Hall tunneling of vortices in high-temperature superconductors

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Quantum tunneling of vortices in very clean type-II superconductors is shown to be governed by the Hall term in the equation of motion. An effective action determining the tunneling probability is calculated. It is argued that high-temperature superconductors may belong to a class of very clean materials, and that they may have Hall tunneling at low temperatures.

At very low temperatures, large temperature-independent magnetic relaxation has been observed in the Chevreul phase,[1] heavy fermion,[2] organic,[3] and high-temperature[4,3] superconductors, suggesting the existence of vortex motion due to quantum tunneling. The tunneling rates of single vortices and of vortex bundles have been determined in the framework of weak collective pinning theory.[5,6] This theory is in reasonable agreement with the experimental data[3] and can explain the quantum creep observed in these "exotic" compounds, whereas "more conventional" superconductors do not show such behavior. The important difference between quantum tunneling and classical thermal activation is the time component of the motion. For thermal activation the time is irrelevant. The probability of the process with exponential accuracy is given by the height of the barrier between the metastable states and the dynamic properties of the vortices influence only the preexponential factor. In contrast, during tunneling the vortex moves under the barrier, the process is virtual, and the longer the time which the vortex has to spend under the barrier, the less chance there is for this process to occur. That is why it is important to understand what are the dynamic equations describing the vortex motion. The simplest possible dynamics is the massive dynamics, for which the quantum decay of a metastable state has been studied extensively. That is probably the reason why the problem of the vortex mass was raised from the beginning of the study of quantum creep.[1,5–7]

The known expressions for the vortex mass were obtained either from different kinds of time-dependent Ginzburg–Landau theories[8] (which definitely are not applicable at low temperature where quantum tunneling was observed) or from various qualitative estimates.[5–7] There is no microscopic derivation of the vortex mass. From microscopic theory of superconductivity it is possible to deduce that the vortex motion should be dissipative[9] for not very clean materials. In the quantum collective creep theory the estimates of the vortex mass are given mainly in order to show that the massive term is not important, and that the vortex tunneling is dissipative.

In this case the Euclidean action $-/S_E^{\rm eff}$ which determines the relaxation of the magnetization M: $\partial \ln M/\partial \ln t \approx -\hbar/S_E^{\rm eff}$ is given by

$$\frac{S_E^{\rm eff}}{\hbar} \simeq \frac{\hbar}{e^2} \frac{L_c}{\rho_n}. \tag{1}$$

Here ρ_n is the normal state resistivity extrapolated to zero temperature and L_c is the collective pinning length which can be expressed in terms of the coherence length ξ and the depairing and critical current densities j_0 and j_c, $L_c \simeq \xi (j_0/j_c)^{1/2}$.

However, there should be no dissipation for very clean superconductors at very low temperatures. In this limit the flux motion should be similar to the motion of the vortices in superfluid helium, in which the vortices are dragged by the superfluid flow. The criteria distinguishing dissipative versus nondissipative vortex motion were given by Kopnin and Kravtsov.[10] Starting from microscopic theory of superconductivity they derived the following equation of motion in the flux flow regime:

$$\eta \mathbf{v}_L + \alpha [\mathbf{v}_L \times \mathbf{n}] = \frac{\Phi_0}{c} [\mathbf{j} \times \mathbf{n}]. \tag{2}$$

Here j is the transport current density, $\mathbf{v}_L$ is the velocity of the vortex line, n is the unit vector along the vortex, and η and α are viscous and Hall drag coefficients, respectively. These coefficients are determined by the interaction of the normal excitation, which exists in the bound states[11] at the vortex core with impurities and which depends strongly on the parameter $\omega_0 \tau$, where τ is transport time and ω_0 is the spacing between the low-lying levels in the vortex core, $\omega_0 \simeq \Delta^2/\varepsilon_F$. Viscous flux flow $(\alpha \ll \eta)$ with Bardeen–Stephen's expression for $\eta \simeq \Phi_0 H c_2/\rho_n c^2 \simeq \pi \hbar n \omega_0 \tau$ corresponds to $\omega_0 \tau \ll 1$, whereas the opposite limiting case $\omega_0 \tau \gg 1$ corresponds to nondissipative flow like in helium-II with $\alpha = \pi \hbar n_s$, where n and n_s are electron density and density of superconducting electrons, respectively, $j_s = e n_s \mathbf{v}_s$.

