Pis'ma v ZhETF, vol. 78, iss. 11, pp. 1203 – 1206

Dark matter from $SU(4)$ model

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Submitted 16 October 2003

Resubmitted 24 October 2003

The left-right symmetric Pati-Salam model of the unification of quarks and leptons is based on $SU(4)$ and $SU(2)\times SU(2)$ symmetry groups. These groups are naturally extended to include the classification of families of quarks and leptons. We assume that the family group (the group which unites the families) is also the $SU(4)$ group. The properties of the fourth-generation of fermions are the same as that of the ordinary-matter fermions in first three generations except for the family charge of the $SU(4)_F$ group: $F=(1/3,1/3,1/3,-1)$, where $F=1/3$ for fermions of ordinary matter and $F=-1$ for the fourth-generation fermions. The difference in F does not allow the mixing between ordinary and fourth-generation fermions. Because of the conservation of the F charge, the creation of baryons and leptons in the process of electroweak baryogenesis must be accompanied by the creation of fermions of the 4-th generation. As a result the excess n_B of baryons over antibaryons leads to the excess $n_{4\nu}=N-\bar{N}$ of neutrinos over antineutrinos in the 4-th generation with $n_{4\nu}=n_B$. This massive neutrino may form the non-baryonic dark matter. In principle the mass density of the 4-th neutrino $n_{4\nu}m_N$ in the Universe can give the main contribution to the dark matter, since the lower bound on the neutrino mass m_N from the data on decay of the Z-bosons is $m_N>m_Z/2$. The straightforward prediction of this model leads to the amount of cold dark matter relative to baryons, which is an order of magnitude bigger than allowed by observations. This inconsistency may be avoided by non-conservation of the F-charge.

PACS: 12.10.–g, 12.60.Rc, 95.35.+d

1. Introduction. Three-family structure of fermions is one of the most puzzling features of the Standard Model. The other puzzles comes from the baryonic asymmetry of the Universe and from astrophysical observations of the dark matter, whose particles are most probably beyond the Standard Model. Here we make an attempt to connect these three phenomena in the framework of the $SU(4)$ scheme with four families of fermions. We discuss why the classification scheme based on the $SU(4)$ groups is preferrable and how it leads to the non-baryonic dark matter in the Universe if the Universe is baryonic asymmetric.

Breaking of the $SU(4)$ symmetry between four generations to its $SU(3)\times U(1)_F$ subgroup separates the fermions of the fourth generation from the fermions of the other three families. If the charge F – the generator of the $U(1)_F$ group – is conserved in the process of baryogenesis, as it occurs for example in electroweak baryogenesis, the formation of the baryon asymmetry automatically leads to formation of dark matter made of the massive fourth-generation neutrinos. In such scenario the mass density of the dark matter in the Universe essentially exceeds the mass density of the baryonic matter.

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2. Pati–Salam model. In the current Standard Model three families (generations) of fermions have identical properties above the electroweak energy scale. Each family contains 16 Weyl fermions (8 left and 8 right) which transform under the gauge group $G(213)=SU(2)_L\times U(1)_Y\times SU(3)_C$ of weak, hypercharge and strong interactions respectively. In addition they are characterized by two global charges: baryonic number B and leptonic number L.

The Grand Unification Theories (GUT) unify weak, hypercharge and strong interactions into one big symmetry group, such as $SO(10)$, with a single coupling constant. There is, however, another group, the minimal subgroup of $SO(10)$, which also elegantly unites 16 fermions in each generation, a type of Pati–Salam model [1 – 3] with the symmetry group $G(224)=SU(2)_L\times SU(2)_R\times SU(4)_C$. This left–right symmetric group preserves all the important properties of $SO(10)$, but it has advantages when compared to the $SO(10)$ group [4–6]. In particular, the electric charge is left–right symmetric:

$$Q=\frac{1}{2}(B-L)+T_{R3}+T_{L3}, \qquad (1)$$

where $B-L=(\frac{1}{3},\frac{1}{3},\frac{1}{3},-1)$ is the difference between baryon and lepton numbers (the generator of $SU(4)_C$

color group), and T_{R3} and T_{L3} are left and right isotopic spins (generators of the groups $SU(2)_R$ and $SU(2)_L$). Also the G(224) group nicely fits the classification scheme of quantum vacua based on the momentum-space topology of fermionic propagators [7].

