[4]O. V. Tarasov, A. A. Vladimirov, and A. Yu. Zharkov, Phys. Lett. B **93**, 429 (1980).
[5]K. G. Chetyrkin, A. L. Kataev, and F. V. Tkachov, Phys. Lett. B **85**, 277 (1979); Nucl. Phys. B **174**, 345 (1980); Preprint INR P-0170, 1980.
[6]F. V. Tkachov, Phys. Lett. B **100**, 65 (1981); K. G. Chetyrkin and F. V. Tkachov, Nucl. Phys. B **192**, 159 (1981).
[7]S. G. Gorishny, A. L. Kataev, S. A. Larin, and L. R. Surguladze, Preprint Ecole Polytechnique A021.1190, 1990; Phys. Lett. B (submitted); in *Proceedings of the International Seminar Quarks-90*, May 15–19, Telavi, World Scientific, to be published.
[8]M. Veltman, "SCHOONSCHIP, A CDC 6600 Program for Symbolic Evaluation of Algebraic Expressions," CERN Report, 1967.
[9]S. G. Gorishny, S. A. Larin, L. R. Surguladze, and F. V. Tkachov, Comput. Phys. Commun. **55**, 381 (1989).
[10]G. D'Agostini, W. De Boer, and G. Grindhammer, Phys. Lett. B **229**, 160 (1989).
[11]R. Marshall, Z. Phys. C **43**, 595 (1989).
[12]K. G. Chetyrkin, N. V. Krasnikov, and A. N. Tavkhelidze, Phys. Lett. B **76**, 83 (1978).
[13]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979).
[14]S. I. Eidelman, L. M. Kurdadze, and A. I. Vainshtein, Phys. Lett. B **82**, 278 (1979).
[15]A. G. Grozin and Yu. F. Pinelis, Mod. Phys. Lett. A **3**, 725 (1988).
[16]E. Braaten, Phys. Rev. Lett. **60**, 1606 (1988); Phys. Rev. D **39**, 1458 (1989).
[17]S. Narison and A. Pich, Phys. Lett. B **211**, 183 (1988).
[18]W. Marciano and A. Sirlin, Phys. Rev. Lett. **61**, 1815 (1988).

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Condensation of gauge bosons in the high-temperature phase of non-Abelian models of a unified interaction with a finite fermion external-charge density

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The curve of $\rho_c(T)$, which is the boundary of the condensate of gauge bosons in the phase of a symmetric Higgs vacuum, is found at $T \gtrsim T_c$ and also far from T_c for non-Abelian unified interaction models with a finite external fermion charge density ρ. At $\rho < \rho_c$, the appearance of a condensate is ruled out, and the shape of the phase diagram depends strongly on the parameter $k = M_H/M_W$.

The phase transitions associated with W condensation in unified interaction models constitute an important physical problem, which has some interesting implications for both a grand unified theory (GUT) and cosmology. A condensation of charged gauge bosons arises for non-Abelian fields with a finite fermion charge density if it is simultaneously required that the complete electrical neutrality of the overall system is conserved. This phenomenon was originally predicted in Refs. 1 and 2 for quantum-mechanical models. It was subsequently generalized to the case of finite temperatures by several other authors.[3,4] However, in Refs. 3 and 4 and all subsequent papers (e.g., Refs. 5 and 6) a study was made of the properties of the W condensate for GUT models in the phase of an asymmetric Higgs vacuum, and only qualitative estimates were derived for a phase with a restored symmetry. Calculations on the high-temperature phase, with a symmetric Higgs vacuum, are also worthwhile, since they determine the right-hand boundary of the region of the W condensate and thus generally make it possible to draw a closed picture of this phenomenon over the entire range of temperatures and external charge densities.

In the present letter we examine a simplified model of the electroweak interaction (the Weinberg-Salam model), using the same multiplet set of fields as in Refs. 5 and 6:

$$S = -\frac{1}{4}(G^a_{\mu\nu})^2 - \frac{1}{4}F^2_{\mu\nu} - |(\partial_\mu - ig\frac{\tau^a}{2}W^a_\mu - i\frac{\tilde{g}}{2}B_\mu)\phi|^2 - \bar{\psi}_L\gamma_\mu(\partial_\mu$$
$$-ig\frac{\tau^a}{2}W^a_\mu + i\frac{\tilde{g}}{2}B_\mu)\psi_L - \bar{e}_R\gamma_\mu(\partial_\mu + i\tilde{g}B_\mu)e_R - \frac{\lambda^2}{2}(\phi^+\phi - \frac{\alpha^2}{2\lambda^2})^2. \quad (1)$$

However, we immediately discard several unimportant terms in (1). All the calculations are carried out in the symmetric phase without a Higgs condensate. We make use of several self-consistent expressions which determine the spectrum of gauge bosons[7] and of scalar fields[8] near the critical point for Higgs condensation, T_c, and also far from this point at $T \gg T_c$. The quantum-mechanical action of model (1) is constructed in the standard way,[9] by supplementing expression (1) with terms which fix the gauge and with terms which introduce fictitious (ghost) fields and their interaction with material fields in the theory. For definiteness, we follow Ref. 6. In particular, we fix the R_ξ gauge, and (a more important point) we introduce all the necessary chemical potentials in model (1) in the standard way (Refs. 4 and 5, for example) and then construct a gauge-invariant version of this model, making use of the additional terms found in Ref. 6.