The condition for the nondissipative flow $\omega_0 \tau \gg 1$, expressed in terms of the mean free path, $l \gg \varepsilon_F/\Delta$, is much stronger than the usual condition for clean limit, $l \gg \xi$, and is virtually never realized in ordinary superconductors, where $\varepsilon_F/\Delta \sim 10^3$. However, high-$T_c$ superconductors have much larger value of Δ with $\varepsilon_F/\Delta \sim 10$. An estimate for ω_0 gives $\omega_0 \sim 10$ K and $\xi \varepsilon_F/\Delta \sim 10$–$20$ *nm*. Extrapolation of the normal state resistivity to zero temperature gives a mean free path $l \sim 70$ *nm*. It is therefore very likely that HTSC are in a "superclean" limit $l \gg \xi \varepsilon_F/\Delta$.

Note that these simple estimates result in a very strong conclusion that the Hall angle in the flux-flow regime in HTSC is $\Theta_H = \arctan(\alpha/\eta) \simeq \pi/2$, which has not yet been confirmed experimentally. However, the measurements of the Hall resistivity which we are aware of have been carried out at high temperatures. Because of flux pinning, the Hall angle goes to zero by lowering the temperature.[12] It is very important, therefore, to perform the measurements of the Hall effect in the flux-flow regime at a very low temperature. A useful tool in this case can be the AC measurements at very high frequencies, where the pinning effects are unimportant.

Let us consider quantum tunneling in the nondissipative case. For simplicity we start with the 2D case where the vortices are point-like objects. The generalization to the 3D case is trivial and will be done later. The classical equation of motion is

$$\alpha [\mathbf{v} \times \mathbf{n}] = -\nabla U(\mathbf{r}), \tag{3}$$

where $\alpha=\pi\hbar n_s^{(2)}$ (here $n_s^{(2)}$ is the superfluid density per superconducting layer), and $U(\mathbf{r})$ is the pinning potential. The same equation describes the motion of a charge with zero mass in a magnetic field B (in this case, α is equal to eB/c). Writing (3) in the components

$$\alpha\frac{dx}{dt}=\frac{\partial U(x,y)}{\partial y};\quad \alpha\frac{dy}{dt}=-\frac{\partial U(x,y)}{\partial x}, \tag{4}$$

we immediately see that the system (4) is of Hamiltonian form. Rescaling $\sqrt{\alpha}x = q$, $\sqrt{\alpha}y = p$, the potential energy U transforms to the Hamiltonian $U(x,y) = H(q,p)$. The action which produces Eqs. (4) has a form

$$S=\int[\alpha\dot{x}y-U(x,y)]dt. \tag{5}$$

The first term is the same as $(e/c)\mathbf{vA}$ term for a particle in a magnetic field with the gauge $A_x=By$. The coordinates x and y are canonical variables with a commutator

$$[x,y]=\frac{i\hbar}{\alpha}=\frac{i}{\pi n_s^{(2)}}. \tag{6}$$

The same approach was used in Ref. 13 to calculate the probability for the quantum creation of vortices in a superfluid helium, and in Ref. 14 to study tunneling in a high magnetic field. From the uncertainty relation we see that the quantum fluctuations of the vortex position are on the order of the average distance between the electrons, $\langle\Delta r^2\rangle Q\sim 1/n_s^{(2)}$.

