Magnetization versus external magnetic field in layered superconductors

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(Submitted 23 May 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 12, 614–618 (25 June 1991)

For an anisotropic layered superconductor we have calculated the magnetization versus the absolute value $\mathscr{H}$ and orientation of the external magnetic field. The curve displays a cusp at a field $\mathscr{H} = \mathscr{H}_1(\theta)$, where θ is the angle between the field and the layers, and a maximum at a certain characteristic field $\mathscr{H} = \mathscr{H}_3(\theta) \propto (\sin\theta)^{-1}$, in agreement with the experimental measurements by N. V. Zavaritsky and V. N. Zavaritsky. We predict the existence of an intermediate critical field $\mathscr{H} = \mathscr{H}_2(\theta) \propto (\sin\theta)^{-1}$, at which the component of the magnetic induction perpendicular to the layers first penetrates the sample. The magnetic susceptibility $\chi = \partial M/\partial\mathscr{H}$ has a jump at $\mathscr{H} = \mathscr{H}_2(\theta)$. The jump is very small at $\theta \gg \gamma^{-1}$, where γ is the anisotropy coefficient. In this range of angles one can observe a cusp on a graph of χ vs $\mathscr{H}$ at $\mathscr{H} = \mathscr{H}_2$. This prediction of the theory is in good agreement with the experiment.[2]

In isotropic type-II superconductors the magnetization vs the magnetic field has a maximum at the first critical field $\mathscr{H}_{c1}$, as it was first shown by Abrikosov.[1] At lower fields the field does not penetrate the superconductor (the complete Meissner effect). In a recent experiment by Zavaritsky and Zavaritsky[2] the magnetization $M_{\parallel}$ vs the

 0021-3640/91/120637-06$01.00

magnetic field has been measured in a strongly anisotropic, layered superconductor Bi 2-2-1-2. The magnetization revealed a cusp at lower critical field and an additional maximum at a higher field, depending on the angle θ between the external magnetic field and the layers. In this article we explain this phenomenon on the basis of a model of a homogeneous anisotropic superconductor. The layered structure leads to a cusp in the graph of the magnetization vs the magnetic field at an intermediate value of the magnetic field. However, this is a weakly pronounced cusp in the total range of angles, except at very small angles. Instead, one can observe a strongly pronounced cusp in a graph of the magnetic susceptibility.

For a homogeneous anisotropic superconductor the magnetization vs the magnetic field was calculated by Buzdin and Simonov.[3] They have found the lower critical field H_{c1} and the maximum of magnetization at a higher field. In contrast with their numerical calculations, we account for the layered structure and use a simplified version of the free energy, which enables us to make a straightforward analysis. Feinberg and Villard[4] were first to predict the locking of kinks in a tilted magnetic field. Unfortunately, their analysis did not incorporate the demagnetizing factors, which are crucial for this problem. In addition, they have not calculated the magnetization explicitly.

We start with an approximate expression for the free energy of an anisotropic layered superconductor[5]

$$F = \frac{\mathbf{B}^2}{8\pi} + \frac{H_1\sqrt{B_x^2 + \gamma^2 B_z^2}}{4\pi} + \frac{H_2 \mid B_z \mid}{4\pi}, \tag{1}$$

where B_x and B_z are the components of the magnetic induction $\mathbf{B}$ parallel and perpendicular to the layers, respectively; H_1 and H_2 are the characteristic magnetic fields which vary logarithmically with the magnetic field and angle, and $\gamma^2 = m_c/m_{ab}$ is the ratio of effective masses. The second term on the right side of Eq. (1) is due to Campbell *et al.*[6] Roughly speaking, this is a contribution of a homogeneous anisotropic superconductor. The third term is due to kinks on the tilted vortices (Ivlev *et al.*[7]). The free energy (1) has a logarithmic accuracy $(\ln \lambda/\xi)^{-1}$ at low magnetic fields, $\propto H_2 \gamma H_1$, and a much higher accuracy $\propto H_2/\mathscr{H}$ in the intermediate range of fields, $H_2 \ll \mathscr{H} \ll H_{c2}$.

The internal field $\mathbf{H}$ can be found by differentiating:

$$\mathbf{H} = 4\pi \frac{\partial F}{\partial \mathbf{B}}. \tag{2}$$

For an ellipsoidal shape of a sample the magnetostatic problem can be solved explicitly, giving the following relationship between the vector H of the external field and the magnetic induction $\mathbf{B}$:

$$\mathscr{H} = \hat{n}\mathbf{B} + (1 - \hat{n})\mathbf{H} = \mathbf{B} - 4\pi(1 - \hat{n})\mathbf{M}\ . \tag{3}$$

Further, we consider a symmetric case where the ellipsoid axes coincide with the crystallographic axes. Then the nondiagonal components of the demagnetizing tensor are equal to zero. Differentiating the free energy (1) and substituting Eq. (3), we obtain

$$\mathcal{H}_x = B_x + (1 - n_{xx})\frac{H_1}{\sqrt{1+p^2}}, \tag{4}$$

$$\mathcal{H}_z = B_z + (1 - n_{zz})\left(\frac{\gamma H_1 p}{\sqrt{1+p^2}} + H_2 \operatorname{sign} B_z\right), \tag{5}$$

where $p = \gamma B_z / B_x$.

