Temperature dependence of the penetration depth of a field into a superconductor

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The presence of low-frequency excitations in a phonon spectrum was found to lead to a situation in which the temperature dependence of the penetration depth becomes a power function at $T \ll T_c$.

The recently published experimental (see Ref. 1 and the bibliography cited there) and theoretical papers[2] give us reason to assume that at low temperatures ($T \ll T_c$) the temperature dependence of the penetration depth is a power function. Such behavior is, as will be shown below, a direct consequence of the equations for strong coupling[3] in the presence of low-frequency excitations.

Ignoring the phonon corrections to the electromagnetic vertex, we obtain for the penetration depth an expression which can be written, within terms on the order of

$e^{-\Delta/T}$ (Δ is the gap in the spectrum), in the form

$$\delta_L^{-2} = \frac{4\pi N e^2}{mc^2} \int_0^{+\infty} d\varepsilon \frac{\Delta^{R^2}(i\varepsilon, T)}{Z^R(i\varepsilon, T)[\Delta^{R^2}(i\varepsilon, T) + \varepsilon^2]^{3/2}} , \tag{1}$$

where the integration is over the upper imaginary semiaxis and Δ^R and Z^R are solutions of the strong-coupling equations which are analytically continued to the upper half-plane and which within the same accuracy can be written as follows:

$$[1 - Z(i\varepsilon_n)]\varepsilon_n = -\int_{-\infty}^{+\infty} d\varepsilon' \frac{\varepsilon'}{\sqrt{\varepsilon'^2 + \Delta^2(i\varepsilon')}} \int_0^{+\infty} d\omega \frac{\omega g(\omega)}{\omega^2 + (\varepsilon_n - \varepsilon')^2}$$

$$- 2\pi \int_0^{+\infty} d\omega g(\omega) N(\omega) Re \left\{ \frac{\varepsilon_n + i\omega}{\sqrt{(\varepsilon_n + i\omega)^2 + \Delta^{R^2}(i\varepsilon_n - \omega)}} \right\}, \tag{2}$$

$$\Delta(i\varepsilon_n) Z(i\varepsilon_n) = \int_{-\infty}^{+\infty} d\varepsilon' \frac{\Delta(i\varepsilon')}{\sqrt{\varepsilon'^2 + \Delta^2(i\varepsilon')}} \int_0^{+\infty} d\omega \frac{\omega g(\omega)}{\omega^2 + (\varepsilon_n - \varepsilon')^2}$$

$$+ 2\pi \int_0^{+\infty} d\omega g(\omega) N(\omega) Re \left\{ \frac{\Delta^R(i\varepsilon_n - \omega)}{\sqrt{(\varepsilon_n + i\omega)^2 + \Delta^{R^2}(i\varepsilon_n - \omega)}} \right\} . \tag{3}$$

Here $N(\omega) = [e^{\omega/T} - 1]^{-1}$, $g(\omega) = \alpha^2(\omega) F(\omega)$ is the effective density of states of the phonons, and $\varepsilon_n = (2n+1)\pi T$, $n \geqslant 0$. It follows from Eqs. (2) and (3) that

$$Z(i\varepsilon, T) = Z(i\varepsilon, 0) + 2\pi \int_0^{+\infty} \frac{d\omega g(\omega) N(\omega)}{\sqrt{\varepsilon^2 + \Delta^2(i\varepsilon, 0)}} + O(T^3 g(T)), \tag{4}$$

and the expansion of $\Delta\ (i\varepsilon,\ T)$ begins with the terms such as $\int_0^{+\infty} d\omega \omega^2 g(\omega) N(\omega)$, i.e., it contains an additional small term T^2. We note that the frequency and temperature dependences in the correction to the value of Z are separated on the imaginary axis, while the power of T on the real axis depends on the relationship between ε and Δ [after an analytic continuation (2)], consistent with the results obtained in Ref. 3.

Evaluating $\Delta(i\varepsilon, 0) \equiv \Delta$ and $Z(i\varepsilon, 0) \equiv 1$, we find the following expression for Debye phonons $(g(\omega) = \lambda(\omega/\omega_0)^2 \theta(\omega_0 - \omega))$:

$$\frac{\delta_L(T) - \delta_L(0)}{\delta_L(0)} = \frac{\pi^2}{2} \zeta(3) \lambda \frac{T}{\Delta} \left(\frac{T}{\omega_0}\right)^2 \tag{5}$$

Measurement of the exponent in the temperature dependence of δ_L at low temperatures thus would yield information on the low-frequency behavior of the effective spectral phonon density [$g\ (\omega)$]. In the development of superconductivity, the interaction of electrons with a subsystem of the two-level-center type, such as that analyzed in Ref. 4, may play a role, in addition to the electron–phonon coupling. Taking this coupling into account leads to the appearance in $g(\omega)$ of a temperature-dependent component $g(\omega) \to g(\omega) + \tilde{g}(\omega)\tanh(\omega/2T)$. At low temperatures, this contribution yields in $\delta_L(T)$ a component on the order of $(T/\Delta)^2\,\tilde{g}(T)$ which may become the leading contribution if $\tilde{g}(\omega) = \text{const}$.

In the case of strong coupling which we have considered the phonon corrections to the electromagnetic vertex do not contain a small parameter. This circumstance does not change, however, the temperature dependence of the penetration depth. A more general analysis of the effects associated with this problem will soon be carried out.

[1] V. F. Gantmakher, N. I. Golovko, I. G. Naumenko *et al.*, Physica C **171**, 223 (1990).
[2] O. V. Dolgov, A. A. Golubov, and A. E. Koshelev, Sol. State Commun. **72**, 81 (1989).
[3] G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **39**, 1437 (1960) [Sov. Phys. JETP **12**, 1000 (1960)].
[4] G. M. Éliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 28 (1987) [JETP Lett. **45**, 35 (1987)].

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