Magnitude and anisotropy of the penetration depth in high-temperature superconductors

V. L. Pokrovskiĭ
Institute of Theoretical Physics, Academy of Sciences of the USSR

(Submitted 14 April 1988)
Pis'ma Zh. Eksp. Teor. Fiz. **47**, No. 10, 539–541 (25 May 1988)

The quasi-2D nature of the superconductivity makes it possible to calculate the penetration depth λ near the transition temperature and to evaluate the anisotropy of this depth. The depth λ turns out to be 3400 Å for $YBa_2Cu_3O_7$ and 4300 Å for $La_{1.85}Sr_{0.15}CuO_4$. The ratio of effective masses lies between 30 and 70. The Ginzburg number is no less than 0.07 for the yttrium superconductor and no less than 0.11 for the lanthanum superconductor.

There are many pieces of experimental evidence pointing to a quasi-2D nature of the superconductivity in the yttrium and lanthanum superconductors. Noteworthy among these pieces of evidence is the essential independence of the superconducting transition temperature from the substitution of rare-earth elements for the Y or the La (Ref. 1). Recent measurements[2] of the magnetic susceptibility of the compound $HoBa_2Cu_3O_z$ shown that the magnetic Ho layers and the superconducting copper layers are essentially independent. On the other hand, it follows from structural studies[3,4] that in both the lanthanum compounds and the yttrium compounds there are plane layers of copper which form a quadratic or nearly quadratic lattice with a period of about 3.8 Å.

Our purpose in the present letter is to point out that one can work from the fact of a quasi-2D nature to calculate the depth (λ) to which the magnetic field penetrates near the point of the superconducting transition and to estimate the anisotropy in λ from the known values of the transition temperature T_c and of the period in the direction perpendicular to the copper layers.

We start from the Kosterlitz-Thouless-Nelson relation[5] between the transition temperature and the density of the superfluid (superconducting) component, n_s:

$$T_c = \frac{\pi}{2} \frac{\hbar^2 n_s}{m}, \tag{1}$$

where m is the mass of the pair. We recall that in the Berezinskii Kosterlitz-Thouless theory the superfluid density appears abruptly below the transition. Equation (1) determines the finite 2D density of the superfluid component which appears below the transition. The 3D density of the superfluid component is found from the 2D density n_s in the obvious way:

$$n_s^{(3)} = n_s \nu / d. \tag{2}$$

Here d is the period of the structure in the direction perpendicular to the layers, and ν is the number of weakly coupled layers over a period d. The situation regarding this number ν is not completely clear. It evidently lies between 1 and 3 for the yttrium systems and is equal to 1 for the lanthanum systems. For the penetration depth λ we used the London formula

$$\lambda^{-2} = \frac{16 \pi n_s^{(3)} e^2}{m c^2}, \tag{3}$$

where the carrier charge has been set equal to $2e$. Comparing (1), (2), and (3), we find the direct relationship between the value of λ at the transition point and the transition temperature:

$$\lambda^2 (T_c) = \hbar^2 c^2 d / 32 T_c e^2 \nu. \tag{4}$$

The quantities n_s and m, which cannot be measured, do not appear in expression (4). For numerical estimates we use the values $T_c \approx 90$ K and $T_c \approx 40$ K for the yttrium and lanthanum superconductors, respectively. We derived values of d from structural data[4,5]: $d = 11.7$ Å for Y... and $d = 6.6$ Å for La ... . Here are the numerical data. For the yttrium superconductors, we give three versions: $\lambda/(T_c) = 3400$ Å$(\nu = 1)$, $\lambda(T_c) = 2400$ Å$(\nu = 2)$, $\lambda(T_c) = 1950$ Å$(\nu = 3)$. For the lanthanum superconductors, we have $\lambda(T_c) = 4300$ Å. The weak coupling between planes results in a 3D behavior of n_s and λ^{-2} near the transition temperature: Both of these properties vanish at $T = T_c$. We estimate the width of the region of 3D behavior, ΔT, and of the crossover from the 3D behavior to the 2D behavior from

$$\Delta T / T_c \approx [\ln(m_\perp / m_\parallel)]^{-2}, \tag{5}$$

where $m_\parallel$ and $m_\perp$ are the effective masses for motion along and across the plane.

