Effect of the zero-charge phenomenon on the structure of a nonsingular vortex in ^{3}He-A

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An analytic solution is derived for a nonsingular vortex at low temperatures in the approximation of a large zero-charge logarithm. The properties of the new vortex are discussed.

The low-temperature behavior of superfluid ^{3}He-A has several distinguishing features which stem from the vanishing of the gap in the spectrum of fermion excitations at two points on the Fermi surface. These features have a strict analogy with the chiral anomaly and the zero-charge effect in elementary particle physics.[1] This circumstance has stimulated experimental studies of ^{3}He-A at $T/T_c \ll 1$. In the present paper we focus on one particular consequence of these features, which can be observed experimentally: the effect of the analog of zero charge on the structure of a nonsingular vortex with two circulation quanta.

In quantum electrodynamics with massless electrons, the polarization of vacuum leads to a logarithmic decay of the effective charge at low frequencies ω (or at large distances)[2]:

$$e_{eff}^2 = e^2 \frac{3\pi}{\ln \frac{\Lambda^2}{\omega^2}} , \quad (1)$$

where Λ is the ultraviolet cutoff parameter. A further consequence is a logarithmic growth of the electromagnetic energy:

$$\frac{1}{8\pi e_{eff}^2} F_{\mu\nu} F^{\mu\nu} = \frac{1}{24\pi^2 e^2} F_{\mu\nu} F^{\mu\nu} \ln \frac{\Lambda^2}{\omega^2} . \quad (2)$$

In ^{3}He-A, whose low-energy structure corresponds to quantum electrodynamics, the role of the vector potential $\mathbf{A}$ of the electromagnetic field is played by the orbital vector $\mathbf{l}$ multiplied by the Fermi momentum: $\mathbf{A} = k_F \mathbf{l}$. A logarithmic divergence of the electromagnetic energy corresponds in ^{3}He-A to a logarithmic divergence in the expansion of the hydrodynamic energy of the liquid in the gradients of the vector $\mathbf{l}$. In the limit $T \to 0$, the hydrodynamic energy is[3]

$$F = \int d^3x \left\{ \frac{1}{2} \rho \mathbf{v}_s^2 - \frac{1}{2} C_0 (\mathbf{l} \cdot \mathbf{v}_s)(\mathbf{l} \text{ curl } \mathbf{l}) + \frac{1}{2} \mathbf{v}_s \text{curl} \left(\frac{1}{2} \rho \mathbf{l} \right) + K_s (\vec{\nabla} \cdot \mathbf{l})^2 \right.$$
$$\left. + K_t (\mathbf{l} \text{ curl } \mathbf{l})^2 + K_b [\mathbf{l}, \text{curl } \mathbf{l}]^2 + \frac{K_t}{\xi_d^2} [\mathbf{l}, \mathbf{d}]^2 \right\} , \quad (3)$$

where

$$C_0/\hbar = \frac{k_F^3}{3\pi^2} \approx \frac{\rho}{m_3} , \quad K_s = \frac{\hbar^2 k_F^3}{96\pi^2 m^*} , \quad K_t = \frac{\hbar^2 k_F^3}{96\pi^2 m_3} \left(1 + \frac{2}{3} \frac{m_3}{m^*} \right) ,$$

and there is a logarithmic divergence of the type in (2) in the parameter K_b:

$$K_b = \frac{\hbar^2 k_F^3}{96\pi^2 m_3} + \frac{\hbar^2 k_F^3}{24\pi^2 m^*} L ,$$
$$L = \begin{cases} \ln \frac{\Delta_0^2}{T^2} , & T^2 > \Delta_0 v_F |[\mathbf{l} \text{ curl } \mathbf{l}]|, ^3 \\ \ln \frac{\Delta_0}{v_F |[\mathbf{l} \text{ curl } \mathbf{l}]|} , & T^2 < \Delta_0 v_F |[\mathbf{l} \text{ curl } \mathbf{l}]| . ^1 \end{cases} \quad (4)$$

Here Δ_0 is the maximum of the gap in the spectrum, which corresponds to the cutoff parameter Λ in (1); m_3 is the mass of the ^{3}He atom; m^* is the effective mass in the Fermi liquid; the last term in (3) is the dipole interaction of the orbital vector $\mathbf{l}$ with the unit vector $\mathbf{d}$ which describes the spin degrees of freedom in ^{3}He-A; $\xi_d \sim 10^{-3}$ cm is the dipole length; and the vector $\mathbf{d}$ can be assumed to remain constant in a vortex, as we will see below.

