Effect of resonant tunneling on the current-voltage characteristics of SIN tunnel junctions

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(Submitted 8 August 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 5, 929–933 (10 September 1990)

Noninteracting localized centers in the insulating layer of an SIN junction cause a current deficiency on the current-voltage characteristic of the junction at high voltages. This factor explains the current-voltage characteristics observed experimentally for structures with a high-T_c superconducting electrode and makes it possible to determine the modulus of the order parameter of this electrode.

It has been established experimentally (Refs. 1–3, for example) that a current deficiency occurs on the current-voltage characteristics of SIN tunnel junctions with high-T_c electrodes at voltages $V > \Delta/e$, where Δ is the modulus of the order parameter of the S electrode. A "current deficiency" here means that the difference $I - V/R_N$, where I is the current through the junction, and R_N is the normal resistance of the junction, assumes a constant negative value. This current deficiency is predicted by the simple tunneling model of junctions based on the BCS theory. This fact was attributed by van Bentrun *et al.* to charge effects which arise, for example, when there are insulated regions with a metallic conductivity in the insulating layer. The mechanism for the suppression of the current by charge effects does not depend on whether the electrodes of the tunnel junction are in the normal or superconducting state. If one of the electrodes of the structure is a superconductor, however, then (as we will show below) a consideration of the resonant tunneling of electrons through noninteracting localized centers in the insulating interlayer leads to a similar current-voltage characteristic in the absence of charge effects.

In proving this assertion, we assume that the density of the localized centers in the insulating layer of the SIN tunnel junction is small enough that the interaction of the electrons localized in this layer is unimportant, and the Hamiltonian of the system can be written in the form[4,5]

$$H = H_l + H_r + H_t + H_e, \tag{1}$$

where $H_{l,r}$ are the Hamiltonians of the left and right electrodes, which are given by

$$H_r = \sum_{p,\sigma} \left[\epsilon_{p,\sigma} b^+_{p,\sigma} b_{p,\sigma} + \Delta b^+_{p,\sigma} b^+_{p,\sigma} + \text{ H.a. } \right],$$

$$H_l = \sum_{n,\sigma} \epsilon_{n,\sigma} a^+_{n,\sigma} a_{n,\sigma} .$$

The term H_t describes the tunneling of electrons through the localized centers and is given by

$$H_t = \sum_{n,\sigma} T_n a^+_{n,\sigma} d_\sigma + \sum_{p,\sigma} T_p b^+_{p,\sigma} d_\sigma + \text{H.a.} \ .$$

The quantity H_e is the Hamiltonian of a localized center when the Coulomb repulsion of electrons is taken into account (U is the Coulomb repulsion energy):

$$H_e = \sum_\sigma \left[\epsilon_\sigma d^+_\sigma d_\sigma + U d^+_\sigma d_\sigma d^+_{-\sigma} d_{-\sigma}\right].$$

Here the operators d^+_σ, $b^+_{p,\sigma}$, and $a^+_{n,\sigma}$ create electrons at an impurity and at the right and left electrodes, respectively; ϵ_σ, $\epsilon_{p,\sigma}$, and $\epsilon_{n,\sigma}$ are the energies of the electrons in these states; and T_n and T_p are the constants of the hybridization of localized centers with the electrodes. In the calculation of the current flowing through the junction, we restrict the discussion to the case (of practical importance) in which the impurity level has a small width Γ:

$$\Gamma = \Gamma_l + \Gamma_r \ll T, \ \ \Gamma_{l,r} = \pi N_{l,r} < T^2_{n,p} >, \ \ < T^2_{n,p} >= T^2_0 \exp\{\pm 2\kappa z\}. \qquad (2)$$

Here T is the temperature; T^2_0 is the square of the hybridization matrix element, averaged over the Fermi surface, for the case of a symmetric impurity distribution; κ is the reciprocal radius of the localized center; z is the coordinate reckoned from the midpoint of the insulating layer in the direction perpendicular to the plane of the junction; and $N_l = N_l(0)$ is the density of states of the left (N) electrode. The density of states of the right electrode (S) is nonzero at $|\epsilon| \geqslant \Delta$, having the value $N_r = N_r(0)\epsilon/(\epsilon^2 - \Delta^2)^{1/2}$.