The penetration depth has been measured in experiments on the depolarization of μ mesons by superconducting single crystals.[6,7] Comparing the numbers which we found with the experimental results, we note that even for $\nu = 1$ the value which follows from $\lambda(T_c)$ from the 2D behavior is only 2.6 times the experimental value of $\lambda(0)$ for $YBa_2Cu_3O_7$ and 2.15 times greater than that for $La_{1.85}Sr_{0.15}CuO_4$. These results show that the Ginzburg number Gi, which tells us whether the theory of a self-consistent field is applicable, is not too small here. The Ginzburg number for the 2D case is $\mathrm{Gi} = T_c b/\pi ca$, where a, b, and c are the coefficients of $|\psi|^2$, $|\psi|^4$, and $|\widehat{\nabla}\psi|^2$ in the Ginzburg-Landau free energy. The value of a is chosen far from the transition point. If the pairing involves only a narrow layer Δ on the Fermi surface, then we have $\mathrm{Gi} \approx 0.57\, \Delta/\epsilon_F$. In the case of a small value of Gi, Eq. (1) determines the Berezinskiĭ-Kosterlitz-Thouless transition temperature, which is close to the value found for this temperature from the theory of a self-consistent field, T_c^{GL}:

$$|(T_c^{GL} - T_c^{BKT})/T_c| = 2\mathrm{Gi}. \qquad (6)$$

The ratio $\lambda^2(0)/\lambda^2(T)$ at the Berezinskiĭ-Kosterlitz-Thouless transition point (more precisely, as the region of 3D behavior is approached) reaches the same value, 2Gi. It can thus be concluded that the value of Δ/ϵ_F is no less than 0.11 in $YBa_2Cu_3O_7$ and no less than 0.20 in $La_{1.85}Sr_{0.15}CuO_4$. These values contrast sharply with the values in ordinary superconductors, again verifying that there is an extremely strong coupling in a pair. It may be that the Bardeen-Cooper-Schrieffer picture of the superconducting state is completely inapplicable here, and we should think in terms of a Bose condensation of a charged liquid of preexisting pairs. Pronounced fluctuation effects associated with an appreciable Ginzburg number have been noted in measurements of H_{c2} in oriented films of $YBa_2Cu_3O_7$ (Ref. 8).

Figure 1 shows a plot of $\lambda^2(0)/\lambda^2(T) = n_s(T)/n_s(0)$ as a function of T/T_c for a $YBa_2Cu_3O_7$ single crystal. Here we have used data from Ref. 7. The crosses show the positions of the points for which we have $\lambda = \lambda(T_c)$, as calculated from the theory for $\nu = 1$, 2, and 3. Using expression (5), we can estimate the anisotropy of the effective masses. These estimates are rather crude, since slight errors in the temperature measurements lead to sharp changes in $m_\perp/m_\parallel$. The various values of ν and the allowance for the errors lead to a spread of the $m_\perp/m_\parallel$ values from 30 to 70. These figures agree fairly well with experimental measurements of this quantity,[9] which yield $m_\perp/m_\parallel \sim 50$, and with measurements of the anisotropy of the critical field (see Ref. 8 in the bibliography).

To what extent can these estimates be used? The correlation function for the

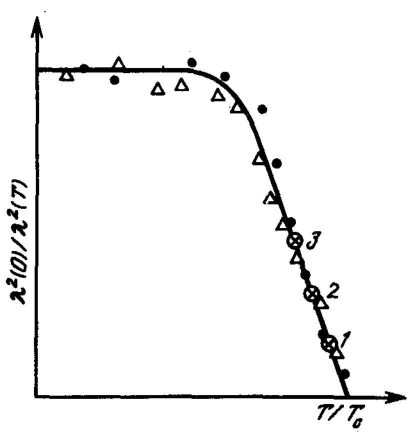

FIG. 1. See the text proper for an explanation. Filled circles—According to the data of Kossler *et al.*; triangles—according to the data of Harshman *et al.*[7]

order parameter, $G^{-1}(k,q)$ near the transition point is determined by the equation