How does the logarithmic divergence in K_b affect the structure of a nonsingular vortex in ^{3}He-A?

In a nonsingular vortex (see the review by Salomaa and Volovik[4]), the quantity δ, functional singularity in the curl (or rotor) of the superfluid velocity $\mathbf{v}_s$, is smoothed through the formation of a texture in the field of the vector $\mathbf{l}$, as described by the Mermin-Ho relation[5]

$$\mathrm{curl}\, \mathbf{v}_s = \frac{\hbar}{4m_3} e_{ijk} l_i [\vec{\nabla} l_j, \vec{\nabla} l_k]. \tag{5}$$

The structure of the vortex is found by minimizing energy (3) under constraint (5) and from the condition that the circulation of $\mathbf{v}_s$ around the vortex along a remote contour constitutes two circulation quanta: $\oint d\mathbf{x}\cdot\mathbf{v}_s = \hbar/m_3$ (the quantum of circulation is $\hbar/2m_3$). The latter assertion means that according to (5) the vector $\mathbf{l}$ in the texture traces out an entire sphere of unit radius.

The logarithmic divergence in K_b has the consequence that the vortex structure with the lowest energy will be that in which bending deformations of the field $\mathbf{l}$ (i.e., deformations with $[\mathbf{l}\,\mathrm{curl}\,\mathbf{l}] \neq 0$) are at a minimum. Such a structure represents a cylindrical domain of radius a, in which the central region with $\mathbf{l} = \hat{\mathbf{z}}$ ($\hat{\mathbf{z}}$ is the direction of the vortex axis) is separated by a thin domain wall of width b (which is $\ll a$, as we will see below) from an external region with $\mathbf{l} = \hat{\mathbf{z}}$. The domain wall is a torsional soliton[6]:

$$\mathbf{l} = \hat{\mathbf{z}} \cos \chi(r) + \vec{\hat{\varphi}} \sin \chi(r), \tag{6}$$

where z, r, and φ are cylindrical coordinates, and the function $\chi(r)$ is shown in Fig. 1. In a planar torsional soliton we would have $\mathrm{curl}\,\mathbf{l} \| \mathbf{l}$; i.e., there would be no bending. A bending arises only by virtue of the curvature of the cylindrical surface. Let us find a and b in the logarithmic approximation, i.e., under the assumption that the logarithm L in (4) is a large number.

In the logarithmic approximation, the energy of a vortex is the sum of the energy of the domain wall, F_w, and of the kinetic energy (F_k) of the liquid flow around the

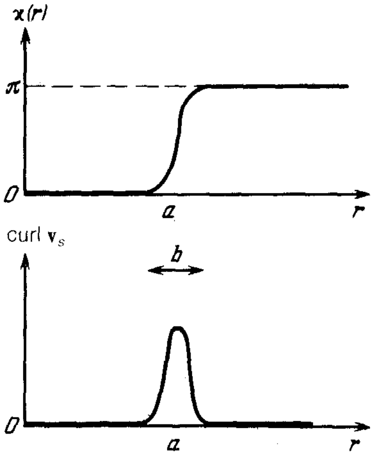

FIG. 1. In a low-temperature nonsingular vortex, the vorticity $\mathrm{curl}\,\mathbf{v}_s$ and the texture in the field of the vector $\mathbf{l}$ are concentrated in a narrow cylindrical shell of width b and radius $a \gg b$. This shell constitutes a torsional soliton in which $\mathbf{l}$ reverses direction, rotating in the plane of the soliton.

