Structure of the vortex lattice in a platelet of an anisotropic superconductor

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The equilibrium shape of an isolated vortex filament is studied. The structure of a lattice of vortices in a superconducting platelet in an oblique magnetic field is also studied. The in-plane component of the field curves the vortices and thereby substantially alters the vortex–vortex interaction. The results derived here give a quantitative description of the vortex structures which are observed at the surface of a $YBa_2Cu_3O_x$ single crystal [P. L. Gammel *et al.*, Phys. Rev. Lett. **68**, 3343 (1992)].

The most striking structural features of magnetic fluxes in the high-Tc superconductors are the mutual repulsion of vortices and the alignment of vortices in chains. The distance between vortices in such a chain is essentially constant and much smaller than the distance between chains. The latter varies with the magnetic induction as $1/B$. This effect was predicted theoretically[2] and confirmed experimentally through the observation of vortex structures on the surfaces of $YBa_2Cu_3O_x$ (Ref. 1) and $Bi_{2.1}Sr_{1.9}Ca_{0.9}Cu_2O_x$ (Ref. 3) single crystals by a decoration technique using dispersed ferromagnetic particles. On the other hand, there is a substantial discrepancy between theory and experiment. For example, it was shown in Ref. 2 that the chains can be observed if the external magnetic field makes an angle of less than 5° with the anisotropy axis. Actually, the chains are observed at much larger angular deviations of the magnetic field. Furthermore, in this interval of small deviation angles one sees only a hexagonal lattice of vortices, the existence of which is totally at odds with the theory.[4] We believe that this contradiction is rooted in a difference between the entities being studied. The experiments are always carried out on platelet-shaped samples, while the theory is derived for unbounded superconductors. In the present letter we describe a vortex structure in an anisotropic superconducting platelet of thickness $d > \lambda$, where λ is the magnetic-field penetration depth.

The platelet occupies the spatial band $-d/2 < x_3 < d/2$; the anisotropy axis and the x_3 axis of the Cartesian coordinate system are oriented along the normal to the platelet surface. The magnetic field of the vortices is found from the solution of the combination of Maxwell's equations in the volume around the platelet and the London equation in the superconductor. In the latter, the shape of the vortex lines is specified by an arbitrary function $\mathbf{x} = \mathbf{l}(x_3)$.

A vortex lattice in an external field $\mathbf{H}$ is described by a Gibbs potential

$$G=\frac{B}{\phi_0}\int_{-d/2}^{d/2} dx_3\left\{F_0(x_3)+\Sigma_R U(\mathbf{R})-\frac{\phi_0 H_2}{4\pi}\left[1-\frac{\cosh(x_3)}{\cosh(d/2)}\right]\right\}. \qquad (1)$$

Here and below, lengths are expressed in units of $\lambda_a = \lambda\mu_a^{1/2}$, where μ_a is the anisotropy parameter of the superconducting platelet.

The oblique field $\mathbf{H} = \mathbf{B} + \mathbf{H}_2$ affects the vortex structure in the platelet in the following way. The normal component $\mathbf{B}$ penetrates into the superconductor in the form of vortices and determines the average induction. The in-plane component $\mathbf{H}_2$ (for definiteness, we direct it along the x_2 axis) penetrates into the platelet in the form of a Meissner field. The interaction of the vortices with this field curves the vortices and distorts the vortex–vortex interaction.

The eigenenergy of a vortex segment,

$$F_0(x_3) = \left(\frac{\phi_0}{4\pi}\right)^2 \left\{ \frac{(1+\mu_a^3\kappa^2)\ln(\xi_a^{-1})}{(1+\mu_a^3\kappa_1^2)^{1/2}(1+\mu_a^3\kappa_2^2)^{1/2}} + \frac{\kappa^2}{2}[\exp(|x_3|-d/2)+(1-\mu_a^3)E_1(d/2-|x_3|+\xi_a)] \right\}, \quad (2)$$

depends on the depth of the segment in the platelet, i.e., the coordinate x_3, and on the deviation of the segment from the anisotropy axis, i.e., $\kappa = dl(x_3)/dx_3$, $\kappa^2 = \kappa_1^2 + \kappa_2^2$; here $E_1(x)$ is the integral exponential function, and ξ_a is the coherence length. The equilibrium shape of a vortex filament is found from the equation