According to Terazawa [8] the 16 fermions of each generation can be represented as the product Cw of 4 bosons and 4 fermions in Fig.1. The Terazawa scheme is

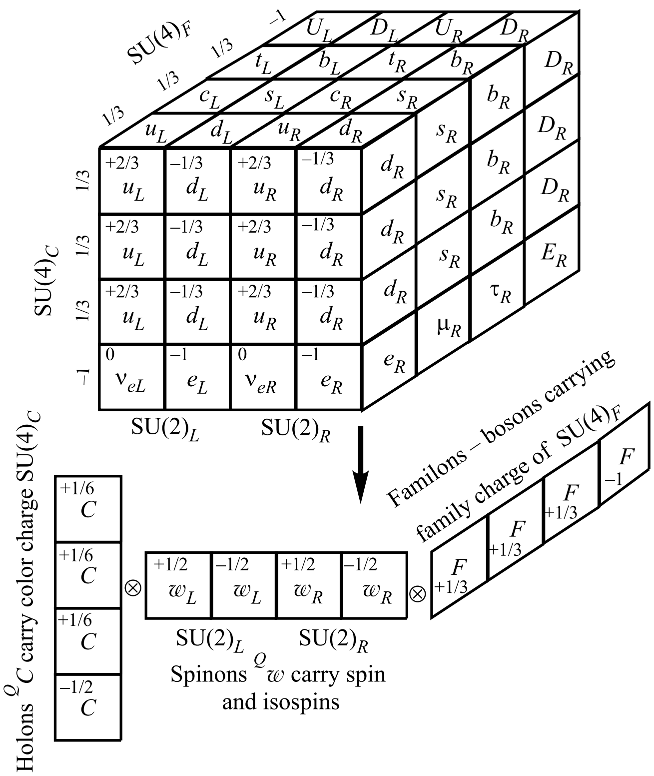

Fig.1. Terazawa scheme of composite fermions wC as bound states of w-fermions and C-bosons. Numbers show the electric charge

similar to the slave-boson approach in condensed matter, where the spinons are fermions which carry spin and holons are bosons which carry electric charge [7]. Here the "holons" C form the color $SU(4)_C$ quartet of spin-0 $SU(2)$-singlet particles with $B-L$ charges of the $SU(4)_C$ group and electric charges $Q = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, -\frac{1}{2})$. The "spinons" are spin-$\frac{1}{2}$ particles w, which are color $SU(4)_C$ singlets and $SU(2)$-isodoublets.

3. Family charge and fourth generation. What is missing in this scheme is fermionic families. The natural extension, which also uses the $SU(4)$ symmetry, is to introduce the group $SU(4)_F$ in the family direction (Fig.2). This requires the introduction of additional 4-th generation of fermions. Then the family charge F comes as one of the generators of $SU(4)_F$ in the same way as the $B-L$ charge comes as one of the generators of $SU(4)_C$

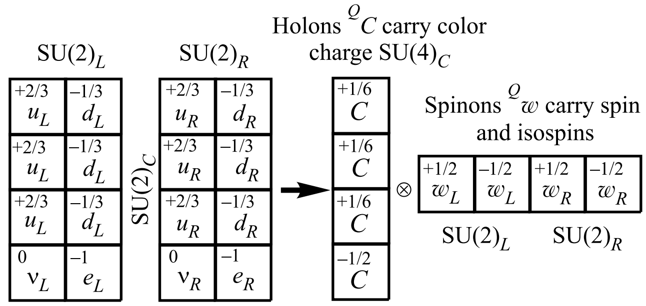

Fig.2. Extended Terazawa scheme of composite fermions wCF as bound states of w-fermions and C- and F-bosons

color group: $F = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1)$. Here $F = \frac{1}{3}$ is assigned to ordinary matter in three generations, while $F = -1$ corresponds to the fourth generation.

