Possible existence of nonsingular-vortex in UPt_3

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A lattice of nonsingular two-quantum vortices can exist in superconducting compounds of the UPt_3 type with a nontrivial pairing. An explanation is offered for the magnetic transition in UPt_3: It is interpreted as a transition from a nonsingular lattice near H_{c2} to a lattice of singular single-quantum vortices in weak fields.

Possible theoretical explanations of the phase transition in magnetic fields $H \sim 0.6H_{c2}$ in superconducting UPt_3 have been discussed in the literature recently.[1-7] In the present letter we show that this transition may be a transition from a nonsingular vortex lattice to a singular vortex lattice. The question of nonsingular vortices has been examined in detail previously for superfluid ^3He (Ref. 8) and also for triplet superconductors with weak spin-orbit coupling.[9] Nonsingular vortices in supercon-

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ductors with an s pairing and with a d-type order parameter which arises in vortex cores have also been studied.[6]

There are pieces of experimental evidence which indicate that superconducting classes arise from the two-dimensional representation E_1 of the D_6 group for UPt_3 (Ref. 10). In this case the order parameter would be a two-component vector $\vec{\eta} = (\eta_x, \eta_y)$. Let us write the corresponding Ginzburg-Landau functional for the case in which a magnetic field is directed along the sixfold symmetry axis, z:

$$F = \int(-a\eta_i\eta_i^* + \beta_1(\eta_i^*\eta_i)^2 + \beta_2|\eta_i\eta_i|^2 + K_1 p_i^*\eta_j^* p_i\eta_j$$

$$+ K_2 p_i^*\eta_i^* p_j\eta_j + K_3 p_i^*\eta_j p_j\eta_i)dV, \tag{1}$$

where $\mathbf{p} = i\hbar\nabla - (2e/c)\mathbf{A}$, $a = \alpha(T_c - T)$,

$$\beta_1 > 0; \quad \beta_2 > -\beta_1; \quad K_1 + K_2 + K_3 > |K_2|; \quad K_1 > |K_3|. \tag{2}$$

In the absence of a magnetic field, a phase of hexagonal symmetry apparently exists in UPt_3, according to Ref. 10. We would then have $\beta_2 > 0$ and $\vec{\eta} \sim (1, \pm i)$. We have switched to the functions

$$\Psi_1 = \sqrt{\frac{\beta_1}{2a}}(\eta_x - i\eta_y); \qquad \Psi_2 = \sqrt{\frac{\beta_1}{2a}}(\eta_x + i\eta_y) \tag{3}$$

in (1).

The solution of the Ginzburg-Landau equations for a single Abrikosov vortex is

$$\Psi_1 = R_{1m}(\rho)e^{im\theta}; \qquad \Psi_2 = R_{2n}(\rho)e^{in\theta}, \tag{4}$$

where θ is the polar angle in the (xy) plane, and ρ is the distance to the center of the vortex. At large values of ρ, one of the functions R_{1m}, R_{2n} is 1, while the other falls off to zero over distances on the order of $\xi = \hbar\sqrt{(2K_1 + K_2 + K_3)/a}$. If, for example, the phase Ψ_1 is nonzero at large distances ρ, the vortex will contain m magnetic flux quanta. In this case the numbers m and n are related by $m + 2 = n$. This relationship in fact follows from symmetry considerations analogous to those used in Ref. 6. Significantly, one of the phases does not vanish at the vortex axis in the special cases $m = -2$, $n = 0$ and $m = 0$, $n = 2$, so nonsingular two-quantum vortices are possible. As $\rho \to \infty$, there can be only nonsingular vortices with a magnetic induction $\mathbf{B}$ directed either along or opposite the z axis, depending on which of the phases (Ψ_1 or Ψ_2) is nonzero. For a superconductor with a Ginzburg-Landau constant $\kappa \gg 1$, such as UPt_3, two-quantum vortices near H_{c1} are unfavorable from the energy standpoint. In fields close to H_{c2}, however, we can show that there is a wide region of parameter values in which a lattice of nonsingular two-quantum vortices is more favorable than a lattice of singular single-quantum vortices. It is possible that a transition between these two types of lattices was observed in the experiments of Refs. 1–3.

