Vortex relaxation in a nonlocal electrodynamics of Josephson junctions

Yu. M. Aliev, V. P. Silin, and S. A. Uryupin
P. N. Lebedev Physics Institute, Russian Academy of Sciences, 117924, Moscow

(Submitted 5 January 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **57**, No. 3, 187–188 (10 February 1993)

Analytic solutions of the nonlocal electrodynamic problem describing the relaxation of a vortex are derived for a Josephson junction with pronounced damping. The length scale of the vortex is assumed to be smaller than the Josephson length λ_J.

In Ref. 1 we derived an equation which serves as the foundation of a spatially nonlocal electrodynamics of Josephson junctions:

$$\frac{1}{\omega_J^2}\frac{\partial^2\varphi}{\partial t^2}+\sin\varphi=\frac{\lambda_J^2}{\pi\lambda}\frac{\partial^2}{\partial z^2}\int_{-\infty}^{\infty}dz'K_0\left(\frac{|z-z'|}{\lambda}\right)\varphi(z',t), \qquad (1)$$

where λ is the London depth, λ_J is the size of the Josephson vortex, ω_J is the Josephson plasma frequency, and $K_0(z)$ is the modified Bessel function. For a spatial variation of the phase which is smooth at the scale of λ, this equation reduces to the ordinary sine-Gordon equation. In the opposite limit, the integral nature of the right side of Eq. (1) is important, as was shown in Ref. 1.

Gurevich[2] has derived a steady-state asymptotic solution of Eq. (1) for $\lambda\gg\lambda_J$, in which case the approximation $K_0(z)\approx\ln(2/x)-C$, where $C=0.577$ is Euler's constant, can be used. That solution is

$$\varphi(z)=\pi+2\arctan\frac{\lambda z}{\lambda_J^2}. \qquad (2)$$

In the present letter we report some results on a time-dependent resistive relaxation of a vortex. This relaxation is described by the customary[3,4] addition of another term to the left side of Eq. (1). In the limit of strong dissipation, $\beta\gg\omega_J$, we then find

$$\frac{\beta}{\omega_J^2}\frac{\partial\varphi}{\partial t}+\sin\varphi=\frac{\lambda_J^2}{\pi\lambda}\frac{\partial^2}{\partial z^2}\int_{-\infty}^{\infty}dz'K_0\left(\frac{|z-z'|}{\lambda}\right)\varphi(z',t), \qquad (3)$$

where $\beta=(RC_s)^{-1}$, R is the resistance, and C_s is the capacitance per unit area of the Josephson junction. In the nonlocal asymptotic limit $\lambda\gg\lambda_J$, we find the following time-varying solution of Eq. (3):

$$\varphi(z,t)=\pi+2\arctan\frac{z}{l(t)}. \qquad (4)$$

The time dependence here is given by

$$l(t)=l(0)\exp(-t\beta^{-1}\omega_J^2)+\frac{\lambda_J^2}{\lambda}[1-\exp(-t\beta^{-1}\omega_J^2)]. \quad (5)$$

Solution (4), (5) describes the relaxation of a vortex in which the length scale $l(t)$ varies with the time, from some initial value $l(0)$ to the steady-state value derived in Ref. 2. We see, in particular, that if the vortex is initially singular, i.e., if $l(0)\ll\lambda_J^2\lambda^{-1}$, then a nonsingular steady-state vortex is established over a time $\sim\beta\omega_J^{-2}$.

The magnetic field of time-varying vortex (4) has a structure similar to that discussed in Ref. 2:

$$H_y(x,z,t)=-\frac{\hbar c}{4e\lambda^2}\left\{\frac{1}{2}\ln\frac{4\lambda^2}{[|x\mp d|+l(t)]^2+z^2}-C\right\}, \quad (6)$$

where $2d$ is the width of the transition layer, the argument $x-d$ corresponds to the region $x>d$, and the argument $x+d$ corresponds to $x<-d$.

Equations (4)–(6) describe the temporal relaxation of an asymptotic vortex in the nonlocal electrodynamics of a Josephson junction with strong damping—a relaxation which results in the formation of a steady-state vortex. If, instead of (4), we use another asymptotic solution of Eq. (3),

$$\varphi(z,t)=2\arctan\frac{z}{l(t)}, \quad (7)$$

it, too, will correspond to the magnetic-field structure described by Eq. (6). However, in this case the length scale of the vortex, $l(t)$, increases in accordance with

$$l(t)=\exp(t\beta^{-1}\omega_J^2)\left[l(0)+\frac{\lambda_J^2}{\lambda}\right]-\frac{\lambda_J^2}{\lambda} \quad (8)$$

as time elapses. The increase in the length scale of the vortex is described by the latter expression only under the condition $l(t)\ll\lambda$. In this case it is legitimate to use a logarithmic approximation of the modified Bessel function.

Yet another time-varying resistive solution of Eq. (3) is

$$\varphi(z,t)=\pi+2\arctan\frac{(\lambda\lambda_J^{-2}z)^2+a^2(t)}{b^2(t)}. \quad (9)$$

In this case the functions $a(t)$ and $b(t)$ obey the equations

$$\beta\omega_J^{-2}\frac{da^2}{dt}=\sqrt{2}\sqrt{\sqrt{a^4+b^4}+a^2},$$

$$\beta\omega_J^{-2}\frac{db^2}{dt}=\sqrt{2}\sqrt{\sqrt{a^4+b^4}-a^2-b^2}. \quad (10)$$

After a sufficiently long time $t\gg\beta\omega_J^{-2}$, according to (10), the temporal relaxation of the vortex may go into a regime with $a^2\gg b^2$, $a(t)\approx\beta^{-1}\omega_J^2t$, and $b^2(t)\approx t\times\exp(-\beta^{-1}\omega_J^2t)$. This regime corresponds to a damping of the vortex.

In summary, the first time-varying analytic solutions describing a resistive relaxation of vortices in the nonlocal electrodynamics of Josephson junctions have been reported in this letter.

[1] Yu. M. Aliev, V. P. Silin, and S. A. Yryunin, Sverkhprovodimost' (KIAE) **5**(2), 230 (1992) [Superconductivity **5**(2), 228 (1992)].
[2] A. Gurevich, Phys. Rev. B **46**, 3187 (1992).
[3] A. A. Abrikosov, *Fundamentals of the Theory of Metals,* Elsevier, New York, 1988.
[4] A. Barone and G. Paternò, *Physics and Applications of the Josephson Effect,* Wiley, New York, 1982.

Translated by D. Parsons