Three chemical potentials have been introduced in model (1) by the method of Refs. 5 and 6. That method is simpler to implement but completely equivalent to the usual scheme for canonical quantization in statistical physics. Two chemical potentials (here, μ_1 and μ_3) are required for conserving electric charge and the weak neutral charge. The third, μ_2, is introduced directly in (1) as a dual potential of the lepton-density operator,

$$N_L = \bar{e}\gamma_4 e + \frac{1}{2}\bar{\nu}\gamma_4(1+\gamma_5)\nu\,. \quad (2)$$

This potential leads to the conservation of the quantity in (2) in accordance with the external conditions selected. The simplest way to introduce the chemical potentials μ_1 and μ_3 in this model is to assume that there exist nonvanishing vacuum values for several gauge fields,

$$\begin{aligned} < W^3_\mu > &= \frac{i}{g}\left[\mu_1 - \mu_3\frac{2\cos^2\theta}{\cos 2\theta}\right]u_\mu, \\ < B_\mu > &= \frac{i}{\tilde{g}}\left[\mu_1 + \mu_3\frac{2\sin^2\theta}{\cos 2\theta}\right]u_\mu, \end{aligned} \quad (3)$$

and to introduce these quantities in (1) by shifting the corresponding fields in accordance with (3).

We have studied the spectra of all elementary excitations of model (1) at $T \gg T_c$, primarily the spectra of charged W-gauge fields, which may condense in a certain temperature interval under the condition $\mu_i \neq 0$. The equation for W condensation takes its standard form,[6]

$$M_W = \pm \mu_W , \tag{4}$$

and should be solved along with the equations for an extremum of the thermodynamic potential. The latter equations determine the behavior of μ_W as a function of the temperature and the external density (in this case, the lepton charge density ρ). Solving this problem is an exceedingly laborious part of the larger problem in which we are interested here, even though we know at the outset that we have

$$\mu_W = \rho\eta^2/T^2 \tag{5}$$

at $T \gtrsim T_c$. A self-consistent calculation of the coefficient η will be reported later; at this point we assume that at $T \gtrsim T_c$ the coefficient η is a weak function of the temperature, and we assume that we can estimate it on the basis of Ref. 6, which gives $1/\eta^2 = 73/36$. In other words, we are hoping that the function $\mu(\rho)$ does not undergo any substantial change at the boundary between phases and is instead more likely to be a continuous and smooth function of ρ over all temperature intervals.

We use an expression from Ref. 7 for the effective mass of the W boson at $T \gtrsim T_c$:

$$M_W(T) = \frac{T}{T_c} M(T_c) - \frac{g^2}{8\pi} m(T) T_c / M(T_c). \tag{6}$$

Here $m(T)$ is the effective mass of the scalar particles, taken from Ref. 8. The use of this expression for M_W is a key part of this paper; it is only through the use of this expression that it becomes possible to draw a physically plausible picture of the phenomenon of interest here. At $T \gg T_c$ the expression for M_W simplifies (as was shown in Ref. 10), and for model (1) it becomes

$$M_W(T) = \sqrt{5/18}\, gT, \tag{7}$$

where it is assumed that $g^2 \lesssim 1$. In particular, it is useful in the calculations to replace g^2 in (7) by $g^2_{\text{eff}}(T)$, constructed by the standard recipes (Ref. 11, for example).

To determine the right-hand boundary of the region of W condensation [the $\rho_c(T)$ curve], we need to solve Eq. (4), using expression (5) and also (6) or (7). At $T \gg T_c$ [where (7) is used] the asymptotic curve of $\rho_c(T)$ is the cubic parabola

$$\eta^2 \rho_c(T) = \sqrt{5/18}\, gT^3. \tag{8}$$

In contrast with previous estimates (e.g., those in Refs. 4 and 6), however, the $\rho_c(T)$ curve does not reach zero at $T = T_c$. According to (6) [in which we have $m(T) = 2\pi/3\ (T - T_c)$ near T_c], as in Ref. 8, we find

