Role of confinement in spontaneous breaking of chiral invariance

Yu. A. Simonov
Institute of Theoretical and Experimental Physics, 117259, Moscow

(Submitted 10 December 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 1, 10–12 (10 January 1991)

In a QCD vacuum consisting of statistically independent configurations with $Q = 1$, the addition of configurations with $Q = 0$ spreads out the density of zero modes and causes the quark condensate to vanish. However, configurations which lead to confinement restore a nonzero quark condensate.

1. The spontaneous breaking of chiral symmetry in a QCD vacuum is related in an obvious way to the presence of quark zero modes, since the chiral condensate $\langle \bar{q}q \rangle$ is, as is well known, proportional to the density of zero modes, $\nu(0)$: $\langle \bar{q}q \rangle = -(\pi/V_4)\nu(0)$. Here V_4 is the 4D volume. It was shown in Ref. 1 that these "global" zero modes may arise from a collectivization of "elementary" zero modes of instantons and anti-instantons in the model of an instanton vacuum. This mechanism was recently generalized[2] to the case of an ensemble of arbitrary field configurations containing elementary regions with quasizero quark modes. A necessary condition for the operation of this mechanism and for the occurrence of a spontaneous breaking of chiral symmetry is a statistical independence of the elementary fields, i.e., an independence of their spatial positions and color orientations.

In the present letter we show that in the absence of confinement quasizero modes overlap to such an extent that the global spectrum is spread out, and $\nu(0)$ (and thus the spontaneous breaking of chiral symmetry) disappears. When confinement is taken into account, i.e., when there are corresponding configurations which lead to confinement in the vacuum,[3] the spontaneous breaking of chiral symmetry is restored.

Following the method of Ref. 2, we write an expression for the density of global zero modes, $\nu(\Lambda)$, for the case in which there is only a single quasizero mode in each elementary spectrum. A "Wigner semicircle"[2,4] arises for statistically independent elementary regions:

$$\nu(\Lambda) = \frac{1}{\pi V^2}(2NV^2 - \Lambda^2)^{1/2}\theta(2NV^2 - \Lambda^2), \tag{1}$$

where N is the number of elementary regions in the volume V_4, and V^2 is the mean square matrix element of the overlap of the elementary zero modes $u_i(x)$ and $u_k(x)$ from neighboring regions. This mean square value is given by

$$V^2 \equiv <|V_{ik}|^2>, \quad V_{ik} = \int d^4x u_i^+(x)(-i\hat{\partial})u_k(x). \tag{2}$$

It can be seen from (1) that the spontaneous breaking of chiral symmetry, i.e., a nonzero $\langle \bar{q}q \rangle$, is related to V^2 by the relation

$$< \bar{q}q >= -\frac{1}{V_4}\left(\frac{2N}{V^2}\right)^{1/2} \tag{3}$$

and that $\langle\bar{q}q\rangle$ disappears in the limit $V^2\to\infty$. The matrix element V^2 itself is given by the following expression after a color averaging[1,2] and when the averaging over the positions of the centers R_i, R_k is taken into account:

$$V^2 = \int \frac{d^4x d^4x'}{N_c V_4^2} d^4R_i d^4R_k u^+(x-R_i)\hat{\partial}u(x-R_k)u^+(x'-R_k)\hat{\partial}u(x'-R_i). \tag{4}$$

The region of large values of x and x^1 in (4) is a dangerous one from the standpoint of divergences. One of the integrals dR_i, dR_k can be evaluated in an elementary way since the integrand is uniform with respect to shifts, and one volume, V_4, cancels out. For instantons, $u(x)$ falls off as x^{-3}, and the integral in (4) converges. In general, however, the typical behavior of any quasizero solution can be found easily from the operator $-\partial_\mu^2$ with $d=4$; it is

$$u^L(x) \sim |x|^{-L-2}, \tag{5}$$

where L is the angular momentum for $d=4$.

