Anomalies in the quantum excitation spectrum of a magnetic material with a strong quasiparticle interaction

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It is shown in a nonlinear theory that the quantum spectrum of excitations becomes analogous to the spectrum of excitations of a quantum Bose liquid in the case of a highly anisotropic ferromagnet, with a large number of interacting quasiparticles.

The nature of the ground state and the spectrum of excitations of the magnetic subsystem of high-T_c superconductors are presently being studied actively. The idea of a possible realization of a spin-liquid state has been brought up in some of the corresponding papers.[1-3] Important roles are played here by the two-dimensional nature of the system, the antiferromagnetic nature of the interaction, the presence of frustrated bonds, and the low spin $S = 1/2$. Acting at the same time, these factors give rise to highly developed quantum zero-point vibrations, so it is no longer sufficient to work in the harmonic approximation. Since the role played by quantum zero-point vibrations is of fundamental importance in the nonlinear theory, it is natural to take up this problem first in the case of a model amenable to calculations whose accuracy can be monitored.

In the present letter we use the particular case of the nonlinear theory of an easy-plane ferromagnetic with $S = 1/2$ (this is a nontrivial example of a system with quantum zero-point vibrations) to demonstrate that anomalies arise in the excitation spectrum as the intensity of the zero-point vibrations increases. An important role is played here by the incorporation of not only the dynamic interaction of Bose quasiparticles but also the kinematic interaction which stems from the finite number of physical states. For this purpose, we use the formalism of an indefinite metric, and we propose a form for the metric operator which is convenient in practice. We find that the presence of quantum zero-point vibrations in the system leads to a contribution from nonphysical states, which is small in a power-law fashion $\propto (\xi/I)^2$ (where ξ is the correlation length, and I is the exchange integral), rather than in an exponential fashion $\propto \exp(-T_c/T)$, as has been assumed previously.

We write the Hamiltonian of the easy-plane ferromagnetic in the following form:

$$\mathcal{H} = -\frac{1}{2}\sum_{fg}\{I^{\parallel}_{fg}S^z_f S^z_g + I^{\perp}_{fg}(S^x_f S^x_g + S^y_f S^y_g)\} - H\sum_f S^x_f. \tag{1}$$

To go over to a Bose description, we use the Dyson–Maleev formalism, supplemented by the procedure of Ref. 4 for introducing an indefinite metric. It can be shown that the exact Bose analog of Hamiltonian (1) is

$$\mathscr{H}_{\mathrm{B}} = F^{\otimes}\mathcal{H}_{D-M}, \tag{2}$$

where $\mathscr{H}_{\text{D-M}}$ is Hamiltonian (1) in the Dyson–Maleev representation, and $F^{\otimes}$ is the metric operator. In the $S = 1/2$ cases, this operator is conveniently sought in the form

$$F^{\otimes} = \Pi_f^{\otimes} F_f, \quad F_f = 1 + \sum_{n=2}^{\infty} A_n (a_f^+)^n a_f^n. \tag{3}$$

The coefficients A_n satisfy the system of equations

$$1 + \sum_{m=2}^{n} [n(n-1)(n-2)...(n-m+1)] A_m = 0; \quad n = 2, \ 3, ..., \tag{4}$$

which can be solved easily: $A_2 = -1/2$, $A_3 = 1/3$, $A_4 = 1/8$, $A_5 = 1/30, \ldots$. We wish to stress that incorporating the metric operator in (2) restores the Hermitian nature of the Bose analog of the Hamiltonian:

$$\mathscr{H}_{\text{B}} = E_0 + \mathcal{H}_{(2)} + \mathcal{H}_{(4)} + \mathcal{H}_{(6)} + ..., \quad (\mathcal{H}_{(n)})^+ = \mathcal{H}_{(n)}. \tag{5}$$

On the other hand, $\mathscr{H}_{\text{D-M}}$ is not Hermitian. Note also that the operator $F^{\otimes}$ in $\mathscr{H}_{(4)}$ and (especially) in $\mathscr{H}_{(6)}$ gives rise to a large number of additional terms, which influence the properties of this magnetic material. Going over to momentum space, using a u–v transformation $a_{\vec{p}} = u_{\vec{p}} \alpha_{\vec{p}} + v_{\vec{p}} \alpha^+_{-\vec{p}}$, and then putting the operator expressions in (5) in a form with the normal structure in terms of $\alpha^+_{\vec{p}}$ and $\alpha_{\vec{p}}$ (Ref. 5), we find a system of integral equations for $u_{\vec{p}}$, $v_{\vec{p}}$:

$$A_{\vec{p}} \sinh 2\varphi_{\vec{p}} - B_{\vec{p}} \cosh 2\varphi_{\vec{p}} = 0, \quad u_{\vec{p}} = \cosh \varphi_{\vec{p}}, \quad v_{\vec{p}} = \sinh \varphi_{\vec{p}},$$

$$A_{\vec{p}} = \epsilon_{\vec{p}} + \frac{2}{N} \sum_{\vec{q}} (\xi_{\vec{p}} + 2\xi_{\vec{q}}) u_{\vec{q}} v_{\vec{q}}$$

$$+ \frac{4}{N} \sum_{\vec{q}} \Gamma_0(\vec{q}\vec{p}; \vec{q}\vec{p}) v_{\vec{q}}^2 + \frac{2}{N} \sum_{\vec{k}\vec{q}} \{\Gamma_0(\vec{q}, -\vec{q}, \vec{p}; \vec{p}, \vec{k}, -\vec{k})\}_{\text{symm}} u_{\vec{q}} v_{\vec{q}} u_{\vec{k}} v_{\vec{k}},$$

$$B_{\vec{p}} = \xi_{\vec{p}} - \frac{2}{N} \sum_{\vec{q}} \Gamma_0(\vec{q}, -\vec{q}; \vec{p}, -\vec{p}) u_{\vec{q}} v_{\vec{q}}. \tag{6}$$

