing due to a negative pole of the effective potential at the transition surface to $f < 0$. This pole still does not guarantee instability, and further studies are necessary; but even if such wormholes can exist, their "remote mouths" are located in antigravitational regions with $f < 0$. So, they cannot connect different parts of our Universe but can only be bridges to other universes (if any) with very unusual physics.

AS was partially supported by the Russian Foundation for Basic Research, grant # 05-02-17450, and by the Research Programme "Astronomy" of the Russian Academy of Sciences. He also thanks the Galileo Galilei Institute for Theoretical Physics for the hospitality and the INFN for partial support during the period when this project was started. KB acknowledges partial support from FAPESP (Brazil) and RFBR grant # 05-02-17478 and thanks the Institute of Physics, University of São Paulo for kind hospitality.

1. N. S. Kardashev, I. D. Novikov, and A. A. Shatskiy, astro-ph/0610441 (2006).
2. F. S. N. Lobo, Phys. Rev. D **71**, 084011 (2005).
3. M. Morris and K. S. Thorne, Am. J. Phys. **56**, 395 (1988).
4. K. A. Bronnikov, Acta Phys. Pol. B **4**, 251 (1973).
5. H. G. Ellis, J. Math. Phys. **14**, 104 (1973).
6. J. P. S. Lemos, F. S. N. Lobo, and S. Q. de Oliveira, Phys. Rev. D **68**, 064004 (2003); S. V. Sushkov, Phys. Rev. D **71**, 043520 (2005).
7. J. M. Cline, S. Geon, and G. D. Moore, Phys. Rev. D **70**, 043543 (2004).
8. A. A. Starobinsky, Pisma v Astron. Zh. **7**, 67 (1981) [Sov. Astron. Lett. **7**, 36 (1981)].
9. K. A. Bronnikov and S. V. Grinyok, Grav. & Cosmol. **10**, 237 (2004) (gr-qc/0411063).
10. A. Vikman, Phys. Rev. D **71**, 023515 (2005).
11. F. Cannata and A. Yu. Kamenshchik, gr-qc/0603129 (2006).
12. R. Gannouji, D. Polarski, A. Ranquet, and A. A. Starobinsky, JCAP **0609**, 016 (2006).
13. K. A. Bronnikov and J. C. Fabris, Phys. Rev. Lett. **96**, 251101 (2006).
14. A. G. Riess, A. V. Philippenko, P. Challis et al., Astron. J. **116**, 1009 (1998); S. Perlmutter, G. Aldering, G. Goldhaber et al., Astroph. J. **517**, 565 (1999).
15. V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000); P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); T. Padmanabhan, Phys. Rep. **380**, 235 (2003); E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 000 (2006) (hep-th/0603057).
16. V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **15**, 000 (2006) (astro-ph/0610026).
17. A. G. Riess, L.-G. Strolger, J. Tonry et al., Astroph. J. **607**, 665 (2004).
18. U. Alam, V. Sahni, and A. A. Starobinsky, JCAP **0406**, 008 (2004); D. Huterer and A. Cooray, Phys. Rev. D **71**, 023506 (2005); Y. Gong, Class. Quant. Grav. **22**, 2121 (2005); D. A. Dicus and W. W. Repko, Phys. Rev. D **70**, 083527 (2004); S. Hannestad and E. Mortsell, JCAP **0409**, 001 (2004).
19. P. Astier, J. Guy, N. Regnault et al., Astron. Astroph. **447**, 31 (2006).
20. D. N. Spergel, R. Bean, O. Dore et al., astro-ph/0603449 (2006).
21. S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **72**, 123519 (2005); U. Alam and V. Sahni, Phys. Rev. D **73**, 084024 (2006); H. K. Jassal, J. S. Bagla, and T. Padmanabhan, astro-ph/0601389 (2006); G. Zhao, J. Xia,

B. Feng, and X. Zhang, astro-ph/0603621 (2006); U. Seljak, A. Slosar, and P. McDonald, JCAP **0610**, 014 (2006).

22. B. Boisseau, G. Esposito-Farese, D. Polarski, and A. A. Starobinsky, Phys. Rev. Lett. **85**, 2236 (2000).
23. L. Perivolaropoulos, JCAP **0510**, 001 (2005); S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **73**, 103511 (2006).
24. K. A. Bronnikov, S. B. Fadeev, and A. V. Michtchenko, Gen. Rel. Grav. **35**, 505 (2003).
25. K. A. Bronnikov and S. V. Grinyok, Grav. & Cosmol. **11**, 75 (2005) (gr-qc/0509062).
26. D. Hochberg and M. Visser, Phys. Rev. D **56**, 4745 (1997).
27. D. Hochberg and M. Visser, Phys. Rev. D **58**, 044021 (1998).
28. K. A. Bronnikov, Phys. Rev. D **64**, 064013 (2001).
29. K. A. Bronnikov, J. Math. Phys. **43**, 6096 (2002).
30. C. Barceló and M. Visser, Class. Quantum Grav. **17**, 3843 (2000).
31. K. A. Bronnikov and S. V. Grinyok, In: *Festschrift in honour of Prof. Mario Novello*, Rio de Janeiro, 2002; gr-qc/0205131.