Critical dynamics of weakly fluctuating degrees of freedom

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A procedure is proposed for incorporating the effect of fluctuations in the order parameter on the dynamics of noncritical long-wave degrees of freedom. The dynamic characteristics of such degrees of freedom are unambiguously related to the generalized susceptibility of the system with respect to the field which is the conjugate of the square of the order parameter. Explicit formulas for making a detailed comparison of theory and experiment are proposed for a wide range of transitions.

In this letter we take a theoretical look at dynamic critical phenomena.

It has now been established that the nature of the singular behavior of *static* physical quantities near a second-order phase transition is determined primarily by whether this transition belongs to some universality class or other, the number of which is limited. It is for this reason that one can describe the static critical properties of various substances by a common theoretical model,[1,2] which explains the basic behavior of the substance near the phase transition: the scaling, the relationship among the various critical exponents, and so forth.

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With regard to critical dynamics, the situation is less satisfactory. Despite the large number of studies of this problem, there is no overall picture of dynamic critical phenomena. The primary reason for this situation is that in the dynamic case the critical effects are manifested in a far more complex way than in the static case. Moreover, substances whose static critical behavior can be described within a common universality class may differ completely in critical dynamics.

The theoretical studies of critical dynamics that have been carried out can be divided into two groups. The first is made up of semiempirical studies, in which calculations are carried out from expressions which do not have a really solid foundation. The second group is made up of studies of simple dynamic models on the basis of the conventional theory of phase transitions. The basic results which have been established in this direction are described by Haplerin and Hohenberg.[3]

This approach has led to some important results on the actual dynamics of the order parameter. The hypothesis of dynamic scaling has been verified for several specific examples by a renormalization-group procedure. The primary difficult which arises along this approach is that one cannot incorporate in the renormalization-group procedure the dynamics of degrees of freedom other than the critical one.

In the present letter we demonstrate how this difficulty can be overcome. We have managed to develop a procedure for relating the structure of the correlation functions of hydrodynamic variables, which carry comprehensive information on the long-wave dynamics of the system, with the dynamic correlation function of the order parameter. This relationship can be written out explicitly for a wide range of second-order phase transitions.

Here are the basic steps which must be taken in this procedure.

First, it is necessary to construct a complete system of nonlinear dynamic levels for the long-wave variables. This system includes both an equation for the order parameter ψ and equations for weakly fluctuating variables. Among these variables are the densities of conserved quantities (mass, energy, and momentum) and variables associated with a spontaneous symmetry breaking (the director in a nematic liquid crystal, the displacement vector in a crystal, etc.). The basic nonlinearities are then distinguished in the dynamic equations.

Second, an effective action I is constructed from the nonlinear dynamic equations. This action makes it possible to formulate a diagram technique for calculations on dynamic fluctuation effects (see, for example, Ref. 4). One can then effectively eliminate from consideration the weakly fluctuating quantities, by integrating the distribution function $\exp(iI)$ over them. Since these variables are weakly fluctuating, it is sufficient to retain in the action I terms of up to second order in the deviations of these variables from their equilibrium values. The integration of $\exp(iI)$ over weakly fluctuating variables then reduces to the replacement of I by an extremum in terms of these variables.

The action obtained as a result describes the actual dynamics of the order parameter ψ. Its structure of course depends on the nature of the transition under consideration. If this action is renormalizable, the correlation functions of the order parameter will satisfy dynamic scaling laws. Halperin and Hohenberg[3] have carried out studies of

this sort for several models.

Third, the correlation functions of the weakly fluctuating quantities must be expressed in terms of the correlation functions of the order parameter. One can do this by retaining in the action the dependence on the "currents" which are the conjugates of the weakly fluctuating variables as these variables are being eliminated and by then expanding a generating functional in terms of these currents. We wish to stress that the dynamic correlation functions of weakly fluctuating variables will no longer satisfy any simple scaling laws.

Below we write out some explicit formulas which have been derived by the procedure outlined above. We are thinking of such phase transitions as orientational phase transitions in liquid crystals, phase transitions to a ferromagnetic or superfluid state, etc. In these cases, the expansion of the energy in the order parameter ψ contains only even terms. Along with them, the expansion of the energy should contain terms of an expansion in deviations of the weakly fluctuating variables from the equilibrium values.