These equations are written within terms $\sim (\xi_0/I_0)^2$, where $\xi_{\vec{p}} = (I^{\parallel}_{\vec{p}} - I^{\perp}_{\vec{p}})/4$ and $I_{\vec{p}} = (I^{\parallel}_{\vec{p}} + I^{\perp}_{\vec{p}})/2$. The quantity Γ_0 represents seed scattering amplitudes for scattering involving four and six bosons. The excitation spectrum is determined by $\Omega_{\vec{p}} = \{A^2_{\vec{p}} - B^2_{\vec{p}}\}^{1/2}$. In the approximation linear in ξ, with $B_{\vec{p}} = \xi_{\vec{p}}$ and $A_{\vec{p}} = \epsilon_{\vec{p}} = H - \xi_0 + (I_0 - I_{\vec{p}})/2$, we find the standard result of the linear theory: $\omega_{\vec{p}} = [(H - \xi_0 + I_0 - I_{\vec{q}})^2 - \xi^2_{\vec{q}}]^{1/2}$. In this case, however, quantum zero-point vibrations are being ignored. Their effect will be taken into account if we solve system (6) to within terms quadratic in ξ. In the nearest-neighbor approximation we then find the following result for a simple cubic lattice:

$$\Omega_{\vec{q}} = \left\{ \Delta^2 + [H - \xi_0 + 2(w-1)\left(\frac{\xi}{I}\right)^2 I_0](I_0 - I_{\vec{q}}) \right.$$

$$\left. + c(\xi, H)(I_0 - I_{\vec{q}})^2 - \frac{w-1}{2}\left(\frac{\xi}{I}\right)^2 \frac{I_{\vec{q}}(I_0 - I_{\vec{q}})^2}{\omega_{\vec{q}}} \right\}^{1/2}, \tag{7}$$

where the gap $\Delta = \{H[H - 2\xi_0 + 4(\omega - 1)\xi_0^2/I_0]\}^{1/2}$ tends toward zero as $H \to 0$ according to the Goldstone theorem, and w is the Watson integral. The last term in the expression in the radical in (7) arises because of the incorporation of the metric operator. This result means that the presence of quantum zero-point vibrations leads to a finite contribution from nonphysical states even at $T = 0$.

Figure 1 shows the excitation spectrum of an easy-plane ferromagnet with $S = 1/2$ in a zero magnetic field when quantum zero-point vibrations are taken into account (solid lines) and also in the harmonic approximation (dashed lines) for three values of the anisotropy parameter $|\xi|/I$: (1) 0.2, (2) 0.4, (3) 0.5. The vector $\vec{q}$ is directed along the [111] axis. We see that when the quantum zero-point vibrations are taken into account, the increase in the anisotropy leads to a qualitative change in the dispersion curve (compare solid lines 1 and 3), while in the harmonic approximation there are no such changes (there is no qualitative difference between dashed lines 1

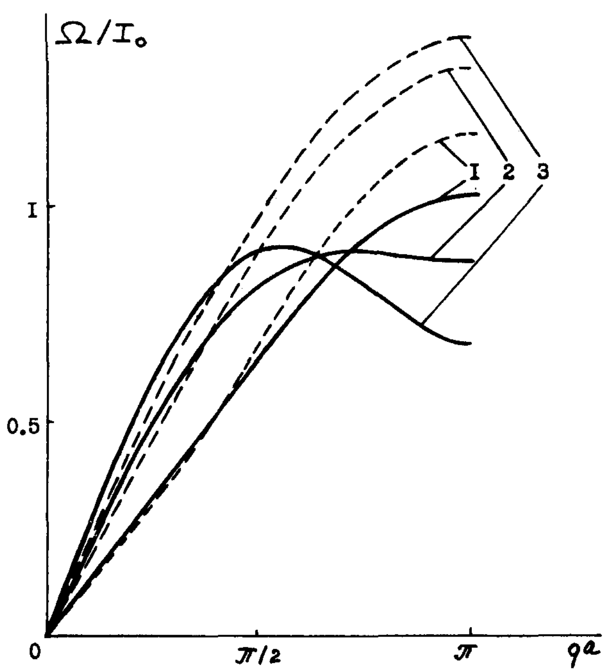

FIG. 1.

and 3). Since the interaction of the magnons is an attraction in the case $q \sim 1/a$, the incorporation of this interaction lowers the excitation energy. In the case $qa \ll 1$, on the contrary, in which the interaction is a repulsion, the zero-point vibrations renormalize the spectrum in such a way that we have[6] $\Omega_{\vec{q}} > \omega_{\vec{q}}$. At large values of $|\xi|/I$, at which the zero-point vibrations are fairly well developed, and there are a significant number of interacting seed quasiparticles, the $\Omega_{\vec{q}}$ spectrum is qualitatively the same as the excitation spectrum of a quantum Bose liquid. In particular, the form of $\Omega_{\vec{q}}$ at the boundary of the Brillouin zone (dashed line 3) suggests the appearance of new quasiparticles.

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