Pis'ma v ZhETF, vol. 85, iss. 1, pp. 79 – 83 © 2007 January 10

Local correlations of different eigenfunctions in a disordered wire

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Submitted 30 November 2006

We calculate the correlator of the local density of states $\langle\rho_\varepsilon(\mathbf{r}_1)\rho_{\varepsilon+\omega}(\mathbf{r}_2)\rangle$ in quasi-one-dimensional disordered wires in a magnetic field, assuming that $|\mathbf{r}_1-\mathbf{r}_2|$ is much smaller than the localization length. This amounts to finding the zero mode of the transfer-matrix Hamiltonian for the supersymmetric σ model, which is done exactly by mapping to the three-dimensional Coulomb problem. Both the regimes of level repulsion and level attraction are obtained, depending on $|\mathbf{r}_1-\mathbf{r}_2|$. We demonstrate that the correlations of different eigenfunctions in the quasi-one-dimensional and strictly one-dimensional cases are dissimilar.

PACS: 73.20.Fz, 73.21.Hb

Modern physics knows a number of paradigmatic models which describe properties of various, seemingly unrelated, phenomena. Anderson localization in disordered systems with one-dimensional geometry is one of such models. Historically, it played an important role in setting down the concept of quantum localization. The model of a one-dimensional (1D) chain was the first example of the system where localization takes place at arbitrary weak disorder [1, 2]. Later on Thouless [3] argued that all the states are localized also in disordered *quasi-one-dimensional* (Q1D) systems, supporting a large number $N \gg 1$ of propagating transverse modes.

During the last three decades it had been realized that many problems of condensed matter physics and quantum chaos can be mapped onto Q1D localization. The most natural example is particle propagation in a disordered wire, both in the limit of weak [4] and strong (granular) [5] disorder. Besides that, the problem of random banded matrices with a large bandwidth can be mapped [6] onto the same model. The problem of the quantum δ-kicked rotor [7, 8] whose evolution operator looks like a quasi-random banded matrix also belongs [9, 10] to the Q1D universality class. Recently it was argued [11] that dynamic localization in quantum dots which can be described by time-dependent random matrices is equivalent to Q1D localization as well.

Despite the fact that mathematical description of strictly 1D disordered systems is rather involved [12, 13], our knowledge about them is quite complete. In this case, the localization length $\xi_{\rm 1D}$ coincides with the mean free path l. The low-frequency conductivity follows the Mott-Berezinsky law: $\operatorname{Re}\sigma(\omega) \propto \omega^2\ln^2(1/\omega\tau)$ [14, 12], where τ is the elastic time. The correlation function of the local density of states (LDOS) was considered by Gor'kov, Dorokhov and Prigara [15] who have shown that in the limit of small energy separation, $\omega\tau \ll 1$, the eigenstates are uncorrelated at $r \gg \xi_{\rm 1D}\ln(1/\omega\tau)$, exhibit nearly perfect level repulsion at $\xi_{\rm 1D} \ll r \ll \xi_{\rm 1D}\ln(1/\omega\tau)$, with a tendency to level attraction at $r \ll \xi_{\rm 1D}$ (for a discussion of these results using Mott's arguments, see Ref. [16]).

In the Q1D case, the localization length $\xi \sim Nl$ [17] is much larger than the mean free path, that allows to formulate the problem of Q1D localization on the language of the nonlinear supersymmetric σ-model [4, 18] describing the physics of interacting diffusive modes. The latter problem can be solved using the transfer-matrix technique introduced by Efetov and Larkin [19]. The idea of the method is to reduce the evaluation of the functional integral over the superfield $Q(x)$ to the solution of a differential equation, in analogy with constructing the Schrödinger equation from the Feynman path integral. However the resulting transfer-matrix equation at finite frequencies ω appears to be too complicated (even in the simplest unitary case), so that no results about correlations of *different* eigenfunctions had ever been obtained in the nonperturbative localized regime, $\omega < \Delta_\xi$, where Δ_ξ is the level spacing at the scale of the localization length. Instead, one can solve the transfer-matrix equation in the limit of $\omega \to +i0$ thus extracting information about statistics of *the same* wave function. In this way one can find the localization length for various symmetry classes [19] and calculate the whole distribution function of a certain eigenfunction's intensity $|\psi_n(\mathbf{r})|^2$ [20]. Similarly, the low-frequency behavior of the dissipative conductivity in the Q1D case is not known up to now.