Now the problem is reduced to one-dimensional quantum mechanics with the Hamiltonian $U(x,y)$. Note, however, that the Hamiltonian $U(x,y)$ is, in general, a symmetric and random function of the canonical "coordinate" x and "momentum" y. It gives some beauty to the problem, but does not help to solve it. It would therefore be worthwhile to consider a model potential, for which the problem can be solved exactly:

$$U(x,y)=U_0\left(\frac{y^2}{r_p^2}+\frac{x^2}{r_p^2}-\frac{x^3}{r_p^3}\right). \tag{7}$$

This is not an artificial potential. If the current $j\|y$ is applied (in such a way that the Lorentz force is directed along x), then near criticality the generic form of the energy potential is given by Eq. (7). After the transformation to the canonical variables $(y\to p,\ x\to q)$ the Hamiltonian (7) describes the one-dimensional motion of the particle with a mass $m=r_p^2/2U_0$ in the potential $U^{(1)}(x)=U(x,y=0)$. In the quasi-classical approximation (i.e., when $r_p\gg 1/\sqrt{n_s}$) the tunneling probability $W\propto\exp(-S^{\text{eff}}/\hbar)$ can be calculated straightforwardly:

$$\frac{S^{\text{eff}}}{\hbar}=\frac{8}{15}\frac{r_p^2\alpha}{\hbar}=\frac{8}{15}r_p^2 n_s^{(2)}. \tag{8}$$

We see that the result for the effective action (8) does not depend on the strength of the pinning potential U_0. In 1D quantum mechanics the action which determines the tunneling probability is equal to $2\int p \cdot dq$ taken along the classically forbidden path. In our problem it corresponds to

$$S^{\text{eff}} = \alpha \oint y dx,$$

[cf. Eq. (5)] which is nothing but α times the area spanned by the vortex trajectory (note that, contrary to the usual massive or viscous tunneling, the vortex trajectory in our case has no turning point—it is not the "bounce" but the orbit with nonzero area). Thus, because of the "geometrical" quantization (the canonical variables are just coordinates), the effective action depends on the length scale of the potential, rather than on its magnitude.

Generalization of this result for the three-dimensional case is simple:

$$\frac{S^{\text{eff}}}{\hbar} = \frac{\alpha}{\hbar} V = \pi n_s V, \tag{9}$$

where V is the volume spanned by the trajectory of the vortex line or the vortex bundle. In the collective pinning picture r_p is on the order of the vortex core size ξ, whereas the length of the tunneling vortex segment is L_c (in the single vortex pinning limit). Hence, the result for the tunneling exponent is

$$\frac{S^{\text{eff}}}{\hbar} \sim n_s^{(2)} \xi^2 \quad (\text{at } D=2); \quad \sim n_s \xi^2 L_c \quad (\text{at } D=3). \tag{10}$$

For the usual BCS-type superconductors at low temperatures in the clean limit n_s coincides with the carrier density n. Results (10) give an order of magnitude estimate for the tunneling exponent $\sim 10^2$, in agreement with the experimental data.[3,4] In the 2D case, the tunneling exponent does not depend on the strength of the pinning potential, in agreement with the results of Ref. 15, where the quantum creep in $Bi_2Sr_2CaCu_2O_8$ with columnar defects was studied, and where quantum relaxation rate was shown to be only weakly affected by the columnar defect, while the critical current substantially increased. The reason for such a behavior is that columnar defect in the Bi-compound (where pinning, especially at low T, is of 2D nature) has a radius close to ξ, so the space scale of the potential was not modified with respect to the unirradiated sample.

This experiment, however, cannot be considered as evidence against dissipative vortex tunneling:[5] For a 2D dissipative tunneling we can use Eq. (1), where L_c is replaced by d, and the result is again independent of the pinning strength. Actually, the results (10) can be obtained qualitatively with the same kind of dimensional estimates, as it was done in Ref. 5 by replacing the viscosity η with the Hall coefficient α in the formulas for the dissipative tunneling. However, such a simple transformation is valid for one-parameter estimates like (10) only, while the dependences of the tunneling rate on the temperature, on the external current, etc. are quite different in the dissipative case and the Hall case. In particular, low-temperature corrections to the result (10) are exponentially small [$\propto \exp(-T_0/T]$, whereas in the dissipative case they are $\propto T^2$. Also, when we study tunneling near the critical force, the length scales along x and along y in Eq. (7) are different. Because of smearing of the potential near criticality, there is an additional factor $[(j_c-j)/j_c]^{1/2}$ in front of the x^2/r_p^2 terms in (7) and the hopping distance, $x_{\rm hop}\sim r_p[(j_c-j/J_c]^{1/2}$, goes to zero. In this case, for tunneling with the dissipation $S_d^{\rm eff}\propto(j_c-j)$, whereas for the Hall tunneling the result is similar to the massive result, and $S_H^{\rm eff}\propto(j_c-j)^{5/4}$ (see Ref. 6). This means that dissipation becomes more important near criticality. The approach developed above can be generalized to the case in which the dissipation and Hall motion are important. The Euclidean action can be written in the form

