The evaluation of the branching ratios of the decays $\phi(1020) \to 2\pi^+2\pi^-\pi^0$, $\pi^+\pi^-3\pi^0$

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Combining the Okubo-Zweig-Iizuka rule in the decay $\phi \to \rho\pi \to \pi^+\pi^-\pi^0$ with the $\rho \to 4\pi$ decay amplitudes, we calculate the $\phi \to 2\pi^+2\pi^-\pi^0$ and $\phi \to \pi^+\pi^-3\pi^0$ ones. The partial widths of the above ϕ decays are evaluated, and the excitation curves in e^+e^- annihilation are obtained, assuming reasonable particular relations among the parameters characterizing the anomalous terms of the HLS Lagrangian. The evaluated branching ratios $B_{\phi\to\pi^+\pi^-3\pi^0} \approx 2\cdot 10^{-7}$ and $B_{\phi\to 2\pi^+2\pi^-\pi^0} \approx 7\cdot 10^{-7}$ are such that with the luminosity $L = 500$ pb^{-1} attained at DAΦNE ϕ factory, one may already possess about 1685 events of the decays $\phi \to 5\pi$.

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The decay of the $\phi(1020)$ meson to the state 3π occurs due to the violation of the Okubo-Zweig-Iizuka rule (OZI). The decay $\phi \to 5\pi$, expected to proceed due to the violation of this rule, should be dominated by the process $\phi \to \rho\pi$ followed by the decay $\rho \to 4\pi$ whose amplitudes and decay width were calculated in, say [1]. See the diagram in Fig.1a. The ρ meson in these diagrams is resonant. Indeed, choosing the averaged pion energy from the condition of 'equilibrium' as $\langle E_\pi \rangle = m_\phi/5$, one finds that the invariant mass of four pions emitted in the transition $\rho \to 4\pi$ is $m_{4\pi} \simeq m_\rho$. The seemingly resonant diagrams Fig.1b and d do not, in fact, possess this property, because three pions produced either from the transition $\pi \to \pi\pi\pi$ or directly from the $\phi \to \rho\pi\pi\pi$ transition, push ρ meson away from the resonance. Indeed, the invariant mass of the pion pair in the transition $\rho \to 2\pi$ evaluated assuming the same average pion energy as above, falls into the interval $2m_\pi \le m_{2\pi} \le 0.41$ GeV, which is far from the resonance value. The contribution of the diagram in Fig.1e containing the OZI-suppressed vertex $\phi\rho\rho\pi$ is, at the ϕ mass, deeply under the threshold of the production of two ρ mesons and by this reason is suppressed, too. The two diagrams in Fig.1c, containing OZI-suppressed point like vertices $\phi \to 5\pi$ and $\phi \to 3\pi$, do not contain the intermediate ρ-meson at all and hence can be neglected in the approximation of the resonant ρ meson. The effect of relaxing these approximations is considered below.

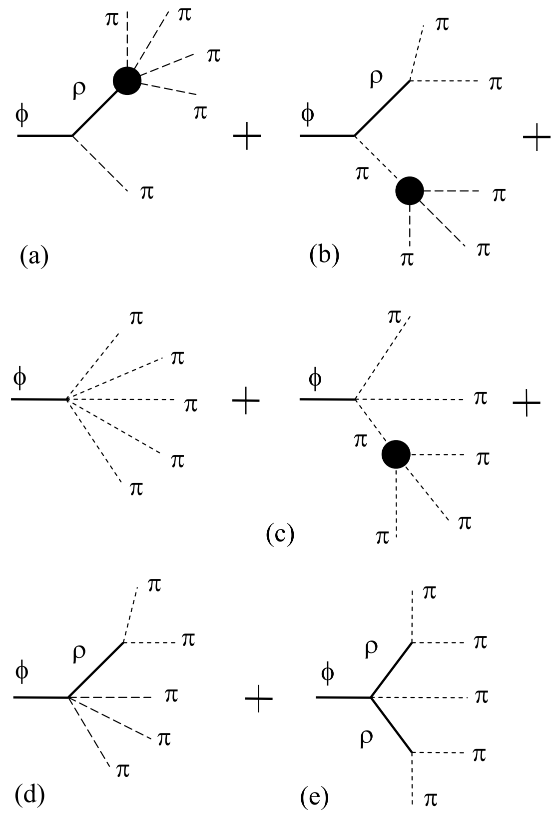

Fig.1. The diagrams describing the amplitudes of the decay $\phi(1020) \to 3\pi$. The total set of diagrams for each isotopic final state $2\pi^+2\pi^-\pi^0$ and $\pi^+\pi^-3\pi^0$ should include all possible permutations of the final pion momenta. The shaded circle in (b), (c) [(a)] refers to the $\pi \to 3\pi$ ($\rho \to 4\pi$) vertex which includes the ρ exchange. See them in Ref. [1].