$$\frac{\delta G}{\delta\kappa(x_3)} = 0. \quad (3)$$

One of the solutions of Eq. (3) is $\kappa_1 = 0$: The vortex line always lies in the plane defined by the vectors $\mathbf{B}$ and $\mathbf{H}_2$. At the boundary of the superconductor we have $\kappa(\pm d/2) = 0$, and the vortex is directed along the normal to the surface. Inside the platelet, the inclination of the vortex is constant and independent of x_3:

$$\kappa_2 = \mu_a^{-3/2}\left[\left(\frac{H_{c1}}{H_2}\right)^2 - 1\right]^{-1/2}, \quad (4)$$

where H_{c1} is the first critical field for nucleation, at the center of the platelet, of vortices parallel to the surface. The transition from the solution $\kappa_2 = 0$ to (4) and the bending of the vortex occur at a distance on the order of λ_a from the surface. The equilibrium shape of the vortex in this region was found through a numerical solution of Eq. (3); the result is in Fig. 1. The orientation of an end segment of a vortex along the normal is determined by the minimum of the energy of the interaction of the vortex with its image. A similar orientation near a surface is assumed by vortices in superfluid helium and by screw dislocations in solids.[8]

The exact expression for the energy of the binary interaction of vortices, $U(\mathbf{R})$, is very unwieldy. It can be simplified substantially by noting that a vortex at a distance less than λ_a from the surface is oriented strictly along the normal, while the inclination (κ_2) of the vortices within the platelet (at a distance greater than λ_a) is constant. We then find

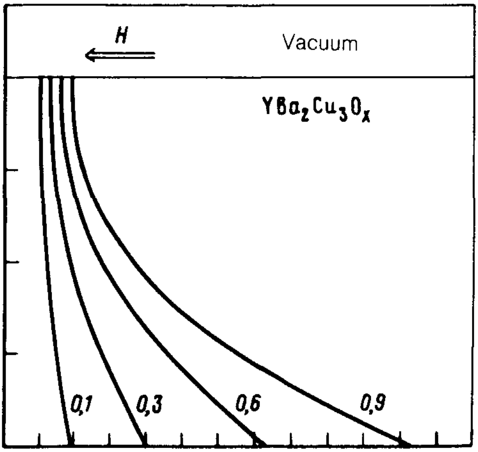

FIG. 1. Equilibrium shape of an isolated vortex near the surface of a $YBa_2Cu_3O_x$ single crystal ($\mu_a = 0.343$) for various strengths of the in-plane field. The strength of the field H_2, in units of H_{c1}, is shown to the right of the vortex line. The distance between scale divisions on the axes is $\lambda_a = 0.14\ \mu$m. The position of the vortex on the external surface is arbitrary.

$$U(\mathbf{R}) = U_1(R) + U_2(\mathbf{R}). \tag{5}$$

Here **R** is the distance between vortices at the surface of the platelet. The long-range interaction of the vortices through vacuum is described by the first term:

$$U_1(R) = 2\left(\frac{\phi_0}{4\pi}\right)^2 \left[\frac{2}{R}[1-\exp(-R)] + \frac{1}{2}\exp(-R)\right].$$

The second term,

$$U_2(\mathbf{R}) = (d-2)\left(\frac{\phi_0}{4\pi}\right)^2 \int_0^\infty \frac{d^2\mathbf{q}}{2\pi q^2} \exp(i\mathbf{qR}) \left[\frac{q^2 + (\mathbf{q}\kappa)^2}{1+q^2+(\mathbf{q}\kappa)^2} + \frac{(\mathbf{q}\kappa)^2}{1+\mu_a^{-3} q^2 (\mathbf{q}\kappa)^2}\right],$$

describes the interaction of vortices within the platelet. A transformation to a coordinate system tied to the vortex axis reduces potential $U_2(\mathbf{R})$ to the known expression for the vortex interaction in an unbounded anisotropic superconductor.[6]

We find the structure of the vortex lattice at the surface of the superconductor and the equilibrium inclination of the vortex within the platelet, κ_2, from the condition that the first variation of the Gibbs potential vanish: $\delta G = 0$.