We also assume that an electron at a localized center is in an S state, with an energy level ϵ_σ near the Fermi energy of the electrons, E_F. We assume that the energy splitting of the levels of the localized center satisfies $U \gg \{T, eV\}$. Going through calculations similar to those in Ref. 6, we find the following expression for the tunneling current through the localized center:

$$I = 4e\frac{\Gamma_l\Gamma_r}{\Gamma}\left[f(\epsilon + eV) - f(\epsilon)\right](1 - < n_\sigma >); \ \ < n_\sigma >= \nu/(1+\nu),$$
$$\nu = \frac{\Gamma_l}{\Gamma} f(\epsilon + eV) + \frac{\Gamma_r}{\Gamma} f(\epsilon), \ \ \epsilon = E_F - \epsilon_\sigma. \qquad (3)$$

Here $f(\epsilon)$ is the Fermi distribution of the electrons in the electrodes, and $\langle n_\sigma \rangle = \langle d^+_\sigma d_\sigma \rangle$ is the expectation value of the number of electrons at a localized center. Under the condition $\epsilon_\sigma + U \simeq E_F$, the current through the junction is the same as that given by the expression derived in Ref. 5 for NIN structures, but with a value of Γ_r which reflects the superconducting properties of the right electrode, from (2).

It is natural to find that the current depends on the average number of electrons at a localized center in (3). When the strong Coulomb repulsion of electrons at a localized center is taken into account, we find that at any given instant the resonant-tunneling channel can be open for only one of the electrons with different spin directions in the state with a given energy ϵ.

To find the current-voltage characteristic, we need to average (3) over the positions and energies of the impurities.[6] Assuming that the localized centers are distributed uniformly in energy and uniformly over the insulating interlayer, with a density $dn/dE = g$, and integrating (3) over the spatial coordinates, we find the following expression for the current-voltage characteristic of the SIN junction:

$$< I >= R_N^{-1} \int\limits_{-\infty}^{\infty} [f(\epsilon - eV) - f(\epsilon)]\, N^*(\epsilon)\ d\epsilon; \qquad (4a)$$

$$R_N^{-1} = \frac{\pi e^2 g S}{\kappa} < T_0^2 > [0,5 N_l(0) N_r(0)]^{1/2} \arctan\left\{\frac{\sqrt{2a}\ \sinh\ (\kappa d)}{1 + 0,\quad 5a}\right\}; \quad (4b)$$

$$N^*(\epsilon) = \frac{(\phi(\epsilon)a^*)^{1/2}}{1 + f(eV + \epsilon)} \arctan\left\{\frac{2\sinh\ (\kappa d)(\phi(\epsilon)a^*)^{1/2}}{a^*\phi(\epsilon) + 1}\right\};\ a = \frac{N_l(0)}{N_r(0)}; \quad (4c)$$

$$a^* = a\frac{1 + f(eV + \epsilon)}{1 + f(\epsilon)};\ \phi(\epsilon) = \frac{|\epsilon|}{(\epsilon^2 - \Delta^2)^{1/2}}\{\theta(\epsilon - \Delta) + \theta(-\epsilon - \Delta)\}. \quad (4d)$$

Here S is the area of the junction, d is the thickness of the insulating layer, and $\theta(x)$ is the step function.

In the temperature region of interest here, $T < T_c$, the function $f(\epsilon)$ can be assumed to be 1 for $\epsilon \leqslant 0$ and 0 for $\epsilon > 0$ in (4c) and (4d). In the energy interval $\Delta \leqslant |\epsilon| \leqslant eV$, which is dominant in (4a), the quantity $N^*(\epsilon)$ is actually independent of V, and an integration in (4a) at $T < T_c$ leads to expression (4b) for R_N. This expression is greater by a factor of $\sqrt{2}$ than the resistance of the junction in the absence of a Coulomb repulsion of electrons at the localized centers.

