Effect of dissipation-free edge currents on the magnetoresistance of a 2D electron gas in a strong magnetic field: Upper limit on magnetoresistance

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It has been shown experimentally that the magnetoresistance $R_{xx}(\nu)$ of long channels in silicon field-effect transistors is equal to the difference between the Hall resistances at noninteger values of the filling factors (ν) of the magnetic levels: $\delta R_{xy} = h/(e^2 j) - R_{xy}(\nu)$, where the integer j corresponds to the closest, well-defined Hall plateau. An upper limit on the magnetoresistance is predicted. Its value would be equal to the difference between neighboring plateaus.

The current distribution in a sample under conditions corresponding to the quantum Hall effect[1] is a question of much interest, since there are two alternative models for the effect. One explains the quantum Hall effect in terms of trapping of electrons and the presence of percolation paths in the interior of the sample. The other attributes the current flow to edge states corresponding to classical skipping paths. One might expect that nontrivial current distributions also exist at parameter values corresponding to transitions between successive values of the quantized Hall resistance $R_{xy} = h/(e^2 i)$ (i is the number of magnetic quantization levels below the Fermi level), and these distributions would be manifested in the resistance of a sample. Evidence for this

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possibility comes from results found previously on the behavior of the average resistivity $\bar{\rho}_{xx} = R_{xx} W/L$ as a function of the distance (L) between the potential electrodes[2,3] and as a function of the width of the sample,[4,5] W.

Working from measurements of the magnetoresistance R_{xx} and the Hall resistance R_{xy} of silicon field-effect transistors, we have shown that channels which support a dissipation-free current flow exist near the edges of the sample at noninteger values of the filling factor $\nu = n_s hc/(eH)$ in the interior of the sample (n_s is the surface carrier density, and H is the magnetic field). The results can be interpreted in either of these two models of the quantum Hall effect, if they are modified appropriately. In the case of the edge states,[6,7] the modification consists of an incorporation of a backscattering of these states.[8,9] In the other approach, which we have proposed, the shunting channels are treated as percolation paths in macroscopic regions under conditions corresponding to the quantum Hall effect. This model predicts the existence of an upper limit on the magnetoresistance and also sharply nonuniform current distributions at noninterger values of ν. A study of the current distribution under these conditions should make it possible to choose between these two approaches.

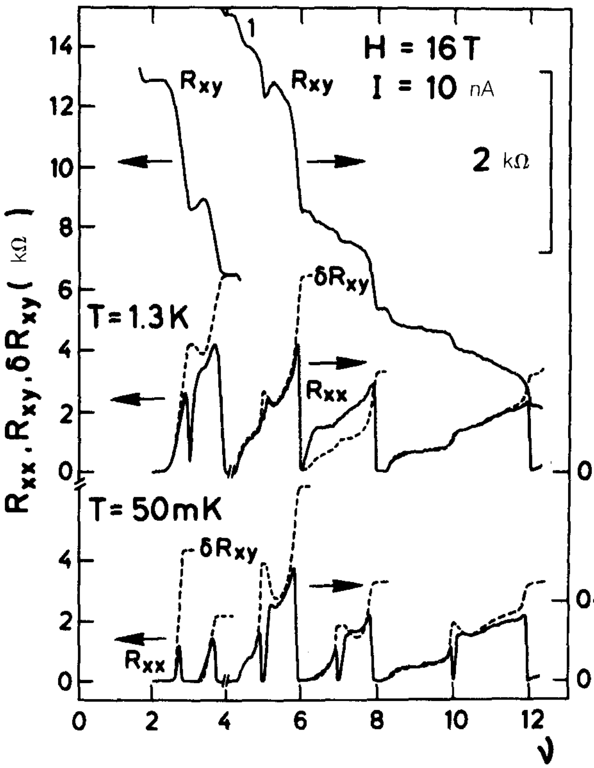

FIG. 1. Results of ac measurements of the Hall resistance R_{xy} and the magnetoresistance R_{xx} of a long sample as a function of ν at two temperatures. The origin of the scale for part (1) of the R_{xy} curve is denoted by 0_1. The magnetic field is $H = 16$ T, and the current amplitude is $I = 10$ nA. Comparison of R_{xx} with the deviation δR_{xy} (dashed line) of the Hall resistance from a quantized value, $\delta R_{xy} = h/(e^2 j) - R_{xy}$.