For any values of the parameters **H**, d, and μ_a, the unit cell of the vortex lattice is rhombic. One diagonal of the rhombus is always oriented along the field $\mathbf{H}_2$. Since the area of the unit cell is ϕ_0/B, the structure of the lattice at an external surface can be described by the single parameter D, which is the distance between the vortices in the chains oriented along $\mathbf{H}_2$.

We have numerically calculated the distance D as a function of the field H_2. To compare the results with the experimental data of Ref. 1, we selected parameter values as follows: $B = 12$ G; λ was varied from 0.20 to 0.40 μm; the platelet thickness d was varied from 2 to 90 μm; and the anisotropy parameter was assigned values of 0.5, 0.343, and 0.25. The in-plane field H_2 was varied from 0 to 10^3 G. Figure 2 shows the relative change in the intervortex distance in a chain, $D(\theta)/D(0)$, for an angular

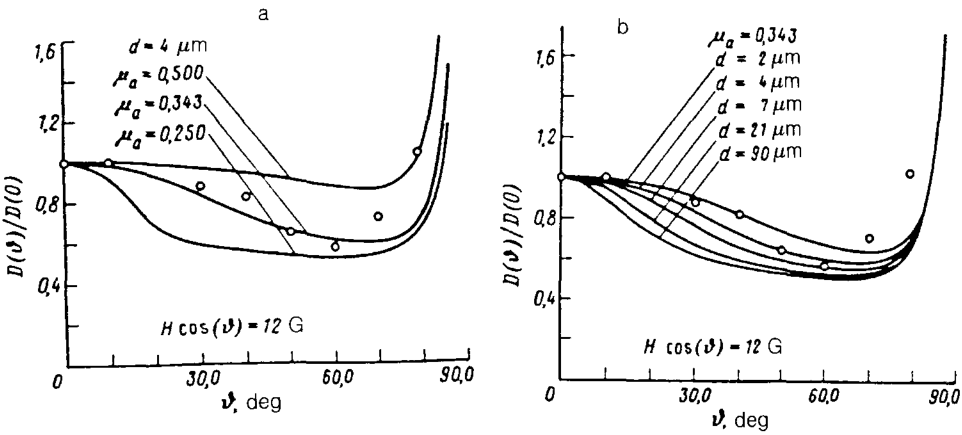

FIG. 2. The distance (D) between the vortices in a chain versus the inclination of the external field **H** at a constant induction. The points are experimental results,[1] and the solid lines are theoretical. *a*—Theoretical results for a platelet of constant thickness (d=4 μm) and for various values of the aniostropy parameter μ_a; *b*—for a constant anisotropy parameter ($\mu_a = 0.343$) and various platelet thicknesses d.

deviation $\theta = \arctan(H_2/B)$ of the field **H** from the anisotropy axis. The best agreement between theory and experiment is reached with the values $d \sim 4$ μm and $\mu_a = 0.343$.

The results in Fig. 2 demonstrate the following transformation of the vortex lattice. In a perpendicular external field ($H_2 = 0$) the vortices are straight and are oriented along the normal ($\kappa = 0$). A centrally symmetric interaction $U(\mathbf{R})$ causes the vortices to become ordered in a regular triangular lattice. When the field **H** deviates from the normal, the vortices in the platelet bend in the direction of H_2 ($\kappa_2 \neq 0$). The interaction between vortices, $U_2(\mathbf{R})$, changes: The repulsion of the vortices in a chain gives way to an attraction. The distance between vortices, D, accordingly decreases. The change in D is governed by a competition between the repulsive forces of the vortices at the surface and their attractive forces in the interior. The latter depend on the platelet thickness d, the anisotropy μ_a, and (implicitly) on H_2 (the inclination of the vortices, κ_2, increases with H_2).

A further increase in H_2 leads to a significant bending of the vortices: $\kappa_2 \gg 1$. The distance between vortices in the platelet decreases to λ_a, and neighboring vortices begin to repel each other. The distance between vortices, $D \sim \lambda_a \kappa_2$, increases with increasing H_2.

The model proposed here gives a quantitative description of the vortex structure observed at the surface of an anisotropic superconductor.[1] The values $\lambda = 0.28$ μm and $\mu_a = 0.343$ which we have found are in convincing agreement with the parameter values found in numerous experiments (e.g., Refs. 7 and 8).

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