Note also that the function $N^*(\epsilon)$ does not have a divergence at $\epsilon = \Delta$. For high-resistance junctions ($\kappa L \gg 1$; a case of practical importance), the sharp increase in this function, $N^*(\Delta) \simeq \exp\{\kappa L\}$, is localized in a narrow energy interval $|\epsilon - \Delta| = \delta \simeq a^2 \Delta \exp\{-4\kappa L\}$. An integration over this interval yields an exponentially small contribution in the calculation of the current-voltage characteristic. In the region $|\epsilon| > \delta$, expression (4c) simplifies substantially, and the function $N^*(\epsilon)$ turns out to be proportional to the square root of the density of states of the superconducting electrode found from the BCS theory.

The current-voltage characteristic calculated for the SIN junction from (4a) (Fig. 1) differs from the corresponding characteristics calculated for junctions, ignoring resonant tunneling (Fig. 2). The difference consists of the appearance of a current deficiency at high voltages. In particular, at $T \ll T_c$ we find the following expression for the current-voltage characteristic from (4):

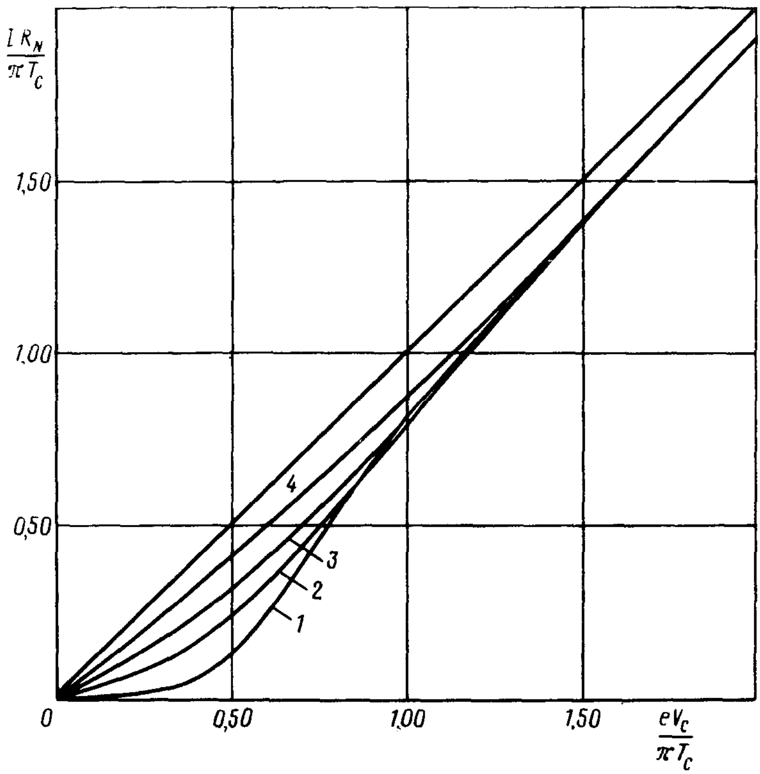

FIG. 1. Current-voltage characteristics of SIN tunnel junctions as calculated from the BCS theory[10] for various temperatures T/T_c: 1–4—0.1, 0.2, 0.3, and 0.5, respectively.

$$I = \frac{\Delta}{eR_N}\left\{\frac{\left[v\sqrt{v^2-1}\right]^{1/2}(2v^2-1)}{1+2v\sqrt{v^2-1}} + \frac{1}{2\sqrt{2}}\left[F(\phi,1/\sqrt{2}) - 2E(\phi,1/\sqrt{2})\right]\right\},$$

$$\phi = \arccos\left\{\frac{1-2v\sqrt{v^2-1}}{1+2v\sqrt{v^2-1}}\right\}, \quad v = eV/\Delta, \qquad (5)$$

where $F(\phi,k)$ and $E(\phi,k)$ are the elliptic integrals of the first and second kinds. At voltages $v \gg 1$, expression (5) takes the simple form

$$IR_N = V - \frac{\Delta}{e\sqrt{2}}\left[2E(1/\sqrt{2}) - K(1/\sqrt{2})\right] \simeq V - 0,60(\Delta/e), \qquad (6)$$

where $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds. This is the type of current-voltage characteristic (Fig. 1) which has been observed experimentally[1–3] at temperatures $T \ll T_c$. The values of Δ which we found with the

