Analysis of some experimental data on high-temperature superconductors

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Some experimental data on new superconductors are analyzed to find an estimate of the width of the fluctuation region near T_c and the parameters of the electron band. The data do not contradict the presence of delocalized electrons. They indicate that the fluctuation region is narrow.

At this point, our theoretical understanding of the new high-temperature superconductors is far from complete. In this letter we are reporting an analysis of experimental data carried out in an attempt to answer two questions: 1) How large are the fluctuations during the superconducting transition? 2) To what extent do the normal and superconducting characteristics of the high-temperature superconductors conform to a Fermi-liquid description? In all the estimates below we use the most characteristic values, although the scatter in the experimental data is quite large.

1. We begin with the neighborhood of the transition temperature. The Ginzburg-Landau free-energy density[1] can be written in the anisotropic case as

$$F = \alpha t |\psi|^2 + \frac{b}{2}|\psi|^4 + \sum_i (4M_i)^{-1} \left|\left(-i\hbar\nabla_i - \frac{2e}{c}A_i\right)\psi\right|^2 , \qquad (1)$$

where $t = T - T_c$. The fluctuational correction to the heat capacity above[1,2] T_c is proportional to $t^{-1/2}$ in the three-dimensional regime, and the fluctuations are small under the condition

$$\frac{|t|}{T_c} \gg Gi_{3D} = \frac{T_c M_1 M_2 M_3 b^2}{4\pi^2 \alpha} . \qquad (2)$$

If the anisotropy is large (if say, $M_3 \gg M_1, M_2$), the correction will be proportional to $t^{-1/2}$. The layered nature of the structure means that an expansion in the gradients along the c axis would not be sufficient as we move away from T_c. The temperature at which the regime changes, T^*, can be estimated by equating the fluctuational corrections to the heat capacity, $4M_3\alpha|t^*| = (2z/c_0)^2$ (z is the number of layers in the unit cell, and c_0 is the length of the cell along the c axis). In the standard BCS theory, the Ginzburg number, Eq. (2), is small because $T_c \ll E_F$ is small.

The parameters α, b, and M_i determine the thermodynamic critical field H_{ct} and the penetration depths. The latter are not known well at all in the high-temperature superconductors, so we will work from measurements from the jump in the heat capacity, ΔC_p,

$$-\frac{\partial H_{ct}}{\partial T}\bigg|_{T=T_c} = (4\pi\Delta C_p / T_c)^{1/2} \tag{3}$$

and the upper critical fields. In the anisotropic case we would have

$$H_{c2}^{\parallel} = \sqrt{2}\,\kappa_{ab} H_{ct}; \qquad H_{c2}^{\perp} / H_{c2}^{\parallel} = (M_c / M_{ab})^{1/2} \tag{4}$$

($\parallel$ and $\perp$ specify the orientation of the magnetic field with respect to the c axis). The penetration depths are related to the critical field

$$\delta_{ab}^{-2} = \frac{2\sqrt{2}eH_{ct}}{c\hbar\kappa_{ab}}; \qquad \delta_c = \delta_{ab}(M_c / M_{ab})^{1/2}. \tag{5}$$

The coherence parameters $\xi_{ab} = \delta_{ab}/\kappa_{ab}$ and $\xi_c = \xi_{ab}(M_{ab}/M_c)^{1/2}$ give the sizes of the pairs in the ab plane and along the c axis in the clean limit.

The characteristic values of ΔC_p are $\Delta C_p/T_c \approx 14$ mJ/[(mole of Cu)$\cdot K^2$] for both the lanthanum and the yttrium ceramics,[3] so we find from (3) the value $-\partial H_{ct}/\partial T \approx 2.2\times 10^2$ G/K for $YBa_2Cu_3O_7$ and $-\partial H_{ct}/\partial T \approx 1.7\times 10^2$ G/K for $La_{0.85}Sr_{0.15}CuO_4$. The width of the resistive transition is usually 0.5–2 K, and it increases with the field, indicating defects in the samples. Accordingly, there is a large scatter in the dependence $H_{c2}(T)$ near T_c. Oh *et al.*,[4] assert on the basis of an analysis of $H_{c2}(T)$ directly in the transition region that the critical region is $t \sim 0.1T_c$. We believe it is more proper to determine the parameters in (1) from the behavior of the quantities away from the diffuseness of the transition. The curves of $H_{c2}^{\parallel}(T)$ exhibit a significant positive curvature near T_c, so the determination of the slope $\partial H_{c2}^{\parallel}/\partial T$ is ambiguous. The values $-\partial H_{c2}^{\parallel}/\partial T \approx 0.37^{(a)} - 0.9^{(b)} T/K$ and $-\partial H_{c2}^{\perp}/\partial T \approx 3T/K$ were found for a $YBa_2Cu_3O_7$ single crystal in Ref. 5. The value $\partial H_{c2}/\partial T \approx -2.5T/K$ has been found[6] for the ceramic $La_{0.85}Sr_{0.15}CuO_4^{(c)}$. For the $LaSrCuO_1^{(d)}$ single crystal, the slope has been measured by only a single group[7]: $-\partial H_{c2}^{\parallel}/\partial T \approx 0.3T/K$ and $-\partial H_{c2}^{\perp}/\partial T \approx 4T/K$. On the basis of these results, we can compile Table I.

Here $\delta(0)$ is related to $\delta(T)$ near T_c by the interpolation formula $\delta(t) - \delta(0)[1 - (T/T_c)^4]^{-1/2}$, and $\xi(0)$ is defined by

TABLE I.

