Two-dimensional diffusion in system with shear

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(Submitted 4 July 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 4, 848–851 (25 August 1990)

The field shear is taken into account in a study of the transport of test particles as a result of fluctuations with a 2D structure in magnetic confinement systems. An analytic study is carried out on the basis of percolation theory, and numerical calculations are also carried out. The diffusion coefficient and the growth rate of the divergence of close orbits are calculated. Despite the presence of a shear potential, the percolation approach can be taken in analyzing diffusion in such systems.

Superthermal electromagnetic fluctuations are generally believed responsible for the anomalously fast energy loss in magnetic plasma confinements systems.[1] Because of the strong longitudinal magnetic field, most of the oscillations which develop in such systems are stretched out along the magnetic field and have a 2D structure ($k_{\parallel} \ll k_{\perp}$). Considerable progress has recently been achieved toward an understanding of the transport mechanism in fields with a 2D turbulence structure thanks to an understanding of the role played by long orbits and a determination of quantitative characteristics of their distribution.[2-4] However, most of the studies in this area, including all on the case of high fluctuation amplitudes, $\widetilde{V} > \Delta\omega/k_{\perp}$ ($\widetilde{V}$ is the particle oscillation velocity, $\Delta\omega$ is the width of the oscillation spectrum, and k^{-1} is a transverse scale dimension of the variation), which is the case of primary interest for practical applications, have ignored the effects of shear. This is true despite the fact that a shear, i.e., a variation in the direction of the main magnetic field with the radius, alters the elongation of the 2D structures, thereby causing a spatial localization of fluctuations and a limitation on their amplitude.

 0021-3640/90/160217-05$01.00

Our purpose in the present study was to learn more about the transport of test particles in a system with shear and fluctuations with a 2D structure. Our analysis is carried out for the particular case of magnetic field diffusion, although the Hamiltonian approach used below makes it a simple matter to examine other classes of systems of this sort, through a simple change in notation.

For the model of a periodic cylinder with a radial coordinate x, an angular coordinate y, and a coordinate z running along the cylinder, with toroidal effects ignored, the evolution of the field lines is described by Hamilton's equations:

$$\frac{d\mathbf{r}_\perp}{dz} = \frac{1}{B_0}\mathbf{e}_z \nabla \Psi , \tag{1}$$

where B_0 is the main magnetic field, and Ψ is the longitudinal component of the vector potential, which is the sum of resonant harmonics and a parabolic shear well, given by

$$\Psi = \Sigma \; \Psi_n + B_0(x - x_0)^2/L_s; \tag{2}$$

The potentials Ψ_n describe perturbations which are stretched out along the magnetic field on resonant surfaces $q(x_n) = m/n$:

$$\Psi_n = \Psi_0(x - x_n)\cos\left(m\frac{y}{x_n} + n\frac{z}{R} + \phi_n\right)$$

$$\approx \Psi_0(x - x_n)\cos(k_y(y + (x_n - x_0)z/L_s) + \phi_n), \tag{3}$$

where x_0 is the radius at which the unperturbed magnetic field is directed along the z axis, x_n is the radius of the resonant surface with toroidal and poloidal wave numbers n and m, $L_s = Rq/s$ is the shear length, $s = d\ln(q)/d\ln(x)$,q^{-1} is the magnitude of the rotational transform on a given surface, and $2\pi R$ is the period of the cylindrical geometry. The radial localization of these oscillations, Δx, is assumed to be greater than the distance between harmonics, $dx = x_{n+1} - x_n$ (Fig. 1), so several oscillations, with a longitudinal correlation length

$$L_\parallel = L_s/(k_\perp \Delta x) \tag{4}$$

are occurring simultaneously at each spatial point. The result is a diffusion of the field lines.

Before we analyze the motion in system of equations (1)–(3), we would like to point out that the percolation approach developed in Refs. 3 and 4 is not, strictly speaking, applicable to this problem. The reason is the parabolic shear increment in the potential, which prevents the formation of orbits which are large in the x direction. If, however, a particle is displaced from its initial position x_0 to some point x_1, it becomes possible to transform to a coordinate system which is rotating at the velocity of the unperturbed magnetic field on the x_1 surface. As a result, the Hamiltonian formally assumes the same form as in (2) and (3), but with x_0 replaced by x_1. The shear parabola can thus be eliminated at any point by transforming to another coordinate system, although it cannot be eliminated over the entire interval. For this reason, despite the circumstance that percolation distributions are not applicable to Eqs. (1)–

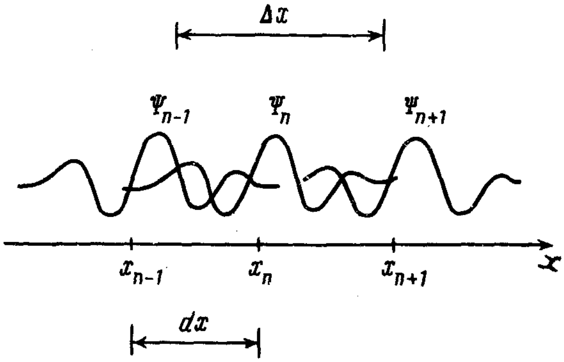

FIG. 1. The potential profile in the system with shear.

(3), this approach can be used as a hypothesis, to be tested subsequently by numerical calculations.