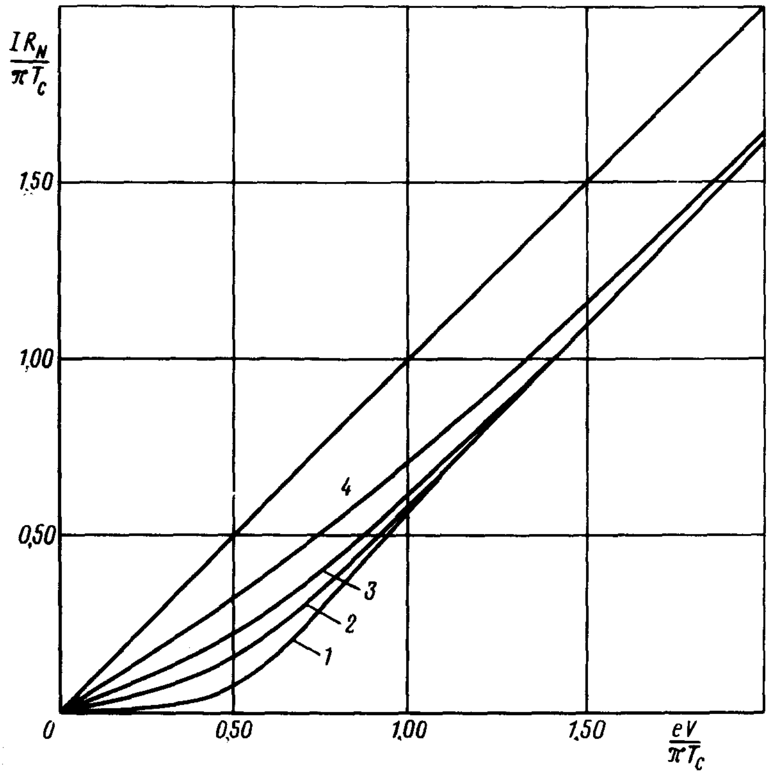

FIG. 2. Current-voltage characteristics of SIN tunnel junctions with localized states in the insulating layer as calculated from expressions (4) for various temperatures T/T_c: 1–4—0.1, 0.2, 0.3, and 0.5, respectively.

help of (6) are essentially the same as the data found by other methods in Refs. 1–3.

It is important to note that Ohm's law follows from expressions (4) in the case $\Delta = 0$, i.e., in NIN junctions. Consequently, the mechanism which has been proposed to explain the shape of the current-voltage characteristic at $T \ll T_c$ can easily be distinguished experimentally from the mechanism involving the existence of charge effects—from the nature of the change in the characteristic with increasing temperature or increasing external magnetic field. This circumstance might provide useful information about the structure of the insulating interlayers in tunnel junctions. In other words, one might be able to determine whether independent localized centers or macroscopically large coupled systems of these centers (clusters or droplets) exist in these layers. Such information is of fundamental importance for the development of a technology of Josephson tunnel structures, since the Coulomb repulsion electrons at localized centers has essentially no effect on R_N, but it leads (as was shown in Ref. 8) to a suppression of the resonant-tunneling channel for superconducting electrons. In other words, it leads to a substantial suppression (in comparison with the result predicted by the Ambegoakar–Baratoff theory[9]) of the characteristic junction voltage $V_c = I_c R_N$.

We wish to thank L. V. Averin and K. K. Likharev for a discussion of these results.

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Translated by D. Parsons