An instanton zero mode is proportional to the mapping $U^+(x)=(x_4-i\vec{x}\vec{\tau}/\sqrt{x^2})$ in a singular gauge and therefore corresponds to $L=1$. However, any solution with a topological charge of zero allows $L=0$ and therefore has the behavior $u^{(0)}(x)\sim|x|^{-2}$. This is the behavior, for example, of quasizero modes in a color-correlated instanton-(anti-instanton) molecule or of the solutions in a truncated self-dual field which were discussed in Ref. 2.

The substitution of $u^{(0)}(x)$ into (4) leads to a divergence of the factor (V_4V^2) as $(V_4)^{1/2}$, while if we take $u^{(0)}$ as u_i and if we take the instanton zero mode $u^{(1)}$ as u_k we obtain the logarithmic divergence $(V_4V^2)\sim\ln V_4$ in (4).

Consequently, the appearance of even a vanishingly low concentration of zero modes $u^{(0)}(x)$ leads to divergent mean values for (V_4V^2) in the limit $V_4\to\infty$ and thus a zero limit for the condensate $\langle\bar{q}q\rangle$. One might say that the zero modes $u^{(0)}$ are analogs of superconducting current carriers, which cause the resistivity to vanish; the analog or the resistivity would be the chiral condensate $\langle\bar{q}q\rangle$.

We now consider confinement configurations, i.e., vacuum, fields B_μ with a zero or noninteger topological charge, which contribute nonzero values of the Kronecker part of the correlation functions[3] $\langle\langle F(x_i)...F(x_n)\rangle\rangle$. It is convenient to single out the B_μ dependence explicitly in (4) with the help of the gauge-covariant factors Φ:

$$u_i(x) = \Phi(x,R_i)s(x,R_i), \tag{6}$$

where s is locally gauge-invariant, and Φ is given by

$$\Phi(x,y) = P\exp(ig\int_y^x B_\mu(z)dz_\mu). \tag{7}$$

Substituting (6) into (4), and incorporating the additional averaging over the fields B_μ in the expression for the matrix element V^2, we obtain an additional Wilson-loop factor in (4):

$$W_{\mu\nu} = \mathrm{tr} < \Phi(R_i, x) D_\mu \Phi(x, R_k) \Phi(R_k, x') D_\nu \Phi(x', R_i) > . \tag{8}$$

The corresponding contour goes through the x, R_i, R_k, x'. According to a cluster expansion[3] and Monte Carlo calculations, W falls off in the case of large areas: $W \sim \exp(-\sigma S_{\min})$, where σ is the string tension, and $S_{\min}$ is the minimum area within the loop. Using the factor $W_{\mu\nu}$, we find that $V_4 V^2$ converges, that we have $V_4 V^2 \sim 1/\sigma$ and that, by virtue of (3),

$$< \bar{q}q > \sim -\mathrm{const}\sqrt{\frac{2NN_c\sigma}{V_4}}, \quad N \sim V_4. \tag{9}$$

The spontaneous breaking of chiral symmetry and confinement are thus related phenomena: When confinement disappears, the chiral condensate also disappears. This effect agrees with lattice calculations.[5]

In addition, it is clear from the discussion above that a spontaneous breaking of chiral symmetry is unstable in an instanton vacuum and is disrupted by entities with $Q=0$.

I wish to thank V. I. Zakharov, A. B. Kaĭdalov, and the participants of the theoretical seminar of the Institute of Theoretical and Experimental Physics for discussions.

[1] D. I. Duakonov and V. Yu. Petros, Nucl. Phys. B **272**, 457 (1986).
[2] Yu. A. Simonov, Preprint TPI-MINN-90/10-T (to be published in Phys. Rev. D).
[3] H. G. Dosh and Yu. A. Simonov, Phys. Lett. B **205**, 339 (1988); Yu. A. Simonov, Yad. Fiz. **48**, 1381 (1988) [Sov. J. Nucl. Phys. **48**, 878 (1988)]; **50**, 213 (1989) [**50**, 134 (1989)]; **50**, 500 (1989) [**50**, 310 (1989)].
[4] E. P. Wigner, Ann. Math. **62**, 548 (1958); F. J. Dyson, J. Math. Phys. **3**, 1199 (1962).
[5] J. Cleymans, R. V. Gravai, and E. Suhonen, Phys. Rep. **230**, 217 (1986).

Translated by D. Parsons