Thus we see that our knowledge about Q1D localization is far from being complete. This is a challenge since it is quasi- rather than strictly one-dimensional localization, which is important for various applications.

The main technical problem lies in the complicated form of the transfer-matrix equations which did not allow to get any results concerning information about different eigenfunctions in the localized regime, $\omega < \Delta_\xi$.

In this Letter we make the first step towards the theory of Q1D localization *at finite frequencies* by calculating the correlation function of the LDOS

$$R(\omega; \mathbf{r}_1, \mathbf{r}_2) = \nu^{-2} \langle \rho_\varepsilon(\mathbf{r}_1) \rho_{\varepsilon+\omega}(\mathbf{r}_2) \rangle \quad (1)$$

in a thick disordered wire in a magnetic field (unitary symmetry); $\rho_\varepsilon(\mathbf{r}) = \sum_n |\psi_n(\mathbf{r})|^2 \delta(\varepsilon - \varepsilon_n)$, and ν is the three-dimensional (3D) density of states for spinless particles. In the present paper we restrict ourselves to the limit of sufficiently small spatial separations $|\mathbf{r}_1 - \mathbf{r}_2| \ll \xi$, when only the zero mode of the transfer-matrix equation is relevant. This zero mode is found exactly using the mapping onto an auxiliary 3D Coulomb problem. The explicit expression (14) for the zero-mode wave function is the main technical achievement of this Letter.

Our result for the LDOS correlation function in the limit $|\mathbf{r}_1 - \mathbf{r}_2| \ll \xi$ has the form

$$R(\omega; \mathbf{r}_1, \mathbf{r}_2) = 1 + A(\omega) + k(\mathbf{r}_1, \mathbf{r}_2) B(\omega). \quad (2)$$

The factor $k(\mathbf{r}_1, \mathbf{r}_2) = \langle \operatorname{Im} G^R(\mathbf{r}_1, \mathbf{r}_2) \rangle^2 / (\pi\nu)^2$ accounts for Friedel oscillations [21, 20]. Its form is not universal and depends on the position of the points $\mathbf{r}_1$ and $\mathbf{r}_2$ with respect to the boundaries of the wire. $k(\mathbf{r}_1, \mathbf{r}_2)$ is equal to 1 at coincident points and decays fast as $|\mathbf{r}_1 - \mathbf{r}_2|$ exceeds the Fermi wavelength $2\pi/k_F$.

The universal functions $A(\omega)$ and $B(\omega)$ determined by slow diffusive modes are insensitive to a particular geometry of the wire. They depend on the single dimensionless parameter ω/Δ_ξ, where $\Delta_\xi = (4\pi^2 D \nu_1^2)^{-1}$ is the level spacing within the localization region (D is the diffusion coefficient, $\nu_1 = \mathcal{A}\nu$, and $\mathcal{A} \sim N k_F^{-2}$ is the wire cross-section). The monotonous functions $A(\omega)$ and $B(\omega)$ are given explicitly by Eq. (16) and (17). The function $A(\omega)$ is negative with $A(\infty) = 0$ and $A(0) = -1/3$, whereas $B(\omega)$ is positive with $B(\infty) = 0$ and $B(0) = 2/3$.