There are two feasible models of the OZI-suppressed $\phi \to \rho\pi$ decay amplitude. The first one is the $\phi\omega$ mixing model [2], where the above decay proceeds due to the small admixture of nonstrange quarks in the flavor wave function of ϕ meson composed mostly of the pair of strange quarks. In the second model ϕ goes to $\rho\pi$ directly, see Ref. [3]. Irrespective of these models the effec-

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tive $\phi \to \rho\pi$ coupling $g^{\rm eff}_{\phi\rho\pi}$ denoted by the shaded circle in the diagram Fig.1a is determined by the branching ratio of the well studied process $\phi \to \rho\pi \to \pi^+\pi^-\pi^0$. Then the contribution to the decay amplitude of the dominant diagram Fig.1a for each isotopic mode, $2\pi^+2\pi^-\pi^0$ and $\pi^+\pi^-3\pi^0$, can be represented as

$$M = -\frac{g^{\rm eff}_{\phi\rho\pi} g_{\rho\pi\pi}}{f_\pi^2}\varepsilon_{\mu\nu\lambda\sigma}\sum\frac{q_\mu \epsilon_\nu q^\pi_\lambda J_\sigma(\rho\to 4\pi)}{D_\rho(q-q^\pi)}, \qquad (1)$$

where the sum is over all possible permutations of the final pion momenta, q and ϵ are the four-momentum and polarization vector of the ϕ meson, q^π is the four-momentum of the final pion in the decay $\phi \to \rho\pi$. The coupling constant $g_{\rho\pi\pi}$ is calculated from the $\rho\to\pi^+\pi^-$ decay width, $f_\pi = 92.4$ MeV is the pion decay constant. The inverse propagator of the ρ meson is

$$D_\rho(q) = m_\rho^2 - q^2 - i\frac{g^2_{\rho\pi\pi}}{6\pi\sqrt{q^2}}\left(\frac{q^2}{4} - m_\pi^2\right)^{3/2}. \qquad (2)$$

The expressions for the $\rho \to 4\pi$ decay currents $J_\sigma(\rho \to \to 4\pi)$ are given in Ref. [1]. The partial width of the decay $\phi\to 5\pi$ is

$$\Gamma_{\phi\to 5\pi}(s) = \frac{1}{2\sqrt{s}(2\pi)^{11}N_{\rm sym}}\int |M|^2 d\mathcal{D}_5, \qquad (3)$$

where s is the total energy squared in the rest frame system of the decaying ϕ meson, the Bose symmetry factor $N_{\rm sym} = 6, 4$ for the decay $\phi\to\pi^+\pi^-3\pi^0$ and $\phi\to 2\pi^+2\pi^-\pi^0$, respectively, and $d\mathcal{D}_5$ given in Ref. [4] is the differential element of the phase space volume of the five pion final state. Note that we take into account the mass difference of the charged and neutral pions both in amplitude and in the phase space volume.

The evaluation of the branching ratios with the resonant diagram Fig.1a gives $B^{\rm resonant}_{\phi\to\pi^+\pi^-3\pi^0} = 2.1\cdot 10^{-7}$ and $B^{\rm resonant}_{\phi\to 2\pi^+2\pi^-\pi^0} = 6.2\cdot 10^{-7}$, where $B_{\phi\to 5\pi} \equiv \equiv \Gamma_{\phi\to 5\pi}(m_\phi^2)/\Gamma_\phi(m_\phi^2)$. The excitation curves for the $\phi\to 5\pi$ decays in e^+e^- annihilation,

$$\sigma_{\phi\to 5\pi}(s) = = 12\pi\left(\frac{m_\phi}{\sqrt{s}}\right)^3\Gamma_{\phi\to e^+e^-}(m_\phi^2)\frac{s\Gamma_{\phi\to 5\pi}(s)}{(s-m_\phi^2)^2 + m_\phi^2\Gamma_\phi^2} \qquad (4)$$

are plotted in Fig.2. We take the fixed width approximation for the ϕ meson propagator because this meson is narrow.