Measurements of R_{xy} and R_{xx} were carried out on transistors with a maximum mobility of about 30 000 $cm^2/(V\cdot s)$ ($T = 1.5$ K) over the temperature interval from 30 mK to 1.5 K in magnetic fields up to 16 T. We compared the results for samples of identical width $W = 40\,\mu m$, with different distances between the potential electrodes: $L_1 = 80\,\mu m$ and $L_2 = 2880\,\mu m$. In the linear regime (at a current $I \leqslant 10$ nA), measurements were carried out at both an alternating current ($f = 13$ Hz) and a direct current. Non-ohmic effects were studied at a direct current. Figure 1 shows some typical experimental results on a long transistor as a function of ν. (The carrier density n_s was varied smoothly by means of the gate voltage.) The experimental results demonstrate that the values of R_{xx} (ν) and $\delta R_{xy}(\nu) = R_{xy}(j) - R_{xy}(\nu) = h/e^2 j - R_{xy}(\nu)$ are approximately the same at noninteger value of ν lying in the intervals (j,k) chosen in the following way: (2,4), (4,6), (6,8), and (8,12). At $T = 50$ mK, the interval (2,4) is broken up further into subintervals: (2,3) and (3,4). Substantial differences between R_{xx} and δR_{xy} are observed near the upper boundary of these intervals and also near certain integer values of ν in these intervals, at which conditions correspond to the quantum Hall effect prevail. In the latter case, this difference disappears when the conditions for the quantum Hall effect are disrupted. (Compare the corresponding curves near $\nu = 5$ and 10 at $T = 50$ mK and $T = 1.3$ K.) The primary effect of an increase in the current is a significant increase in the magnetoresistance R_{xx} at noninteger values of ν (Fig. 2), while the changes in δR_{xy} are very slight.

Turning to a comparison of the results on the long and short samples, we first note that these results are similar to some results published previously.[2,3] While the resistances differ by a factor of 36 (i.e., are proportional to L) in a zero magnetic field, in a strong field the difference is by a factor of only 2. The Hall resistances, on the other and, agree within 5% for the two types of samples.

The difference of more than an order of magnitude in the resistivities of the long and short samples clearly indicates that the greater part of a long sample is short-circuited by a dissipation-free channel. In principle, this channel could be created by edge states which do not undergo a backscattering.[8,9] Our analysis of such a model has revealed that it is capable of explaining our results if the phenomenological parameters involved in the model are chosen appropriately. In particular, the probability for the passage of edge states is a nonmonotonic function of the filling factor ν. We will also demonstrate the possibility of another approach, which starts from an interpretation of the quantum Hall effect as a bulk effect.

Our model is based on these assumptions: (1) Following the ideas of Ref. 3, we assume that the potential varies smoothly near the edges of the sample, so there are macroscopic regions there with all integer filling factors $i < \nu$. (2) We assume that the regime of the quantum Hall effect is established in all these regions as a result of the position of the Fermi level in the gaps between magnetic levels. We characterize these regions by the resistivity tensor components ρ_{xyi} and ρ_{xxi}, where $\rho_{xyi} = h/(e^2 i)$, while ρ_{xxi} is close to zero. (3) Since one can expect an increase in the carrier density, i.e., a downward swing of the potential, near the potential electrodes, we assume that the edge regions with $i < \nu$ are ruptured near the electrodes (Fig. 3). (4) We estimate the resistance corresponding to the current flow through an edge region with a filling factor i from $R_i = \rho_{xxi} L / W_i + [\rho_{xyi} - \rho_{xy}(\nu)]$, where W_i is the effective width of the

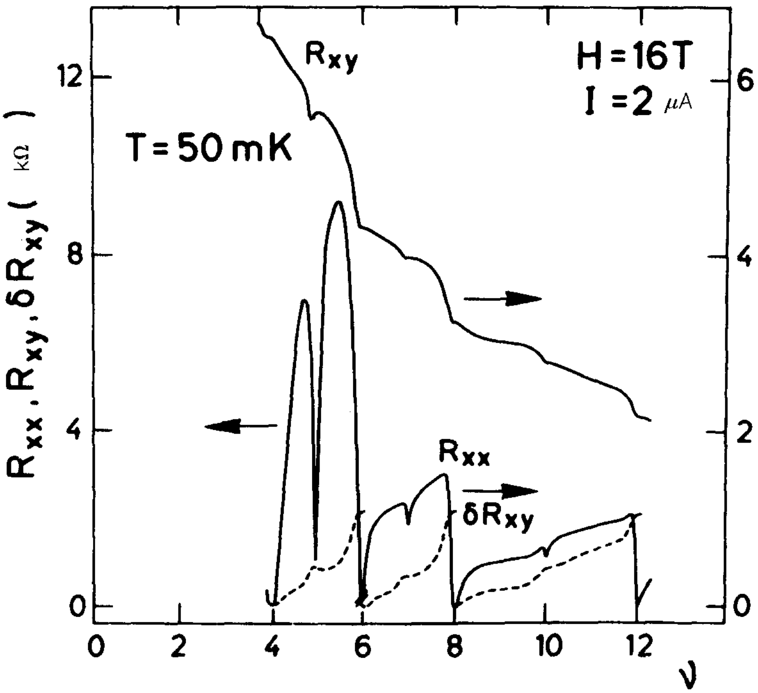

FIG. 2. Experimental results on R_{xy} and R_{xx} as a function of ν for a long sample according to dc measurements under nonlinear conditions ($I = 2$ μA). $H = 16$ T, $T = 50$ mK. Comparison of R_{xx} and δR_{xy} (dashed line).