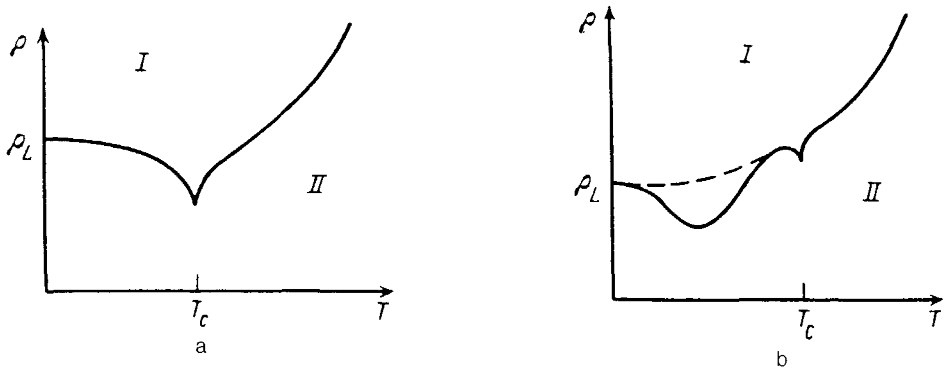

FIG. 1. Phase diagram of W condensation (the lower part of the curve is taken from Ref. 6). The W condensate exists in region I, and the normal phase in region II. The value $\rho_L = (a/k)^3/6\pi^2$ is taken from Ref. 1. a—Case of a light Higgs ($k < 1/\pi$); b—the limit $k^2 \gg 1$. In each case the critical density ρ_c [the minimum on the $\rho_c(T)$ curve] is nonzero. Here $k = 2\lambda/g$.

$$\eta^2 \rho_c(T) = \tilde{M}(T_c)T^3 \left(1 - \frac{g^2}{12} \frac{|T - T_c|}{T\, \tilde{M}^2(T_C)}\right), \tag{9}$$

where $\tilde{M} = M/T$ is the reduced mass (a weak function of the temperature). An important point is that expression (9) is (we believe) valid in a certain temperature interval near T_c (both above and below this point), and the singularity at T_c signals a phase transition in the Higgs sector of the theory. The curve of $\rho_c(T)$ is continuous everywhere; in particular, its behavior near $T = 0$ (which is not being considered in the present paper) can be found from Refs. 5 and 6 and also Ref. 12.

On the whole, the behavior of $\rho_c(T)$ is extremely sensitive to the value of the parameter k^2 [here $k = M_H/M_W$, and the estimate

$$\rho_L/\rho_c(T_c) = g^2(1 + k^2/3)^{3/2}/\pi^5 k^3 \tag{10}$$

is valid], but at any value of k^2 the appearance of a condensate at densities ρ below a certain critical $\rho_c \neq 0$ is ruled out. At $k \lesssim 1/\pi^2$ the critical density ρ_c is definitely lower than ρ_L (Fig. 1a), but in the case of heavy Higgs bosons (with $k^2 \gg 1$) an alternative version in principle is possible (see the dashed line in Fig. 1b). This possibility was pointed out in Refs. 5 and 12, although we assume that the other situation (as shown by the solid line in Fig. 1b) is most likely even in the case $k^2 \gg 1$. We also hope that the results derived here at a qualitative level are model-independent, valid for any non-Abelian GUT. The shape of the $\rho_c(T)$ curve is determined entirely by the temperature dependence of the mass of the vector fields, $M(T)$, which is universal.[7,13]

[1] A. D. Linde, Phys. Lett. B **86**, 39 (1979).
[2] I. V. Krive, Yad. Fiz. **31**, 1259 (1980) [Sov. J. Nucl. Phys. **31**, 650 (1980)].
[3] O. K. Kalashnikov and H. Perez Rojas, Kratkie Soobshcheniya po Fizike, No. 2, 23 (1986).
[4] H. Peres Rojas and O. K. Kalashnikov, Nucl. Phys. B **293**, 241 (1987).
[5] E. Ferrer, V. de la Incera, and A. I. Shabad, Nucl. Phys. B **309**, 120 (1988).

[6] O. K. Kalashnikov, L. V. Razumov, and H. Perez Rojas, Preprint TPI-MINN-89/39-T; Phys. Rev., 1990.
[7] O. K. Kalashnikov and L. V. Razumov, Yad. Fiz. (4), (in press) (1991) [Sov. J. Nucl. Phys., to be published (1991)].
[8] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974).
[9] E. S. Fradkin and I. V. Tyutin Rivista, Nuovo Cimento, No. 4, 1 (1974).
[10] O. K. Kalashnikov and V. V. Klimov, Yad. Fiz. **31**, 1357 (1980) [Sov. J. Nucl. Phys. **31**, 699 (1980)].
[11] O. K. Kalashnikov, Fortschr. Phys. **32**, 525 (1984).
[12] J. I. Kapusta, Preprint TPI-MINN-90/6-T; Phys. Rev., 1990.
[13] A. V. Smilga, Phys. Lett. B **222**, 462 (1989).

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