Let us discuss the role of the remaining diagrams in Fig.1. To this end one should write the effective Lagrangian for the decays $\phi\to 3\pi$ and $\phi \to 5\pi$ as

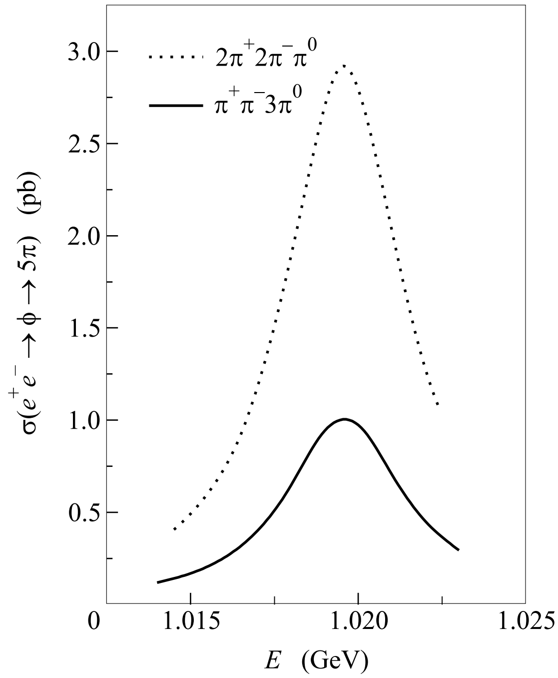

Fig.2. The excitation curve of the decays $\phi\to 5\pi$ in e^+e^- annihilation

$$\begin{aligned}\mathcal{L}^{\rm an}_{\phi,\rho,\pi} &= \frac{1}{2f_\pi^3}(\beta_1-\beta_2-\beta_3)\times\\ &\times\varepsilon_{\mu\nu\lambda\sigma}\phi_\mu\left(\partial_\nu\boldsymbol{\pi}\cdot[\partial_\lambda\boldsymbol{\pi}\times\partial_\sigma\boldsymbol{\pi}]\right) +\\ &+\frac{1}{8f_\pi^5}\left[-\beta_1+\frac{5}{3}(\beta_2+\beta_3)\right]\times\\ &\times\varepsilon_{\mu\nu\lambda\sigma}\phi_\mu\left(\partial_\nu\boldsymbol{\pi}\cdot[\partial_\lambda\boldsymbol{\pi}\times\partial_\sigma\boldsymbol{\pi}]\right)\boldsymbol{\pi}^2 -\\ &-\frac{2\beta_3 g}{f_\pi}\varepsilon_{\mu\nu\lambda\sigma}\partial_\mu\phi_\nu\Big\{\left(\boldsymbol{\rho}_\lambda\cdot\partial_\sigma\boldsymbol{\pi}\right) +\\ &+\frac{1}{6f_\pi^2}\left[\left(\boldsymbol{\rho}_\lambda\cdot\boldsymbol{\pi}\right)\left(\boldsymbol{\pi}\cdot\partial_\sigma\boldsymbol{\pi}\right) - \boldsymbol{\pi}^2\left(\boldsymbol{\rho}_\lambda\cdot\partial_\sigma\boldsymbol{\pi}\right)\right]\Big\} -\\ &-\frac{2g}{f_\pi}(\beta_1+\beta_2-\beta_3)\varepsilon_{\mu\nu\lambda\sigma}\phi_\mu\Big\{\frac{1}{4f_\pi^2}\left(\partial_\nu\boldsymbol{\pi}\cdot\boldsymbol{\rho}_\lambda\right)\times\\ &\times\left(\boldsymbol{\pi}\cdot\partial_\sigma\boldsymbol{\pi}\right) - \frac{g}{4}\left([\boldsymbol{\rho}_\nu\times\boldsymbol{\rho}_\lambda]\cdot\partial_\sigma\boldsymbol{\pi}\right)\Big\}, \qquad (5)\end{aligned}$$

where $\beta_{1,2,3}$ are arbitrary parameters responsible for the violation of the OZI rule in the $\phi\to 5\pi$ decays, and $g = g_{\rho\pi\pi}$. The expression (5) is writen guided by the form of the anomalous contributions in the hidden local symmetry approach [5, 6]. As is evident from Eq. (5), the coupling constant of direct $\phi\to\rho\pi$ transition is

$$g_{\phi\rho\pi} = -\frac{2\beta_3 g}{f_\pi} = 0.8\ {\rm GeV}^{-1}. \qquad (6)$$

