Relaxation of nuclear spin in a 2D electron gas

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The nuclear spin relaxation rate has been calculated. The relaxation occurs as a result of the formation of spin excitons in the presence of a random impurity potential in a strong magnetic field.

The nuclear spin relaxation rate T_1^{-1} was measured in a high-grade heterostructure in a strong magnetic field,[1,2] and the nuclear orientation was achieved by a dynamic polarization in ESR. The polarization subsequently shifted the position of the ESR because of the Overhauser effect, making it possible to measure it. A Korringa relaxation was observed experimentally, i.e., the spin was found to flip in the electron gas, as was initially suggested in Ref. 3. Since the electron Zeeman energy is much higher than the nuclear energy, and since 2D electrons in a magnetic field have discrete energy levels, such a process cannot occur without allowance for the electron energy in the random impurity potential. The Coulomb interaction of electrons, which dramatically changes the effective g-factor, is also of considerable importance. These two circumstances greatly complicate the calculation of the relaxation rate compared with the case of electrons in the absence of a magnetic field. In the calculation of the depolarization rate, carried out by Berg *et al.*,[2] the one-electron density of states was assumed to be known and the interaction was taken into account by introducing the g-factor which depends on the filling of the Landau levels.

We will analyze this problem more systematically, in a slightly idealized formulation, where all the electrons completely fill the Landau spin sublevel (whose spin is opposite that of the nuclear spin). In the more general case of a fractional filling, we will restrict the analysis to the mean-field approximation and calculate the probability for the occurrence of a spin excitation in such a system, i.e., a spin exciton[4] with zero energy. The principal factor determining this process—the density of states of the spin exciton with zero energy (disregarding the nuclear magneton)—always must be taken into account in the expression for the relaxation time.

A general equation can be found for the depolarization rate by making use of the perturbation theory for contact hyperfine interaction (see, for example, Ref. 2). This equation can be written in the variables associated with the production of spin excitons in the form

$$T_1^{-1} = A^2 \nu_+ (1-\nu_-) \int \mathrm{Im} G(\vec{k},\omega)\delta(\omega) L_n\left(\frac{k^2}{2}\right) e^{-k^2/2} \frac{d^2k}{(2\pi)^2} d\omega, \tag{1}$$

where ν_+ and ν_- are the filling factors for spin-up and spin-down electrons in the last filled Landau level n, and L_n is the Laguerre polynomial. We assume $l_h^2 = \cos\hbar/eH = 1$, $\hbar = 1$, A is the hyperfine interaction constant, and $G(\vec{k},\omega)$, the Green's function of the spin exciton, is expressed in the usual way in terms of the operators which create the spin exciton,

$$A^+(\vec{k}) = \frac{1}{\sqrt{N_L \nu_+ (1-\nu_-)}} \sum_p e^{ikx^p} a_+\left(p - \frac{k_y}{2}\right) a_-^+\left(p + \frac{k_y}{2}\right) \tag{2}$$

(a and a^+ are the operators which create electrons in the Landau gauge, and N_L is the number of states in the Landau level), and which are Hermitian conjugate. The normalization was chosen in such a way that the expectation value of the commutator would be ($\langle [A_k^+ A_k] \rangle = 1$), as it should be for a particle.

The particularly simple case[4] $\nu_+ = 1$, $\nu_- = 0$ can be analyzed in detail in the lower order in the ratio of the Coulomb energy $E_c = e^2/\kappa l_h$ (κ is the dielectric constant) to the cyclotron energy $\hbar\omega_c$. Extending this result for the mean-field theory to arbitrary ν, we easily find an effective Hamiltonian for spin excitons in an external field,

$$H_{eff} = |g|\mu H + \frac{p^2}{2m} + \vec{p}[\vec{n}\vec{\nabla} U(\vec{r})], \tag{3}$$

where $1/2m = 1/4\sqrt{\pi/2}(\nu_+ - \nu_-)e^2/\kappa l_h$, $U(\vec{r})$ is the random potential acting on the electrons, and n is a unit vector in the direction of the magnetic field directed perpendicular to the 2D layer. In deriving (3) we assumed the momentum of the exciton to be small and so we expanded it in the momentum, while the electron density, ν_+ and ν_-, was assumed to be uniform.