We denote by φ_a the deviations of these variables from their critical values. Since the variables φ_a are weakly fluctuating, it is sufficient to retain only those terms of the expansion which are linear and quadratic in φ_a. For the transitions under consideration here, they are

$$\frac{1}{2}\varphi_a\beta_{ab}\varphi_b + \frac{1}{2}\psi^2\Xi_a\varphi_a. \tag{1}$$

Here and below, a repeated index implies a summation.

For several transitions the quantities β_{ab} and Ξ_a are simply a set of constants. If the φ_a include such quantities as the displacement vector $\mathbf{u}$ of a crystal lattice, the quantities β_{ab} and Ξ_a are differential operators, since the energy of a crystal may depend only the derivatives $\nabla_i u_k$. All the formulas below hold in both cases, but in the second case β_{ab} and Ξ_a are functions of the wave vector.

The dynamic correlation functions of weakly fluctuating quantities are expressed in terms of the function $F(\omega,\mathbf{q})$, which is the generalized susceptibility of the system with respect to the field conjugate to ψ^2. The function F is analytic in the frequency in the upper half-plane and is related by the following equation to the Fourier component of the correlation function $\langle\psi^2(t,\mathbf{r})\psi^2(0,0)\rangle$:

$$<\psi^2\psi^2>_{\omega,q} = 4iT\omega^{-1}(F(\omega) - F(-\omega)). \tag{2}$$

Here T is the temperature. Relation (2) unambiguously relates the function F and the correlation function $\langle\psi^2\psi^2\rangle$. Expression (2) follows from the fluctuation dissipation theorem.

The critical behavior of the dynamic characteristics of a system can be found by replacing the matrix β_{ab} by the following expression in the linearized seed equations for the quantities φ_a:

$$\tilde{\beta}_{ab}(\omega,\mathbf{q}) = \beta_{ab} - \Xi_a^* F \Xi_b (1 + \Xi_c^* \beta_{cd}^{-1} \Xi_d F)^{-1}. \tag{3}$$

Here Ξ_a and β_{ab} are generally functions of the wave vector $\mathbf{q}$, F depends on both $\mathbf{q}$ and ω, the matrix β_{ab}^{-1} is the inverse of β_{ab}, and $\Xi_a^*(\mathbf{q}) = \Xi_a(-\mathbf{q})$. Expression (3) gives the matrix β_{ab} renormalized because of the fluctuations of the order parameter.

For example, the linear equation for the Fourier component of the velocity $\mathbf{v}$ is

$$\{\rho\omega\delta_{ik} - \omega^{-1} g_{ai}^* g_{bk} \tilde{\beta}_{ab} + i\eta_{ikmn}^0 g_m g_n\} v_k(\omega, \mathbf{q}) = iF_i(\omega, \mathbf{q}). \quad (4)$$

Here ρ is the mass density, η_{ikmn}^0 is a seed viscosity tensor, F_1 is the density of the external force applied to the system, and the quantities g_{ai} figure in the linear dissipation-free equations for the variables φ_a:

$$\omega\varphi_a = -ig_{ai}v_i.$$

The dispersion relations of the dynamic modes of the system are determined by setting the determinant of the matrix in braces (curly brackets) on the left side of (4) equal to zero.

We wish to stress that Eq. (4) holds, regardless of whether there is a mean-field regime or highly developed fluctuations. All the corresponding information is encoded in the particular form of the function $F(\omega,\mathbf{q})$, which determines the renormalized matrix $\tilde{\beta}_{ab}$ according to (3). Equation (4) describes the critical behavior of acoustic and viscous degrees of freedom over the entire region near the phase transition. It can therefore be compared in detail with experimental data. One such comparison was made in Ref. 5, for the case of a (smectic A)–(smectic C) phase transition.

Equation (4) leads to an extremely complex crossover behavior of the dynamic characteristics, as a result of both the crossover behavior of the function F and the complex F dependence of the matrix $\tilde{\beta}_{ab}$. For example, for the low-frequency viscosity coefficients, which determine the sound attenuation in the region of developed fluctuations, there is a crossover from the behavior $\propto |T - T_c|^{-z\nu-\alpha}$ to the behavior $\propto |T - T_c|^{-z\nu+\alpha}$. Here z is a critical dynamic exponent, and ν and α are the critical exponents of the correlation radius and the specific heat.

Equations (3) and (4) above solve the problem of relating the critical behavior of observable dynamic characteristics to the correlation functions of the order parameter which satisfy dynamic scaling relations. This problem was solved in Ref. 6 for the particular case (smectic A)–(smectic C) phase transition.

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