	κ_{ab}	M_c/M_{ab}	$\lvert t^*\rvert/T_c$	$\delta_{ab}(0)$ (Å)	$\delta_c(0)$ (Å)	$\xi_{ab}(0)$ (Å)	$\xi_c(0)$ (Å)	Gi_{3D}
$YBa_2Cu_3O_7$ (a)	12	65	0.4	700	5600	36	4.5	0.4×10^{-5}
(b)	28	11	1	1100	3500	23	6.7	1×10^{-5}
$La_{1.85}Sr_{0.15}CuO_4$ (c)	100	–	–	4000	–	22	–	2×10^{-5}
(d)	12	180	0.4	1400	$2\cdot10^4$	60	4.6	0.6×10^{-5}

$$\xi(T) = 0.84\xi(0)\left(1 - \frac{T}{T_c}\right)^{-1/2}, \quad \xi(0) = \xi_0 \tag{6}$$

in the clean limit (for YBaCuO; see the discussion below) and by

$$\xi(T) = 0.91\,\xi(0)\left(1 - \frac{T}{T_c}\right)^{-1/2}, \quad \xi^2(0) = \xi_0 l \tag{7}$$

in the dirty limit (for LaSrCuO). The fluctuation region in the high-temperature superconductors must therefore be quite narrow, although wider than in ordinary superconductors. The layered nature of the structure has effects rather far from T_c. Independent determinations of the penetration depth yield[8,9] $\delta \sim (1.5\text{–}3)\times 10^3$ Å.

2. The properties of "pure" La_2CuO_4 have so far been poorly reproducible. At a low doping level, a competition is observed between magnetism and superconductivity in this material.[10] Anderson[11] attributes this observation to a doping by a small number of holes in a Hubbard zone. However, the sharp changes in both the magnitude of the resistance and the nature of its temperature dependence do not contradict the suggestion that doping with strontium puts the system in a state with delocalized electrons. Correspondingly, the critical temperature in $YBa_2Cu_3O_x$ is $T_c \approx 90$ K and independent of x over the interval $6.7 < x < 7$. The paramagnetic susceptibility is constant. We take this result as indicating the existence of a delocalized zone. Let us attempt to find its parameters.

In the lanthanum cuprates (one hole per Cu atom) the carrier density is expected to be $n \approx 10^{22}$ cm^{-3}. The Hall effect for $La_{1.85}Sr_{0.15}CuO_4$ yields[12] $n \approx 0.7\times 10^{22}$ cm^{-3}, i.e. ~ 0.7 of a hole per Cu atom. Although estimates based on the Hall effect in weak fields are inaccurate (the average scattering cross section is involved in these estimates), we adopt the value $n = 7\times 10^{21}$ cm^{-3} for LaSrCuO below. In $YBa_2Cu_3O_7$ there are apparently both electrons and holes. For the single crystals of Ref. 13 we have the value $n \approx (7\text{–}9)\times 10^{21}$ cm^{-3}, corresponding to 1.2–1.5 carriers of each sign per unit cell. Below we adopt the value $n = 10^{22}$ cm^{-3} for YBaCuO. Because of the pronounced anisotropy of the resistances and H_{c2}, we assume that the Fermi surface is cylindrical. We then find $p_F/\hbar \approx 5.4\times 10^7$ cm^{-1} $(z = 2)$ and $p_F/\hbar \approx 5\times 10^7$ cm^{-1} $(z = 3)$ for $La_{1.85}Sr_{0.15}CuO_4$ and $YBa_2Cu_3O_7$, respectively. The band mass m_{ab} can be estimated from the plasma frequency, $\omega_p^2 = 4\pi ne^2/m_{ab}$. If we take $\omega_p = 1.7$ eV (Ref. 14) for LaSrCuO, we find $m_{ab}/m_0 = 3.3$ V. In YBaCuO we have[15] $\omega_p = 2.3$ eV and $m_{ab}/m_0 \approx 2.6$. The Pauli susceptibilities calculated on the basis of these band masses (the Fermi-liquid renormalizations are ignored) are respectively 0.66×10^{-4} cm^3/(mole of Cu) and 0.52×10^{-4} cm^3/(mole of Cu) or lower than the observed value[3] by a factor of about 2 or 2.5. For the Fermi energy $E_F = p_F^2/2m_{ab}$ we find 0.33 eV and 0.36 eV, respectively. The values of the ratio $k_B T_c/E_F$ for LaSrCuO and YBaCuO are 0.8×10^{-2} and 2×10^{-2}, respectively. The mean free path is estimated from the resistance $\rho(T_c)$. For LaSrCuO we find $l \approx 32(\rho_0/\rho)$ Å, and for YBaCuO we find $l \approx 21(\rho_0/\rho)$ Å, where $\rho_0 = 100$ $\mu\Omega\cdot$cm. Comparing $l(T_c)$ with $\xi_{ab}(0)$ from Table I, we see that even the better LaSrCuO samples [with a resistivity $\rho(T_c) \approx 300$

$\mu\Omega\cdot$cm] are effectively "dirty." The YBaCuO single crystals with $\rho(T_c)\approx 60\ \mu\Omega\cdot$cm (Ref. 16) can be regarded as clean.

In general, the properties of the high-temperature superconductors in the normal state are consistent with the presence of a fairly wide band. The range $l(T_c)$ is comparable to ξ_0, and the existing differences in the values of T_c for the various specific samples are consistent with the suggestion that the pairing is of a nontrivial nature. The "sizes of the pairs" are not much greater than the interatomic distances; for $YBa_2Cu_3O_7$, for example, we have $\hbar/p_F\xi_0\approx 0.9\times 10^{-1}$. Whether the pairs are bound by the Cooper mechanism or strong electron correlations[11] remains an open question.

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