As was shown in Ref. 5, in the case $D < C^2/(1 + C)$ [where $D = (K_3 - K_2)/2K_1$, $C = (K_2 + K_3)/2K_1$)] the quantities Ψ_1 and Ψ_2 are simultaneously nonzero below H_{c2}, and they have the form of the wave functions of the zeroth and second Landau

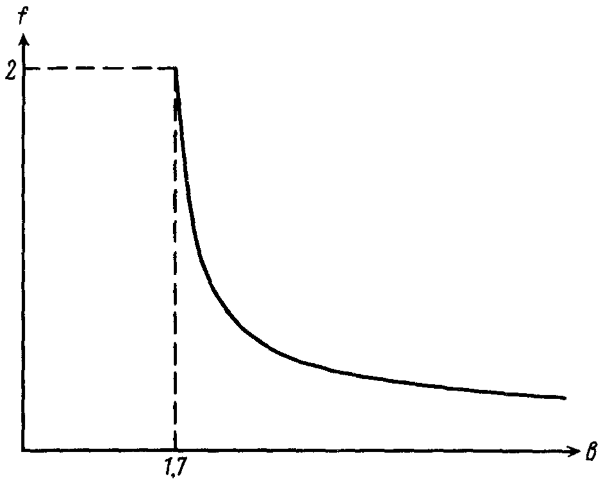

FIG. 1.

levels, respectively. For $\kappa \gg 1$ the energy of the lattice is $\sim -(H-H_{c2})^2/\beta$, where

$$\beta = \frac{\langle |\Psi_1|^4 \rangle + \langle |\Psi_2|^4 \rangle + 2(1+2b)\langle |\Psi_1|^2 |\Psi_2|^2 \rangle}{\langle |\Psi_1|^2 + |\Psi_2|^2 \rangle^2} . \tag{5}$$

The angle brackets here mean an average over the volume; Ψ_1 and Ψ_2 in (5) are the solutions of the linearized Ginzburg-Landau equations at $H = H_{c2}$. The problem of finding β for the particular structure of the vortex lattice with nonvanishing Ψ_1 and Ψ_2 is an exceedingly involved problem, and it will be analyzed separately. In the present

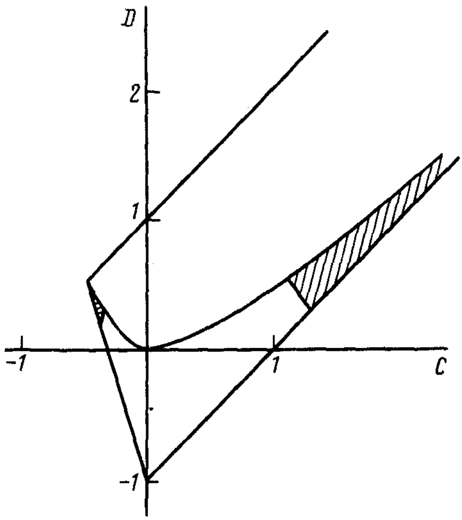

FIG. 2.

letter, we will content ourselves with a calculation of β by an approximate method similar to the Wigner-Seitz method, which yields results in a simple analytic form. For superconductors with a single-component order parameter, this method yields very good results.[11] One can hope that again in our case this method will lead to reasonable estimates for a regular triangular lattice. On the other hand, this method of course ignores possible distortions of the lattice structure. As the unit cell we adopt a lattice of circles through which the flux is equal to the quantum ϕ_0 for a single-quantum lattice and equal to $2\phi_0$ for a two-quantum lattice. We assume that the center of the cell coincides with a site of the vortex lattice at which the order parameter Ψ_1 vanishes. For an approximate description of the functions Ψ_1 and Ψ_2 within the cell, we use the solutions of the linearized Ginzburg-Landau equations for H_{c2} with a definite angular momentum. With the help of those solutions, we can calculate β for the single-quantum and two-quantum cases. We find that a two-quantum lattice is favored from the energy standpoint under the following condition:

$$\left|\frac{C}{1+C-D/2}\right| > f(b); \qquad D < \frac{C^2}{1+C}, \tag{6}$$

where $f(b)$ is shown in Fig. 1, and $b=\beta_2/\beta_1$. The hatched region in the phase diagram in Fig. 2 (cf. Ref. 5) shows the region of values of the parameters C and D for $b=2.5$. At $b<1.7$, the single-quantum lattice is preferred from the energy standpoint for all values of C and D permitted by conditions (2).

[1]V. Muller *et al.*, Phys. Rev. Lett. **58**, 1224 (1987).
[2]Y. J. Qian *et al.*, Solid State Commun. **63**, 599 (1987).
[3]A. Schenstrom *et al.*, Phys. Rev. Lett. **62**, 332 (1989).
[4]G. E. Volovik, J. Phys. C **21**, L221 (1988).
[5]M. E. Zhitomirskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 333 (1989) [JETP Lett. **49**, 379 (1989)].
[6]G. E. Volovik, J. Phys. C **21**, L215 (1988).
[7]S. Sundaram and R. Joynt, Phys. Rev. B **40**, 8780 (1989).
[8]M. M. Salomaa and G. E. Volovik, Rev. Mod. Phys. **59**, 533 (1987).
[9]L. I. Burlachkov and N. B. Kopnin, Zh. Eksp. Teor. Fiz. **92**, 1110 (1987) [Sov. Phys. JETP **65**, 630 (1987)].
[10]Z. Fisk *et al.*, Science **239**, 33 (1988).
[11]D. Ihle, Phys. Status Solidi B **47**, 429 (1971).

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