There is some justification to assume that the electron density of states is determined by a relatively large-scale potential created by the charged impurities which are removed a distance roughly corresponding to that of the spacer. We will assume, therefore, that the random potential is a Gaussian potential with a large correlation radius, and we will use a semi-classical approximation. A zero spin excitation energy can be reached only as a result of relatively rarely encountered random potential fluctuations. This circumstance makes it possible to use the maximum-fluctuation method to calculate[5] $\mathrm{Im}G(\vec{k}, \omega)$. Clearly, the maximum fluctuation corresponds to small values of k^2, $k^2 \approx |g|\mu H/E_c \ll 1$. This result justifies an expansion in the momentum which was used in the derivation of Hamiltonian (3), while T_1^{-1} is actually determined by the density of states. The calculation itself is rather standard, except that the single-particle Hamiltonian has the form (3), instead of the usual sum of the kinetic energy and potential energy.

The largest contribution comes from the maximum fluctuation, in which the spin exciton is localized near a certain point with a small momentum $p^2 = 2g\mu Hm$ with a zero velocity. The nuclear spin relaxation rate in this case is

$$T_1^{-1} \sim \nu_+(1-\nu_-)\exp\frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{\mu|g|H}{R''(0)}(\nu_+-\nu_-)\frac{e^2}{\kappa l_h}, \tag{4}$$

where $R''(0)$ is the second derivative of the pairing correlation function of the random potential at $r=0$. For a continuous random potential, the result thus contains only the mean-square value of the random potential gradient [which is equal to $R''(0)$]. This makes it possible to determine the asymptotic result (4) slightly more precisely if the exciton is considered in a random uniform field ($\nabla U = \text{const}$). The flipping rate in a uniform field can be calculated from perturbation theory and after averaging over various ∇U we obtain.

$$T_1^{-1} = (\nu_+ - \nu_-)\int \frac{1}{l_h^2 k}\exp\left\{\left[\frac{|g|\mu H + \frac{k^2}{2m}}{l_h^2 k}\right]^2 \frac{1}{R''(0)} - \frac{l_h^2 k^2}{2}\right\} L_n\left(\frac{k^2}{2}\right) d^2k. \tag{5}$$

This integral can be calculated by the steepest-descent method or numerically. The results of calculation with the parameters for GaAs, along with the experimental data, are shown in Fig. 1. The adjustable parameters are the scale T_1^{-1} (the common factor) and $R''(0) \approx 9\times 10^{-4}$ (meV/nm)^2.

Despite a qualitative agreement, the region of high magnetic field cannot be reproduced well. The discrepancy can be attributed to the following factors: (a) The key fluctuations in this case are the small-scale fluctuations of the random potential. (b)

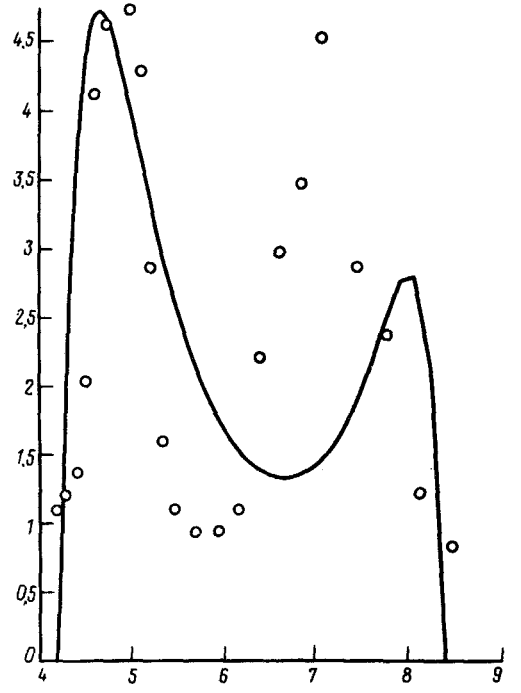

FIG. 1. Experimental points and curves calculated from (7) for $R''(0) = 9\times 10^{-4}$ (meV/nm)^2. The magnetic field is plotted along the abscissa (in Tesla) and T_1^{-1} is plotted along the ordinate (in arbitrary units).

The random potential itself depends on the filling factor of the Landau levels, which actually is the case for the electron density of states.

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