$$S_E=\int dt\left[\int dt'\frac{\eta}{4\pi}\frac{[x(t)-x(t')]^2+[y(t)-y(t')]^2}{(t-t')^2}+i\alpha\dot{x}y+U(z,y)\right]. \qquad (11)$$

The first term in the above expression is the Caldeira–Leggett term,[16] which accounts for the dissipation; in the case where the potential $U(x,y)=U_0y^2/r_p^2+U^{(1)}(x)$, the change of the variable $y\rightarrow -iy$ transforms the two last terms in Eq. (11) to the form corresponding to the classical motion of the particle in a magnetic field and in the "inverted" potential $-U^1(x)$; however, such a simple analogy is not valid for the general form of the potential $U(x,y)$.

Going over to the Fourier representation in (11) and performing the Gaussian integration over the y variable, we obtain

$$S_E=r_p^2\int\frac{d\omega}{2\pi}\left\{\left[\frac{\eta|\omega|}{2}+\frac{\alpha^2\omega^2}{2+\eta\omega}\right]x_\omega^2+x_\omega^2-(x^3)\omega\right\}. \qquad (12)$$

Here we rescaled $x,y\rightarrow xr_p,yr_p$. The problem now reduces to a one-dimensional motion in a cubic potential with a dispersive kinetic term. At $\alpha\gg\eta$, we have again the usual one-dimensional tunneling problem, while in the opposite limit the dissipative tunneling takes place. The characteristic scale of displacement in the y direction is given by $(1+\eta/\alpha)^{-1}$. Minimization of the action (12) is complicated by the presence of the cubic term nonlocal in ω, and the resulting integral equation can be solved only numerically. We see, however, that the general evolution of the optimal trajectory is from circular orbit at $\eta=0$ via its narrowing in the y direction, where η increases until it collapses to a straight line along the axis in the limit $\alpha\ll\eta$. At small η the relative corrections to a purely nondissipative result are on the order of η/α, whereas in the opposite limit the "Hall" corrections to a dissipative result are $\propto(\alpha/\eta)^2$. The last result is due to the fact that the tunneling rate should be independent of the α sign and analytic at small α. A very interesting approach in dealing with the Hall effect in superconductors was proposed by Ao and Thouless[17] (see also Ref. 18, where a similar approach was used for vortices in helium films. They have shown that the Magnus force acting on a vortex line can be obtained by calculating the Berry phase for an adiabatic motion of the vortex along the closed loop. In this approach expression (9) can be interpreted as the Berry phase which is acquired by the vortex during tunneling and which is equal to π times the number of superconductive electrons inside the volume spanned by the trajectory of the vortex line. Note, however, that the microscopic derivation of the Hall coefficient α is still a subject of controversy. It was found in Ref. 10 (see also Ref. 19) that the Magnus force is almost completely canceled in the case of relatively weak disorder (i.e., in the usual "clean," although not "super-clean" regime). The problem of the topological Berry phase dependence on the strength of disorder is very important and clearly needs further investigation. In this paper we considered this problem on a more deeply phenomenological level, starting from a given value of the Hall constant α.

In conclusion, the "vortex mass" problem is not relevant for quantum tunneling of vortices; depending on the disorder strength, the tunneling can be of the dissipative or Hall nature. It is likely that HTSC belong to the class of very clean materials and that the Hall tunneling is realized at low temperatures. In this case the estimates for the tunneling rate are given by Eqs. (9) and (10), in reasonable agreement with the experimental data. More detailed experiments should be performed in order to distinguish between the Hall tunneling and dissipative tunneling in HTSC.

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Note added in proof: When this work was completed, we became aware that this problem was also studied recently by D. Thouless and co-workers.[20] We are grateful to D. Thouless for the discussion that followed.

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