At coincident points, the correlator $R^{\text{coins}}(\omega) = 1 + A(\omega) + B(\omega)$ is always larger than 1 signaling short-scale level attraction (top curve in Fig.1). It starts with $R^{\text{coins}}(0) = 4/3$ in the deeply localized regime, has a maximum in the crossover region $\omega \sim \Delta_\xi$, and slowly (as $\sqrt{\Delta_\xi/\omega}$) reaches the uncorrelated limit $R(\infty) = 1$ in the metallic regime ($\omega \gg \Delta_\xi$). This behavior should be contrasted with the ω-independence of the same correlator $R^{\text{coins}}_{\text{1D}}(\omega) = 1$ for strictly 1D chains [15].

For sufficiently close but different points, $k_F^{-1} \ll |\mathbf{r}_1 - \mathbf{r}_2| \ll \xi$, the correlation function (2) is nearly independent of $|\mathbf{r}_1 - \mathbf{r}_2|$ and equal to $R^{\text{close}}(\omega) = 1 + A(\omega)$ which is smaller than 1 indicating level repulsion (bottom curve in Fig.1). However, this repulsion is weak enough, $R^{\text{close}}(0) = 2/3$, compared to the perfect level repulsion, $R(0) = 0$, in the random matrix theory. Surprisingly, this value of 2/3 exactly coincides with the result [15] obtained for the strictly 1D geometry for $\omega \to 0$ in the equivalent limit $k_F^{-1} \ll r \ll \xi_{\text{1D}}$.

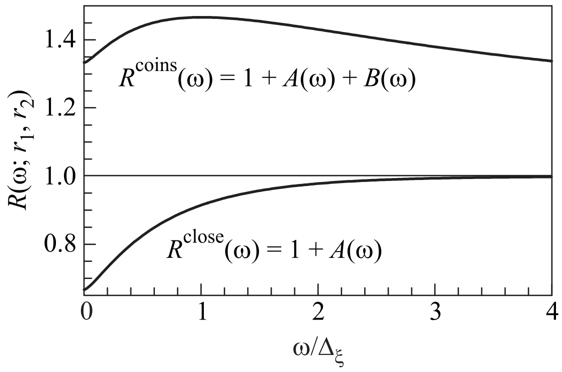

Fig.1. The LDOS correlation function vs. ω/Δ_ξ: at coincident points, $R^{\text{coins}}(\omega)$, and at $k_F^{-1} \ll |\mathbf{r}_1 - \mathbf{r}_2| \ll \xi$, $R^{\text{close}}(\omega)$

We start the technical section of this Letter by representing the correlation function (1) in terms of the retarded and advanced Green functions:

$$R(\omega; \mathbf{r}_1, \mathbf{r}_2) = \frac{1}{2} + \frac{\operatorname{Re} \langle G^R_{\varepsilon+\omega}(\mathbf{r}_1, \mathbf{r}_1) G^A_\varepsilon(\mathbf{r}_2, \mathbf{r}_2) \rangle}{2\pi^2 \nu^2}. \quad (3)$$

Then we write G^R and G^A as the integrals over superfields and perform the standard sequence of steps leading to the Q1D unitary σ-model [18] formulated in terms of the supermatrix field Q acting in the direct product of the Fermi–Bose (FB) and Retarded–Advanced (RA) spaces. The resulting expression has the form

$$R(\omega; \mathbf{r}_1, \mathbf{r}_2) = \frac{1}{2} - \frac{1}{2} \operatorname{Re} \int P[Q] e^{-S[Q]} DQ(x), \quad (4)$$

with the usual diffusive action and the preexponent:

$$S[Q] = \frac{\pi\nu_1}{4} \operatorname{str} \int \left[D(\nabla Q(x))^2 + 2i\omega \Lambda Q(x) \right] dx, \quad (5)$$

$$P[Q] = Q^{\text{RR}}_{\text{BB}}(x_1) Q^{\text{AA}}_{\text{BB}}(x_1) + k(\mathbf{r}_1, \mathbf{r}_2) Q^{\text{RA}}_{\text{BB}}(x_1) Q^{\text{AR}}_{\text{BB}}(x_1), \quad (6)$$