The total number 64 of Weyl fermions, each being described by 2 complex or 4 real functions, satisifies the 2^n rule, which probably reflects the importance of the Clifford algebra in the undelying physics [9]. This is another advantage of classification based on $SU(2)$ and $SU(4)$ groups. This extension also fits the Terazawa scheme which is modified by addition of extra boson. All 64 fermions are now represented as the product FCw in Fig.2, where F is the quartet of bosons with F charges of the $SU(4)_F$.

Let us assume that the breaking of $SU(4)_F$ symmetry occurred in the same way as the breaking of the color group: $SU(4)_C \to SU(3)_C \times U(1)_{B-L}$ and $SU(4)_F \to SU(3)_F \times U(1)_F$, where $U(1)_F$ is the group generated by charge F. Then all the charges of the fourth-generation fermions are the same as that of the ordinary-matter fermions in first three generations except for the family charge F. This separates out the 4-th generation in the same manner as leptons – the fourth color – are separated from quarks. The 4-th generation fermions cannot mix with ordinary fermions in the first three generations in the same manner as leptons cannot mix with quarks. This is the main difference from the democratic scheme of sequential generations, which assumes that all generations are equal. The important consequence of such a discrimination between the generations is that the excess of the baryonic charge in the Universe must be accompanied by the uncompensated neutrinos of the 4-th generation.

4. Dark matter and electroweak baryogenesis. Let us assume that the F charge is strictly conserved. This certainly occurs in the process of electroweak baryogenesis, in which the baryons and leptons are created due to the anomalous non-conservation of the baryonic and leptonic numbers, while the chiral anomaly conserves the charges F and $B-L$ (this does not depend on whether $SU(4)_F$ is a local or global group). The conservation of these two charges gives

$$F = \frac{1}{3}(n_q + n_l) - (n_{4q} + n_{4l}) = 0,$$
$$B - L = \frac{1}{3}(n_q + n_{4q}) - (n_l + n_{4l}) = 0, \quad (2)$$

where n_q and n_l are (algebraic) densities of quarks and leptons belonging to the ordinary matter in three generations; and n_{4q} and n_{4l} are that of the fourth generation. From Eqs.(2) one obtains the densities of the fourth-generation quarks and leptons in terms of the ordinary matter (which mainly belongs to the first generation):

$$n_{4q} = n_l, \quad n_{4l} = \frac{1}{3}n_q - \frac{2}{3}n_l. \quad (3)$$

If in our Universe $n_l = 0$, i.e. the leptonic charge of electrons is compensated by that of antineutrinos, one obtains

$$n_{4q} = 0 \ , \ n_{4l} = \frac{1}{3}n_q = n_B, \quad (4)$$

where n_B is the algebraic number density of baryons in our Universe.

This means that any excess of the baryonic charge in the Universe is accompanied by the excess $n_{4\nu} = N - \bar{N} = n_B$ of the neutrinos over antineutrinos in the fourth generation (we assume that the mass of the 4-th generation neutrino is smaller than that of the 4-th generation electron, $m_N < m_E$). If this is correct, the 4-th generation neutrinos adds to the mass of our Universe and may play the role of the dark matter. The mass m_N of the 4-th generation (dark) neutrino can be expressed in terms of the mass of baryon (nucleon) m_B and the energies stored in the baryonic matter (with fraction Ω_B of the total mass in the flat Universe) and non-baryonic dark matter (with fraction Ω_{DM}):