We find the region of the complete Meissner effect, putting $B_x = B_z = 0$. Let the values $p = \gamma B_z / B_x$ and $\sigma = \operatorname{sign} B_z$ remain undefined and varying in the intervals: $-1 \leqslant p, \sigma \leqslant 1$. This region, designated as region 1, is defined parametrically by the equations

$$\mathcal{H}_x = (1 - n_{xx})\frac{H_1}{\sqrt{1+p^2}},$$

$$\mathcal{H}_z = (1 - n_{zz})\left(\frac{\gamma H_1 p}{\sqrt{1+p^2}} + H_2 \sigma\right).$$

The boundaries of this region are two straight lines,

$$B_x = \pm(1 - n_{xx})H_1, \quad | B_z | \leq (1 - n_{zz})H_2 ,$$

and two ellipses,

$$\frac{\mathcal{H}_x^2}{(1 - n_{xx})^2 H_1^2} + \frac{[\mathcal{H}_z \pm (1 - n_{zz})H_2]^2}{(1 - n_{zz})^2(\gamma H_1)^2} = 1$$

(see Fig. 1). In addition, there exists the region of partial Meissner effect, where the parallel component of the induction B_x penetrates but the perpendicular component B_z does not (region 2). This region is situated outside region 1 and between two straight lines

$$\mathcal{H}_z = \pm(1 - n_{zz})H_2 .$$

In the experiment by Zavaritsky and Zavaritsky[2] the angle θ between the layers and the external magnetic field was fixed and the parallel magnetization $M_{\|}$ vs the absolute value of the magnetic field $\mathscr{H}$ was measured. We present here the analytical

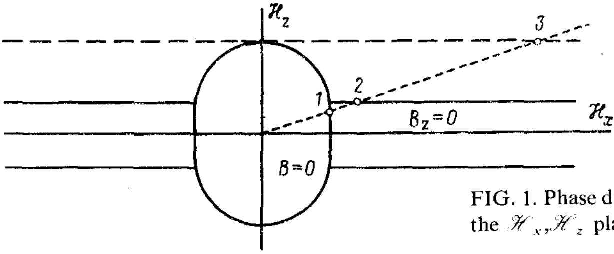

FIG. 1. Phase diagram of a layered superconductor in the $\mathscr{H}_x, \mathscr{H}_z$ plane. See the explanation in the text.

calculations of $M_{\parallel}$ and the perpendicular component of magnetization $M_{\perp}$ based on Eqs. (4) and (5).

In region 1 we obtain the standard formulas

$$M_{\parallel} = -\frac{\mathscr{H}}{4\pi}\left(\frac{(\cos\theta)^2}{1-n_{xx}} + \frac{(\sin\theta)^2}{1-n_{zz}}\right), \tag{6}$$

$$M_{\perp} = \frac{\mathscr{H}\sin\theta\cos\theta}{4\pi}\left(\frac{1}{1-n_{zz}} - \frac{1}{1-n_{xx}}\right). \tag{7}$$

At a fixed angle θ the complete Meissner effect proceeds until $\mathscr{H} = \mathscr{H}_1 = (1-n_{xx})H_1/\cos\theta$, provided that $\theta < \theta_c$, where

$$\theta_c = \arctan\frac{(1-n_{zz})H_2}{(1-n_{xx})H_1}.$$

As can be seen in Fig. 1, a straight line corresponding to a fixed value of $\theta < \theta_c$ crosses first the vertical line $\mathscr{H} = \mathscr{H}_1$ and then the horizontal line $\mathscr{H}_z = (1-n_{zz})H_2$. Between these two crossing points the magnetization obeys the equations

$$M_{\parallel} = -\frac{1}{4\pi}\left(H_1\cos\theta + \mathscr{H}\frac{(\sin\theta)^2}{1-n_{zz}}\right), \tag{8}$$

$$M_{\perp} = \frac{1}{4\pi}\left(-H_1\sin\theta + \mathscr{H}\frac{\sin\theta\cos\theta}{1-n_{zz}}\right). \tag{9}$$

A simple calculation shows that there exists a parallel magnetic susceptibility jump $\Delta\chi$ at $\mathscr{H} = \mathscr{H}_2$:

$$\Delta\chi_{\parallel} = \frac{[H_2(1-n_{zz})\cos\theta - H_1(1-n_{xx})\sin\theta]\sin\theta}{4\pi H_1\gamma^2(1-n_{zz})^2}. \tag{10}$$