where $\operatorname{str}(\dots)$ is the supertrace, $\Lambda = \operatorname{diag}(1, -1)$ is a matrix in the RA space, and $Q^{ab}_{\alpha\beta}$ denotes a single element of the Q matrix. In deriving $P[Q]$ we have assumed that the field Q does not fluctuate between the points x_1 and x_2, which is true provided that $|\mathbf{r}_1 - \mathbf{r}_2| \ll \min(\xi, \xi\sqrt{\Delta_\xi/\omega})$. As we will be mainly concerned with the localized regime, $\omega < \Delta_\xi$, we will refer to this relation as $|\mathbf{r}_1 - \mathbf{r}_2| \ll \xi$.

Since the preexponent (6) depends only on Q in the single point x_1, the functional integral (4) can be written as the integral over the single supermatrix $\mathcal{Q} = Q(x_1)$: $\int P[Q] e^{-S[Q]} DQ(x) = \int P(\mathcal{Q}) \Psi^2(\mathcal{Q})\, d\mathcal{Q}$, where $\Psi(\mathcal{Q}) = \int_{Q(x_1)=\mathcal{Q}} e^{-S[Q]} DQ(x \geq x_1)$. In the transfer-matrix approach, the function $\Psi(\mathcal{Q})$ is obtained as the zero-energy solution of an appropriate supermatrix Hamiltonian [19]. In Efetov's parameterization [4, 18], $\Psi(\mathcal{Q}) = \Psi(\lambda, \lambda_1)$ depends only on two variables: λ and λ_1, parameterizing the FF and BB sectors of the supermatrix $\mathcal{Q}$, respectively. Taking the integrals over the other variables of the parameterization we obtain the following expressions for the functions defined in Eq. (2):

$$A(\omega) = \frac{1}{2} \mathrm{Re} \int \Psi^2(\lambda, \lambda_1)\, d\lambda\, d\lambda_1, \tag{7}$$

$$B(\omega) = \frac{1}{2} \mathrm{Re} \int \frac{\lambda_1 + \lambda}{\lambda_1 - \lambda} \Psi^2(\lambda, \lambda_1)\, d\lambda\, d\lambda_1, \tag{8}$$

where the integrals are taken over the strip $-1 \leq \lambda \leq 1$ and $\lambda_1 \geq 1$.

The function $\Psi(\lambda, \lambda_1)$ is the zero mode ($H\Psi = 0$) of the transfer-matrix Hamiltonian [18, 19]:

$$H = -\frac{(\lambda_1 - \lambda)^2}{2} \left[\frac{\partial}{\partial \lambda} \frac{1 - \lambda^2}{(\lambda_1 - \lambda)^2} \frac{\partial}{\partial \lambda} + \frac{\partial}{\partial \lambda_1} \frac{\lambda_1^2 - 1}{(\lambda_1 - \lambda)^2} \frac{\partial}{\partial \lambda_1} \right] - \frac{i\omega}{4\Delta_\xi} (\lambda_1 - \lambda), \tag{9}$$

with the boundary condition $\Psi(1,1) = 1$ due to supersymmetry.

The Hamiltonian (9) without the potential term $\propto (\lambda_1 - \lambda)$ had been studied in detail [22, 23]. The zero mode in the deeply localized regime, $\omega \ll \Delta_\xi$, had been obtained by Efetov and Larkin [19]. In this limit it depends only on the noncompact bosonic variable λ_1 and has the form $\Psi(\lambda, \lambda_1) \approx \sqrt{-2i(\omega/\Delta_\xi)\lambda_1} \times K_1(\sqrt{-2i(\omega/\Delta_\xi)\lambda_1})$, with K_1 being the MacDonald function.