Since there is no sizeable point like $\phi\to\pi^+\pi^-\pi^0$ contribution, see Refs. [7, 8], one can set

$$\beta_1 - \beta_2 - \beta_3 = 0. \qquad (7)$$

The results of relaxing this conditions are discussed at the end of the paper. After all, the ratio β_1/β_3 remains arbitrary. We set

$$\beta_1 + \beta_2 - \beta_3 = 0, \quad (8)$$

hence $\beta_1 = \beta_3$, $\beta_2 = 0$, so that the $\phi \to 5\pi$ decay amplitudes are determined by the only parameter β_3. The amplitudes in this case are very lengthy and given elsewhere [9]. The results of the evaluation are:

$$B_{\phi\to\pi^+\pi^-3\pi^0}(m_\phi^2) = 2.4 \cdot 10^{-7},$$
$$B_{\phi\to 2\pi^+2\pi^-\pi^0}(m_\phi^2) = 6.9 \cdot 10^{-7}. \quad (9)$$

The evaluation of the non-resonant contribution in Fig.1b–e gives $B^{\rm non-resonant}_{\phi\to\pi^+\pi^-3\pi^0} = 0.34 \cdot 10^{-7}$, which constitutes 16% of the resonant contribution, and $B^{\rm non-resonant}_{\phi\to 2\pi^+2\pi^-\pi^0} = 0.70 \cdot 10^{-7}$ which constitutes about 11% of the resonant one. The above estimates illustrate clearly the dominance of the diagrams with the resonant ρ meson in the intermediate state in the decay $\phi \to 5\pi$, because the resonant and the smaller non-resonant contributions add incoherently in the case of the $\phi \to 5\pi$ decay. Indeed, the phase space averaged relative phase between the resonant and non-resonant contributions calculated with the help of given branching ratios is about $\delta = 91^\circ$ in the decay $\phi \to \pi^+\pi^-3\pi^0$ and $\delta = 89^\circ$ in the decay $\phi \to 2\pi^+2\pi^-\pi^0$. For comparison, opposite situation takes place in the case of the $\omega \to 5\pi$ decay amplitudes where the smaller non-resonant contribution to the decay amplitude adds almost in phase with the resonant one and by this reason is essential [9].

The relaxing the constraint Eq. (8) to $-1 \le (\beta_1 + \beta_2 - \beta_3)/4\beta_3 \le 1$ implies the deviations of $B_{\phi\to 5\pi}$ by less than 1%. Relaxing the constraint of the absence of the point like $\phi \to \pi^+\pi^-\pi^0$ amplitude, Eq. (7), gives the following. Using the KLOE data Ref. [8], one can estimate the combination characterizing the point-like $\phi \to \pi^+\pi^-\pi^0$ vertex as $|3(\beta_1 - \beta_2 - \beta_3)/2\beta_3 m_\rho^2| \simeq 1$. Then the evaluation of $B_{\phi\to 5\pi}$ gives the results deviating by $\pm 8\%$ (depending on the sign of the above combination) from those obtained under the constraint Eq. (7). All the above discussion shows that the branching ratios of the decays $\phi \to \pi^+\pi^-3\pi^0$ and $\phi \to 2\pi^+2\pi^-\pi^0$ are determined within the conservatively estimated accuracy 20% by the well studied OZI rule violating transition of ϕ meson to the $\rho\pi$ state followed by the transition $\rho \to 4\pi$ in the model independent way.

The excitation curves of the decay $\phi \to 5\pi$ in e^+e^- annihilation can be used to evaluate the expected number of these decays at the ϕ peak. With the luminosity $L = 10^{32}$ cm$^{-2}\cdot$s^{-1} the observation of, respectively, 750 (250) $\phi \to 2\pi^+2\pi^-\pi^0$ ($\phi \to \pi^+\pi^-3\pi^0$) decays per month is feasible. Note that the existing upper limit is $B_{\phi\to 2\pi^+2\pi^-\pi^0} < 4.6 \cdot 10^{-6}$ (90% CL) [10]. With the luminosity $L = 500$ pb^{-1} already attained at ϕ factory DAΦNE [11], one could gain about 1685 events of the decay $\phi \to 5\pi$ proceeding via chiral mechanisms considered in the present paper. The possible non-chiral-model background from the dominant decay $\phi \to K_L K_S$, $K_L \to 3\pi$, $K_S \to 2\pi$ is well cut from the considered chiral mechanism by macroscopic distances kaons fly away. Rare decay $\phi \to \eta\pi^+\pi^-$ whose branching ratio was estimated [12, 13] at the level $B_{\phi\to\eta\pi^+\pi^-} \sim 3 \cdot 10^{-7}$, is cut by removing events in the vicinity of the η peak in the three pion distribution observed in the five pion events [10].

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