For $\theta \ll 1$ the ratio $\Delta\chi/\chi \sim (1+\gamma\theta)^{-1}$. Thus the cusp is well pronounced at small $\theta \ll \gamma^{-1}$ and very weakly pronounced at $\theta \gg \gamma^{-1}$. The corresponding value of θ for Bi 2-2-1-2 is about 1°. Disregarding the jump of χ, we find an approximate expression

$$M_{\parallel} = -\frac{1}{4\pi(1-n_{zz})}\left[\frac{\cos\theta}{\gamma}\sqrt{(\gamma H_1(1-n_{xx}))^2 - (\mathscr{H}\sin\theta - (1-n_{zz})H_2)^2} + \mathscr{H}(\sin\theta)^2\right]. \tag{11}$$

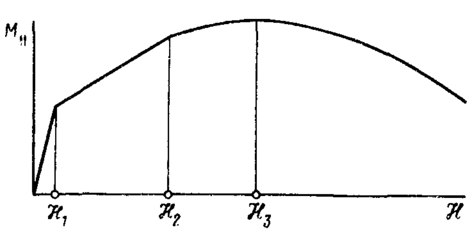

FIG. 2. Schematic diagram of the parallel magnetization vs the magnetic field.

Equation (11) is valid at $\gamma\theta \gg 1$ and in the range of fields $\mathscr{H}_2 \leqslant \mathscr{H} \leqslant \mathscr{H}_3 = (\gamma H_1 + H_2)/\sin\theta$. It displays a spurious singularity at $\mathscr{H} = \mathscr{H}_3$. Expression (11) is invalid in a small neighborhood $\propto (\gamma\theta)^{-1/3}$ of the point $\mathscr{H} = \mathscr{H}_3$. In this range the parallel magnetization peaks and then decreases slowly to $\mathscr{H}_{c2}$ (see Fig. 2).

Returning to the point $\mathscr{H} = \mathscr{H}_2$, we find from Eq. (11) a jump of the parallel susceptibility derivative

$$\Delta\frac{\partial\chi_\|}{\partial\mathscr{H}} = \frac{\cos\theta(\sin\theta)^2}{4\pi[(1-n_{zz})\gamma H_1]^2}. \tag{12}$$

The relative jump $\Delta(\partial\chi_\|/\partial\mathscr{H})/(\partial\chi_\|/\partial\mathscr{H})$ is of the order of unity. For completeness, we write down the expression for the transverse magnetization:

$$M_\perp = \frac{\sin\theta}{4\pi(1-n_{zz})}\left[\mathscr{H}\cos\theta - \frac{\sqrt{(\gamma H_1(1-n_{zz}))^2 - (\mathscr{H}\sin\theta - (1-n_{zz})H_2)^2}}{(1-n_{zz})\gamma}\right].$$

We have analyzed the experimental data[2] for the magnetization vs the field in order to find the magnetic susceptibility $\chi_\|$. The cusps in the graphs of $\chi_\|$ vs $\mathscr{H}$ are clearly seen in the tilt angle range from 15° to 80°, although the general calculation is rather inaccurate. The external magnetic field corresponding to the cusp at a fixed angle θ was multiplied by $\sin\theta$. The result is shown in Fig. 3. According to the theory, it should be a constant equal to $(1-n_{zz})H_2$. We observe a good agreement between theory and experiment. The experiment[2] can therefore be considered as the first clear evidence of the appearance of kinks.

For completeness we show the z component of the external magnetic field at the

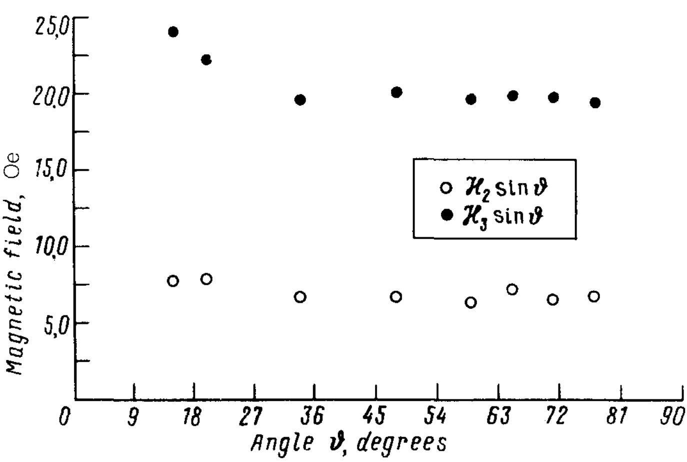

FIG. 3. The critical value of $\mathscr{H}_z$ corresponding to a cusp in the magnetic susceptibility (the open circles) and the value of $\mathscr{H}_z$ corresponding to the maximum of $M_\|$ (the filled circles) obtained by analysis of the experimental data.[2]

maxima of the parallel magnetization $(1-n_{zz})(\gamma H_1+H_2)$ according to Ref. 2. From the experimental data we find $H_1 \approx 3$ Oe and $H_2 \approx 75$ Oe.

We are indebted to N. V. Zavaritsky and V. N. Zavaritsky for stimulating discussions and for providing the original experimental data.

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