Now we show that the zero mode of the Hamiltonian (9) can be found exactly for an arbitrary ω. The region of variation of the parameters λ and λ_1 suggests us to consider them as elliptic coordinates on the half-plane $y > 0$ of an auxiliary plane (x, y): $\lambda = (r - r_1)/2$ and $\lambda_1 = (r + r_1)/2$, where $r = \sqrt{x^2 + y^2}$ and $r_1 = \sqrt{(x-2)^2 + y^2}$, see Fig.2. Changing variables from (λ, λ_1) to (x, y) and introducing a new function Φ: $\Psi = r_1 \Phi$, we obtain

$$H\Psi = -\frac{r_1^2 r}{2} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} + \frac{i\omega}{2\Delta_\xi r} \right] \Phi = 0. \tag{10}$$

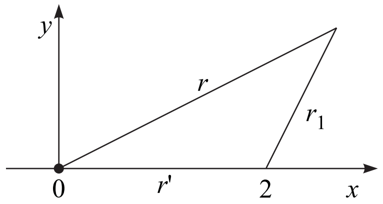

Fig.2. To the construction of elliptic coordinates

The differential part of this operator resembles the Laplace operator written in cylindrical coordinates. Rotating the upper half-plane around the x axis by the angle φ we span the whole 3D space (the function Φ is evidently independent of φ). The initial condition $\Psi(1,1) = 1$ transforms to $\Phi(r_1 \to 0) = 1/r_1$, which can be easily incorporated into the equation for Φ:

$$\left[\nabla_{\mathbf{r}}^2 + i\omega/2\Delta_\xi r \right] \Phi(\mathbf{r}) = -4\pi \delta^{(3)}(\mathbf{r} - \mathbf{r}'), \tag{11}$$

where $\mathbf{r}'$ is the 3D vector pointing from the origin to the point (2,0,0), so that $|\mathbf{r} - \mathbf{r}'| = r_1$. Comparing with the equation $\{\nabla_{\mathbf{r}}^2 - 2\alpha/r + k^2\} G_k(\mathbf{r}, \mathbf{r}') = \delta^{(3)}(\mathbf{r} - \mathbf{r}')$, which defines the Green function in the Coulomb potential α/r at energy $k^2/2$, we immediately conclude that $\Phi = -4\pi G_0(\mathbf{r}, \mathbf{r}')$ is nothing but the zero-energy Green function in the field of an *imaginary* charge $\alpha = -i\omega/4\Delta_\xi$. To avoid possible confusion we emphasize that this 'zero-energy' refers to the auxiliary Coulomb problem, while the frequency ω can be arbitrary. We also note that it is only the zero mode of the Hamiltonian (9) that can be reduced to the Coulomb problem: for a finite E, the equation $H\Psi = E\Psi$ would correspond to the motion in a non-central field $-i\omega/4\Delta_\xi r - E/r r_1$.

Surprisingly, the Coulomb Green function has a very simple form in the real-space representation. According to the seminal result by Hostler and Pratt [24],

$$G_k(\mathbf{r}, \mathbf{r}') = \frac{i\Gamma(1 + i\alpha/k)}{4\pi k |\mathbf{r} - \mathbf{r}'|} (\partial_u - \partial_v) \times W_{-i\alpha/k, 1/2}(-iku)\, M_{-i\alpha/k, 1/2}(-ikv), \tag{12}$$

where $u = r + r' + |\mathbf{r} - \mathbf{r}'|$, $v = r + r' - |\mathbf{r} - \mathbf{r}'|$, and W and M are Whittaker's confluent hypergeometric functions. Taking the $k \to 0$ limit of Eq. (12) with the help of the asymptotic formulae for Whittaker's functions [25], we obtain the Green function at zero energy:

$$G_0(\mathbf{r}, \mathbf{r}') = \frac{(\partial_u - \partial_v) \sqrt{u}\, K_1(2\sqrt{\alpha u})\, \sqrt{v}\, I_1(2\sqrt{\alpha v})}{2\pi |\mathbf{r} - \mathbf{r}'|}. \tag{13}$$