vortex. The interaction between $\mathbf{v}_s$ and the texture, which is described by the second and third terms in (3), is small in this approximation. The flow velocity $\mathbf{v}_s$ is $\hbar/m_3 r$ outside the cylinder, while inside the cylinder it vanishes, because of the screening effect of the domain wall, in which $\mathbf{l}$ takes on all values. The kinetic energy per unit length of the vortex is therefore

$$F_k = \pi\rho \left(\frac{\hbar}{m_3}\right)^2 \ln\frac{R}{a}, \tag{7}$$

where R is the external cutoff radius.

The energy of a domain wall per unit length of the vortex can be written as follows under the condition $b \ll a$ and when we note that d must be directed along $\hat{\mathbf{z}}$:

$$F_w = 2\pi a K_t \int dr \left\{ \left(\frac{d\chi}{dr}\right)^2 + \frac{K_b}{K_t}\frac{\sin^4\chi}{a^2} + \frac{\sin^2\chi}{\xi_d^2} \right\}. \tag{8}$$

As we will see below, since the dipole contribution to the energy [the third term in Eq. (8)] is small in comparison with the second (bending) term, it may be treated as a perturbation. In the lowest approximation we find

$$\chi(r) = \operatorname{arccot}\left(-\frac{r-a}{b}\right), \quad b = a\left(\frac{K_t}{K_b}\right)^{1/2} \sim aL^{-1/2} \ll a. \tag{9}$$

In other words, the wall width b is indeed smaller than the cylinder radius a. Correcting for the dipole term, we find the energy F_w to be

$$F_w = 2\pi^2 (K_t K_b)^{1/2} \left(1 + \frac{b^2}{\xi_d^2}\right). \tag{10}$$

Minimizing $F_k + F_w$ with respect to a, we find the following expression for a:

$$a = \xi_d \left(\frac{\hbar^2\rho}{4\pi m_3^2 K_t}\right)^{1/2} \left(\frac{K_b}{K_t}\right)^{1/4} \sim \xi_d L^{1/4}. \tag{11}$$

Since the wall dimension $b \sim \xi_d L^{-1/4}$ is smaller than ξ_d, the dipole contribution to the energy of the domain wall is indeed small, justifying the assumptions made in the derivation of (9) and (10).

As follows from (4), the vlaue of L at $T=0$ is ξ_d/ξ_0, where $\xi_0 = v_F/\Delta_0$ is the coherence length. Since we have $\xi_d/\xi_0 \sim 10^3$, the use of the logarithmic approximation does not introduce a large error. Furthermore, ^{3}He-A exists at $T=0$ only in the presence of a magnetic field. If the magnetic field, which directs the vector $\mathbf{d}\perp\mathbf{H}$, is not to disrupt the vortex structure which we have found, this field must be directed perpendicular to the rotation axis. The vector $\mathbf{d}$ must then lie along $\hat{\mathbf{z}}$. The deviations of $\mathbf{d}$ from $\hat{\mathbf{z}}$ due to the interaction with the texture of the domain wall are small in the logarithmic limit.

Here are some properties of the vortex structure which has been found:

1. This vortex falls in the category of w vortices,[4] for which parity is broken in such a way that there is spontaneous mass flux along the axis of the vortex within a domain wall.

2. By virtue of the well-defined and rather large radius of the cylindrical domain, a new well-defined oscillation mode arises in a vortex: oscillations of the radius a near its equilibrium value.

3. The low-temperature vortex which has been found differs from a high-temperature vortex in that the vector **l** outside the core of the vortex is directed along the axis, so that if we ignore small deviations of the vector **d**, the vortex is axisymmetric. At intermediate temperatures we would thus expect a phase transition between symmetric and asymmetric states in a nonsingular vortex.

4. The structure of a vortex is highly sensitive to the direction of the magnetic field: When the magnetic field deviates slightly from a transverse direction, the domain wall becomes blurred, and the axial symmetry is lost. This effect should be significant in ultrasonic experiments.

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