Let us estimate the transport coefficients under the assumption that the length distributions of the orbits found by percolation theory are valid. We assume that $\xi = \Psi/\langle\Psi\rangle$ describes the relative height of the level, where $\langle\Psi\rangle$ is the mean square perturbation amplitude. The length of the projection of the orbit in the final plane is then $l_\perp \sim \xi^{-(\nu+1)}/k_\perp$, where $\nu = 4/3$ (Ref. 2). The width of the sheaf of orbits of level ξ and below is thus $\Delta \sim \xi/k_\perp$, and the maximum transverse dimension of such orbits is $\tilde{x} \sim \xi^{-\nu}/k_\perp$. These estimates are standard estimates for the percolation approach[4] and at this point do not reflect the shear in any way. The shear effects the correlation length of the noise which produces the orbit of level ξ. If the size of this orbit, $\tilde{x}$, exceeds the size of the region of radial localization of the oscillations, Δx, then the shear causes the orbit to experience a noise with an effective correlation length shorter than that in (4). Taking this circumstance into account, we can estimate the longitudinal correlation length of the noise for the orbit of level ξ from

$$l_c \sim \xi L_s/((\Delta x + \tilde{x})k_y), \tag{5}$$

where, in contrast with Ref. 4, the circumstance that the larger of the dimensions $\Delta x, \tilde{x}$ is the decorrelating factor has been taken into account. Taking account of the fraction of the surface covered by the orbits of level ξ, i.e., $f \sim \Delta l_\perp/\tilde{x}^2$, we can estimate the diffusion coefficient from

$$D_\xi \propto f\frac{\tilde{x}^2}{l_c} \propto \frac{\Delta l_\perp}{l_c} \propto \frac{l_\perp}{L_s}(\Delta x + \tilde{x}) \propto \frac{1}{L_s k_y^2}\xi^{-(\nu+1)}(\Delta x + \tilde{x}). \tag{6}$$

The total diffusion coefficient is the integral of D_ξ over all levels ξ. It can be estimated easily by taking account of the maximum contribution from the orbits for which the particle manages to traverse the orbit over the orbit decorrelation time $l_\perp \approx \langle\Psi\rangle k_y l_c/B_0$:

$$\xi_m = \begin{cases} R_\Psi^{\frac{1}{\nu+2}} & \text{for } 1 < R_\Psi < (k_y \Delta x)^{1+2/\nu} \\ (R_\Psi k_y \Delta x)^{\frac{1}{2\nu+2}} & \text{for } (k_y \Delta x)^{1+2/\nu} < R_\Psi , \end{cases} \tag{7}$$

where the parameter $R_\Psi = \langle \Psi \rangle k_y L_s / (B_0 \Delta x)$ characterizes the noise amplitude. From these estimates we find the following values for the diffusion coefficient:

$$D_m k_y^2 L_s = \begin{cases} R_\Psi^{7/10} k_y \Delta x & \text{for } 1 < R_\Psi < (k_y \Delta x)^{5/2} \\ R_\Psi^{11/14} & \text{for } (k_y \Delta x)^{5/2} < R_\Psi , \end{cases} \tag{8}$$

where the first case corresponds to the estimate of D_m without allowance for shear effects,[4] and the second incorporates the shear. Under actual conditions, the parameter $k_y \Delta x$ is close to 1. The shear limit, $D_m \propto R_\Psi^{11/14}/(k_y^2 L_s)$, then begins to operate immediately after the quasilinear limit, $D_m \propto k_y \Delta x/(k_y^2 L_s)$, which is valid for $R_\Psi < 1$ (Ref. 5).

To test the percolation hypotheses embodied in estimates (7) and (8), we have carried out a numerical study of Eqs. (1)–(3) by a method similar to one which has been used in a calculation on diffusion in two waves with different phase velocities.[6] Figure 2 shows the diffusion coefficient D_m and the Kolmogorov entropy

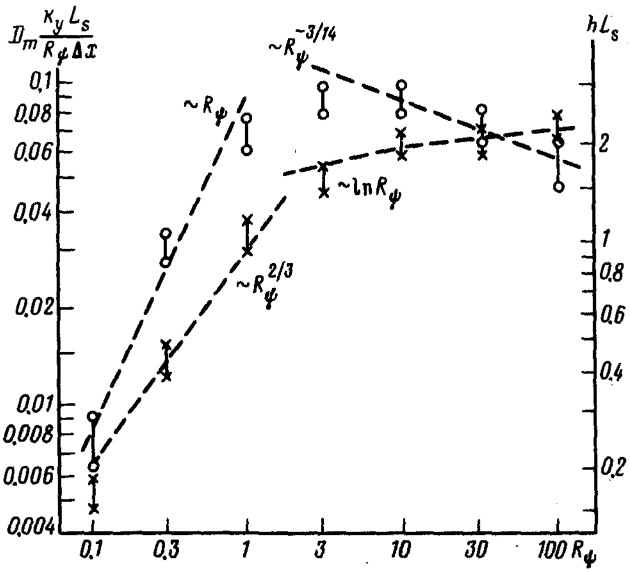

FIG. 2. Comparison of the diffusion coefficient D_m (O) and the Kolmogorov entropy h (×) with theoretical estimates.

$$h = \lim_{t \to \infty} \lim_{d \to 0} \frac{1}{t} \ln \frac{d(t)}{d(0)}$$

in comparison with the theoretical estimates. The entropy h, however, differs in functional dependence from the estimate $h \propto L_s^{-1} R_\psi^{-1/2} \ln(R_\psi)$, found in Ref. 4. It agrees better with the estimate $h \propto L_s^{-1} \ln(R_\psi)$, found from the results of a numerical calculation on diffusion in two waves.[6]

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Translated by D. Parsons