In order to get the function $\Psi(\lambda, \lambda_1)$ one has to multiply $\Phi = -4\pi G_0(\mathbf{r}, \mathbf{r}')$ by r_1 that cancels the factor $|\mathbf{r} - \mathbf{r}'|$ in the denominator of Eq. (13), and express u

and v in terms of λ and λ_1: $u = 2(\lambda_1+1)$, $v = 2(\lambda+1)$. Thus we obtain the explicit form of the zero mode of the Hamiltonian (9) valid for an arbitrary frequency ω:

$$\Psi(\lambda,\lambda_1) = K_0(p)\, qI_1(q) + pK_1(p)\, I_0(q), \tag{14}$$

where we have denoted $p = \sqrt{-2i(\omega/\Delta_\xi)(\lambda_1+1)}$ and $q = \sqrt{-2i(\omega/\Delta_\xi)(\lambda+1)}$. The properties of the Bessel functions ensure proper normalization: $\Psi(1,1) = 1$.

The form of Eq. (14) indicates that the initial coordinates λ and λ_1 were somehow 'more natural' than the auxiliary 3D Coulomb coordinates. Nevertheless we think that the mapping to the Coulomb problem is important as it reveals the high symmetry of the transfer-matrix Hamiltonian (9). This symmetry is related to the fact that all orbits of classical motion in a Coulomb potential are closed due to the presence of an additional integral of motion, the so-called Runge-Lentz vector [26]. In quantum mechanics, the operators associated with the components of the Runge-Lentz vector together with the momentum operators commute as the generators of rotations in the four-dimensional space, leading to the $O(4)$ symmetry of the Coulomb problem [27, 28]. This $O(4)$ rotational invariance was used by Schwinger [29] who found the Coulomb Green function in the momentum representation. Note however that in deriving the real-space representation (12) Hostler and Pratt [24] did not explicitly employ the $O(4)$ symmetry of the problem.

The knowledge of the zero mode $\Psi(\lambda,\lambda_1)$ allows us to calculate $A(\omega)$ and $B(\omega)$ in Eqs. (7) and (8) analytically. The integration for $A(\omega)$ is done in terms of indefinite integrals of the product of two Bessel functions. The integral for $B(\omega)$ is much simplified in the representation of the 3D Coulomb coordinates: $B(\omega) = 4\pi \operatorname{Re}\beta(\omega)$ with

$$\beta(\omega) = \int d^3\mathbf{r}\, G_0^2(\mathbf{r},\mathbf{r}') = -\lim_{\mathbf{r}''\to\mathbf{r}'}\lim_{k\to 0}\frac{\partial G_k(\mathbf{r}'',\mathbf{r}')}{\partial k^2}, \tag{15}$$

which is calculated using the exact expression (12). The results for $A(\omega)$ and $B(\omega)$ read ($\kappa = \sqrt{-4i\omega/\Delta_\xi}$):

$$A(\omega) = (4/3)\operatorname{Re}\Big\{\kappa^2\big[I_1^2(\kappa) - I_0(\kappa)I_2(\kappa)\big]\times \\ \times\big[K_1^2(\kappa) - K_0(\kappa)K_2(\kappa)\big] - I_1^2(\kappa)K_1^2(\kappa)\Big\}, \tag{16}$$

and

$$B(\omega) = (4/3)\operatorname{Re}\big[I_1(\kappa)K_1(\kappa) + 2I_2(\kappa)K_0(\kappa)\big], \tag{17}$$

where we have used the properties of the Bessel functions and the fact that κ^2 is imaginary (ω is real) in order to simplify the result. Exact Eqs. (16) and (17) interpolate between the metallic regime ($\omega \gg \Delta_\xi$), which can be treated perturbatively by expanding in diffusive modes, and the nonperturbative localized regime ($\omega \ll \Delta_\xi$). The asymptotic expressions have the form ($z = \omega/\Delta_\xi$):

$$A(\omega) = \begin{cases} -1/3 + z^2\ln^2(1/z)/3 + \ldots, & z \ll 1, \\ -3/(64z^2) + \ldots, & z \gg 1; \end{cases} \tag{18}$$

$$B(\omega) = \begin{cases} 2/3 - 2z^2\ln(1/z)/9 + \ldots, & z \ll 1, \\ \sqrt{1/2z} + \ldots, & z \gg 1. \end{cases} \tag{19}$$