region, and $\rho_{xy}(\nu)$ is the Hall component in the interior of the sample. The first term in this expression incorporates a possible finite resistance of the region, while the second describes the "contact" resistance corresponding to the current flow between regions with different Hall components.[10] In our case, this "contact" resistance arises from the current flow out of the interior of the sample into the edge region and back near the potential electrodes (Fig. 3). The effect of the edge regions on the resistance of the sample is substantial if $min\{R_i\} = R_j$ is lower than the bulk resistance of the sample, $R_b = \rho_{xx}(\nu)L/W$. In samples with a large ratio L/W one would expect the resistance of the sample to be determined by R_j, so we would have $R_{xx} \simeq h/(e^2 j) - \rho_{xy}(\nu)$ with $\rho_{xxj}L/W_j \simeq 0$. A comparison of the measurements of R_{xy} on long and short samples suggest that the edge regions have only a slight effect on this quantity. We can thus assume $R_{xy}(\nu) = \rho_{xy}(\nu)$ and finally find $R_{xx} \simeq \delta R_{xy}$.

If $\rho_{xxi}L/W_i$ is negligible at all i, then we have $j = \mathrm{INT}[\nu]$. We see, however, that the edge region with $i = j$ is dominant in the integer interval (j,k), which also includes other integer values of ν (e.g., $\nu = 5$, 7, and 10). Consequently, the edge regions with the corresponding filling factors are ineffective in the shunting. This circumstance may be the result of comparatively large values of the quantity $\rho_{xxi}L/W_i$ for these regions. It then seems natural to explain the extremely small values of R_{xx} at $T = 50$ mK and

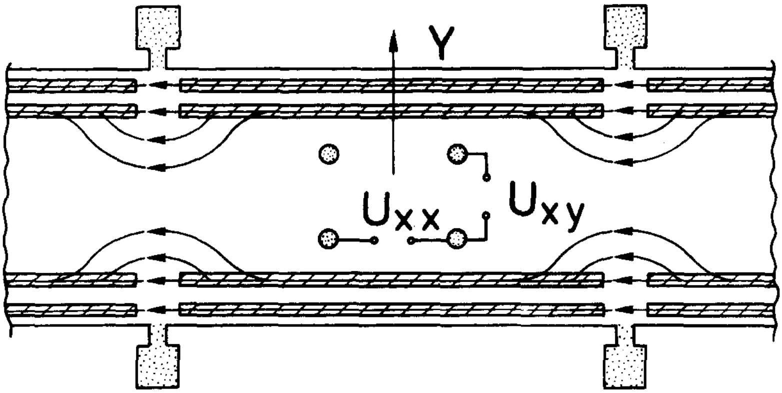

FIG. 3. Schematic diagram of the current distribution in a sample with dissipation-free edge regions (the hatched regions) and with $\nu = 2.5$ in the interior of the sample. The points show potential electrodes. The internal electrodes are discussed in the text proper.

$\nu = 5$, 7, and 10 on the basis of a current flow in the interior of the sample, so we would have $R_{xx}(\nu = i) = \rho_{xxi} L/W \simeq 0$ because of the considerably greater width, $W \gg W_i$, of the current-carrying region. The results found under nonlinear conditions also point to this explanation, since the manifestation of the basic nonlinear effects at noninteger values of ν can be attributed to a considerably higher current density in the edge regions.

On the basis of this model we can make several predictions. (1) In strong magnetic fields the magnetoresistance measured between neighboring potential electrodes will have an upper limit. This upper limit will be independent of the dimensions of the sample and will be equal to the difference between the Hall resistances on neighboring, well-defined Hall plateaus. This maximum value is reached at $L/W \gg 1$. If the sample is divided into parts by the potential electrodes, its magnetoresistance will be the sum of the resistance of these parts. (2) Under the condition $L/W \gg 1$, and at noninteger values of ν, the current far from the potential electrodes will concentrate near the edges of the sample (Fig. 3). In this region the voltages U_{xx} and U_{xy}, measured between internal electrodes (Fig. 3), will be close to zero. At integer value of ν we would expect a current flow in the interior of the sample and, correspondingly, a nonzero Hall voltage U_{xy} between the internal electrodes, as has been observed experimentally.[11,12] (3) Discontinuities in the edge regions near the potential electrodes can be eliminated by artificially reducing the carrier density there (e.g., by applying a gate voltage to heterojunctions[13]). The effect should be to erase the magnetoresistance R_{xx}. (4) The values of L/W at which the edge currents influence the magnetoresistance can be estimated from

$$L/W > \frac{2}{\pi}\frac{(N+1/2)^2 + \pi^2\nu^2/4}{j(N+1/2)} - \frac{\pi}{2}\frac{\nu}{N+1/2},$$

where N is the index of the Landau level. This relation was derived on the basis of theoretical values[14] of the conductivities σ_{xx} and σ_{xy} at half-integer values of ν. For the case (typical of silicon field-effect transistors) $N = 1$, $\nu = 4.5$, the right side of this inequality is approximately 0.8. This estimate shows that edge effects may be important even in short samples.

In summary, we have experimentally established the relationship $R_{xx} \simeq \delta R_{xy}$, and we have proposed a new model to explain it. We have made predictions regarding the magnitude of the magnetoresistance and the current distribution in a sample. Corresponding experimental studies can serve as a test of this model; a study of the current distribution would be the most informative.

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