$$G^{-1}(k,q) \propto k^2\xi_{\parallel}^2(0) + \gamma(1-\cos qd) + \tau; \qquad \tau = (T_c^{GL} - T)/T_c , \tag{7}$$

where k and q are components of the momentum parallel and perpendicular to the copper planes, γ is a dimensionless characteristic of the interplanar bonds, and $\xi_{\parallel}(0)$ is the coherence length in the plane at $T=0$. The weakness of the bonds between planes means that we have $\gamma \ll 1$. At small values of q, we can expand $G^{-1}(k,q)$ in powers of q; in doing so, we find a relationship between d and $\xi_{\perp}(0)$: $\gamma = 2d^2/\xi_{\perp}^2(0)$. Using the value of 2.1 Å found in Ref. 8 for $\xi_{\perp}(0)$, we find the estimate $\gamma \sim 0.06$. At these values of τ, the 2D behavior gives way to a 3D behavior. The point which we found thus lies in the crossover region, so the 2D approach leads to a reasonable result. In the region $\gamma \ll \tau \ll 1$, the superconductivity is of a 2D nature. In the region $[(d/4\xi_{\perp}(0))\mathrm{Gi}]^2 \ll \tau \ll \gamma$, the binding between planes is strong, but the fluctuations are weak.[1)] The 2D fluctuations at $\tau \sim \gamma \sim 0.06$ reduce the value of n_s to about half its average-field value. If we assume $\mathrm{Gi} = 0.07$, we find that the range of applicability of the 3D Ginzburg-Landau theory is extremely narrow: $0.01 < \tau < 0.06$.

I wish to thank L. P. Gor'kov, N. B. Kopnin, and G. V. Uĭmin for discussions.

[1)]The quantity on the left side of the inequality is equal to the 3D Ginzburg number.[10]

[1]Z. Fisk, J. D. Thompson, E. Zirngiebl *et al.*, Preprint, 1987; J. O. Willis, Z. Fisk, J. D. Thompson *et al.*, Preprint, 1987; S. E. Brown, J. D. Thompson, J. O. Willis *et al.*, Preprint, 1987.
[2]J. R. Thompson, D. K. Christen, S. T. Secula *et al.*, Phys. Rev. **B36**, 836 (1987).
[3]J. M. Kosterlitz and D. Thouless, J. Phys. **C6**, 1181 (1973); D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).
[4]V. B. Grande, H. Müller-Bushbaum, and M. Schwazer, Z. Anorg. Allg. Chem. **8**, 120 (1979); J. D. Jorgensen, H.-B. Schüttler *et al.* Phys. Rev. Lett. **58**, 1024 (1987); S. C. Moss, K. Forster, J. 0. Axe *et al.*, Phys. Rev. **B35**, 7195 (1987).
[5]R. J. Cava, B. Battlogg, R. B. Van Dover *et al.*, Phys. Rev. Lett. **58**, 1676 (1987); D. E. Cox, A. R. Moodenbaugh, J. Hurst, and R. H. Jones, J. Phys. Chem. Solids, 1987; J. J. Capponi, C. Chaillout, A. W. Hewat *et al.*, Europhys. Lett. **3**, 1301 (1987); Hoidoo Lou, R. K. McMillan, J. D. Axe, and D. E. Cox, Preprint, 1987.
[6]W. J. Kossler, J. R. Kempton, X. H. Yu *et al.*, Phys. Rev. **B35**, 7133(1987).
[7]D. R. Harshman, G. Aeppli, E. J. Ansaldo *et al.*, Phys. Rev. **B36**, 2386 (1987); W. J. Kossler, J. R. Kempton, and A. Moodenbaugh *et al.*, in: Novel Superconductivity. Proceedings of the International Workshop on Novel Mechanisms of Superconductivity, Berkeley, 1987 (ed. S. A. Wolf and V. z. Kresin), Plenum, New York, 1987.
[8]B. Oh, K. Char, A. D. Kent *et al.*, Preprint, Stanford, 1987.
[9]D. R. Harshman and Shneemeyer (in press).
[10]A. Z. Patashinskiĭ and V. L. Pokrovskiĭ, Fluctuation Theory of Phase Transitions, Nauka, Moscow, 1982.

Translated by Dave Parsons