$$\frac{m_N}{m_B} = \frac{\Omega_{DM}}{\Omega_B}. \quad (5)$$

Masses of the 4-th generation of fermions must be below the electroweak energy scale, since the masses of all the fermions (ordinary and of 4-th generation) are formed due to violation of the electroweak symmetry, as follows from the topological criterion of mass protection (see Chapter 12 of Ref. [7]). There is also the lower constraint on the 4-th neutrino mass which comes from the measured decay properties of the Z-boson: $m_N > m_Z/2 = 45.6$ GeV (see e.g. [10] and somewhat stronger constraint $m_N > 46.7$ GeV from the Z-resonance lineshape in [11]). Then the fourth generation contribution to the dark matter must be

$$\Omega_{DM} = \Omega_B \frac{m_N}{m_B} > 50\Omega_B. \quad (6)$$

Since in our spatially flat Universe $\Omega_{DM} < 1$, this gives the constraint on the baryonic mass in the Universe: $\Omega_B < 0.02$. On the other hand, since the baryonic density in the luminous matter is $\Omega_B \sim 0.004$, the constraint on the dark matter mass is $\Omega_{DM} > 0.2$.

There are, however, the other constraints on the mass of m_N of heavy neutrino obtained from particle and astroparticle implications of the 4-th generation neutrinos [12], from which it follows that m_N must be larger than 200 GeV. For the early limits on heavy neutrino contribution to the density of dark matter, which follow from consideration of neutrino-antineutrino annihilation, see refs [13]. On the other hand the electroweak data fit prefers the 50 GeV neutrinos [14], though the masses as high as 200 GeV are not excluded.

One must also take into account that the dark matter contribution Ω_{DM} is only a fraction of unity, as follows from observational data on CMB angular temperature fluctuations [15]. On the other hand the baryonic content of dark matter is about 10 times bigger than the baryon density in the visible matter, as follows independently both from the structure of acoustic peaks in the angular power spectrum of CMB temperature fluctuations [15] and from the primordial nucleosynthesis theory plus observed abundance of light elements (^{4}He, D, ^{3}He and ^{7}Li) [16]. As a result one finds that $\Omega_{DM}/\Omega_B < 7$. Together with the constraint on m_N this indicates that the excess $n_{4\nu}$ of neutrinos over antineutrinos must be smaller (maybe by one or two orders of magnitude) than n_B. This would mean that the F-charge is not strictly conserved.

5. Conclusion. It appears that the classification scheme based on the $SU(2)$ and $SU(4)$ groups has many interesting features. Here we considered one of them – the discrimination between the ordinary-matter fermions and the fourth generation. If the family charge F is strictly conserved, the fermions of the fourth-generation neutrinos with density $n_{4\nu} = n_B$ must be necessarily present in the baryonic asymmetric Universe to compensate the family charge. Moreover, it is not excluded that it is the asymmetry in the fourth-generation neutrinos which is primary and it serves as a source of the baryonic asymmetry of the Universe.

These massive neutrinos form the stable dark matter with mass density exceeding the baryonic mass density by the ratio of masses of neutrino and baryon m_N/m_B. According to the well known physics of the decay of Z-boson, this factor must be larger than 50. Whether this is an acceptable solution to the dark matter problem depends on details of the G(2244) $= SU(2)_L \times SU(2)_R \times SU(4)_C \times SU(4)_F$ model. The above relation between the baryonic and non-baryonic masses of the Universe is based on the assumption of the conservation of the F-charge. If this requirement is weakened, the dark matter density coming from the fourth generation of fermions will depend on the details of the baryo- or neutrino-genesis and on the further cosmological evolution of ordinary and dark matter.

I am grateful to S. L. Adler for e-mail correspondence, J. C. Pati for encouraging comment, M. Yu. Khlopov and T. Vachaspati for numerous discussions, and L. B. Okun and A. A. Starobinsky for careful reading of the manuscript and valuable suggestions. This work was supported by ESF COSLAB Programme and by the Russian Foundations for Fundamental Research.

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