Now let us compare eigenfunctions' correlations in the 1D and Q1D geometries. It is known that the *single* eigenfunction statistics in the two cases are closely related [20]. Namely, the statistics of the wave function envelopes are precisely the same while the short-scale oscillations are different. Having derived the LDOS correlator in the Q1D case, we are now able to compare correlations of *different* eigenfunctions in the Q1D and 1D cases. The most striking mismatch between the two problems is seen in the LDOS correlation function at coincident points: our result for $R^{\text{coins}}(\omega)$ is always > 1 with a nontrivial ω-dependence (top curve in Fig.1), whereas $R_{\text{1D}}^{\text{coins}}(\omega) \equiv 1$ for the strictly 1D geometry [15]. However since the scales shorter than the Fermi wavelength can hardly be resolved (and with the results being model-dependent), it is more instructive to compare $R^{\text{close}}(\omega)$ at spatial separations $|\mathbf{r}_1 - \mathbf{r}_2|$ larger than k_F^{-1} and smaller than the localization length (ξ for Q1D, and l for 1D). For the 1D geometry, $R_{\text{1D}}^{\text{close}}(\omega) = 1 - (1/3)\int_0^\infty dt\, t^3 e^{-t}/[(\omega\tau)^2 + t^2]$ [13, 15], which is definitely different from our result, $R^{\text{close}}(\omega) = 1 + A(\omega)$. Though $R^{\text{close}}(0) = R_{\text{1D}}^{\text{close}}(0) = 2/3$, already the next term in the small-ω expansion [$\omega^2\ln^2(\Delta_\xi/\omega)$ vs. $\omega^2\ln(1/\omega\tau)$] demonstrates the difference between the two problems even in the deeply localized regime. Thus we conclude that the analogy between 1D and Q1D localization seen in the correlations of single wave functions, *does not extend* to the correlations of different wave functions.

Finally, we mention that the same approach can be employed in calculating the density-density correlation function which determines the kinetic response of the wire. In the limit $|\mathbf{r}_1 - \mathbf{r}_2| \ll \xi$ we get a counterpart of Eq. (2) with $1 + A(\omega)$ and $B(\omega)$ interchanged:

$$\langle \operatorname{Im} G_\varepsilon^R(\mathbf{r}_1,\mathbf{r}_2)\operatorname{Im} G_{\varepsilon+\omega}^R(\mathbf{r}_2,\mathbf{r}_1)\rangle = \\ = (\pi\nu)^2[k(\mathbf{r}_1,\mathbf{r}_2)(1 + A(\omega)) + B(\omega)]. \tag{20}$$

Note however that Eq. (20) is not sufficient to get the conductivity of the wire, which requires a much more complicated analysis of the limit $|\mathbf{r}_1 - \mathbf{r}_2| \sim \xi$ [19, 18].

To conclude, we obtained the explicit expression for the zero mode of the supersymmetric transfer-matrix

Hamiltonian of a thick disordered wire in a magnetic field. This allowed us to get the nonperturbative result for the pair correlation function of LDOS, which describes correlations between different eigenfunctions, in the limit $|\mathbf{r}_1 - \mathbf{r}_2| \ll \xi$. We demonstrate that these correlations are different in the 1D and Q1D cases. Generalization to arbitrary $|\mathbf{r}_1 - \mathbf{r}_2|$ and other symmetry classes will be the subject of further studies.

We thank D.M. Basko, M.V. Feigel'man, I.V. Gornyi, D.A. Ivanov, I.V. Kolokolov, V.E. Kravtsov and A.D. Mirlin for fruitful discussions. The research of M. A. S. was supported by the Program "Quantum Macrophysics" of the Russian Academy of Sciences, RFBR under grant # 04-02-16998, and the Russian Science Support Foundation.

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