Effect of third dimension on the properties of phases with gauge flux (anyons)

I. E. Dzyaloshinskiĭ
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

(Submitted 9 November 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **50**, No. 11, 486–487 (10 December 1989)

Some simple symmetry-based considerations indicate that the third dimension would disrupt a macroscopic gauge flux. The gauge field always has an "antiferromagnetic" structure.

Phases with a gauge flux (anyons) have recently become one of the most popular models of a paramagnetic quantum liquid.[1-3] The two-dimensional nature of the phenomenon is important here, remaining a good approximation for the copper oxide layers in the high-T_c superconductors. In the CP^1 model (Ref. 2, for example), the

 © 1990 American Institute of Physics

behavior of the spins of the CU^{2+} copper ions in the xy plane and in the time τ ($abc = xy\tau$) is thus described by the complex boson field z_α ($\alpha = \pm$) and by the real gauge potential A_a with the action

$$\frac{1}{2g} |(\partial_a - A_a)z|^2 + \kappa \frac{1}{4\pi^2} i\epsilon_{abc} A_a \partial_b A_c. \qquad (1)$$

In the molecular field approximation, it is simple to verify that the second term in (1)–the Chern-Simon term–converts the z bosons into anyons with a gauge flux (with a magnetic field H_g) per particle which is proportional to $1/\kappa$.

The three-dimensional nature of a sample disrupts this simple picture. When the arrangement of spins is generally quite different from a layered arrangement, it can be shown that the only possible anyon is (as one would expect) a fermion.[4] In the quasi-two-dimensional situation, the action given by (1) remains a good approximation. All that is necessary is to add to it the energy associated with the hopping of z particles from one layer to another. The simplest term of this type is a Josephson tunneling through layers which is quadratic in the derivative $\partial / \partial z$, across the layers.

There is, however, an expression which is linear in the transverse derivatives. Structurally, this is again a Chern-Simon term, but in this case it is made up not of xy and τ but of xy and z:

$$\kappa' i\epsilon_{jkl} A_j \partial_k A_l; \quad jkl = xyz. \qquad (2)$$

The new Chern-Simon term given by (2) does not introduce an additional symmetry breaking: PT invariance is in this case broken by the Chern-Simon term in (1).

On the other hand, (2) does lead to a new phenomenon: a current of z particles in the direction transverse with respect to the layers and proportional to the gauge field (flux) H_g:

$$J_z \sim \kappa' H_g. \qquad (3)$$

Since there is no current J_z at equilibrium, relation (3) forbids a macroscopic (in the three-dimensional sense) gauge magnetization H_g. The fluxes and fields in neighboring layers should be oriented in an antiferromagnetic fashion. In particular, one would hardly expect a rotation of the polarization plane.[3]

[1] V. Kalmeyer and R. B. Laughlin, Phys. Rev. Lett. **59**, 2095 (1987); R. B. Laughlin, Science **242**, 525 (1988).
[2] I. Dzyaloshinskii *et al.*, Phys. Lett. A **127**, 112 (1988).
[3] Yi-Hong Chen *et al.*, "*On anyon superconductivity*," Preprint IASSNS-HEP-89/27, 1989.
[4] I. Dzyaloshinskii, Mod. Phys. Lett. B **3**, 1067 (1989).

Translated by Dave Parsons