

# Three-particle anyon excitons

E. I. Rashba

*L. D. Landau Institute for Theoretical Physics RAS, 117940 Moscow, Russia and  
Department of Physics, University of Utah, Salt Lake City, Utah 84112, USA*

M. E. Portnoi

*A. F. Ioffe Physicotechnical Institute, RAS, 194021 St. Petersburg, Russia and  
Department of Physics, University of Utah, Salt Lake City, Utah 84112, USA*

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We propose a model of an anyon exciton (AE) consisting of a hole and several anyons [E. I. Rashba and M. E. Portnoi, *Bull. Amer. Phys. Soc.* **38**, 136 (1993)] and apply it to the spectroscopy of an incompressible quantum liquid (IQL). Fractionalization of the electron charge makes properties of such entities different from those of usual magnetoexcitons. The model describes a number of properties established by few-particle simulations, including an abrupt change in the emission spectra vs electron–hole asymmetry of the system.

Experimental determination<sup>2</sup> in the optical spectroscopy of an IQL,<sup>3</sup> which underlies the fractional quantum Hall effect<sup>4</sup> (FQHE), stimulated recent theoretical activity in this field.<sup>5–8</sup> A remarkable property of IQL's is that their elementary excitation carry fractional charge<sup>3</sup> and that they are anyons,<sup>9,10</sup> i.e., they obey fractional statistics.<sup>11,12</sup> The theory based on simulations<sup>5,8</sup> predicts that fractional charges should manifest themselves as abrupt changes in the electronic density distribution of an exciton against the background of an IQL. These changes set in when the charge asymmetry of the electron–electron and electron–hole interactions is changed, and they are accompanied by dramatic changes in the position and intensity of the emission band vs the charge asymmetry parameter.<sup>5,8</sup> The ratio  $h/l$ , where  $h$  is a distance between the electron and hole confinement planes, and  $l = (c\hbar/eH)^{1/2}$  is the magnetic length, may be chosen as such parameter. Numerical simulations for few-electron systems are accessible only for small values of  $h/l < 2$ . For the opposite limiting case of strongly asymmetric systems,  $h/l \gg 1$ , the anyon concept seems to be most promising. An exciton which appears against the background of an electron IQL is a neutral entity consisting of a valence hole and several anyons. For example if the filling factor  $\nu = 1/3$ , the charge of anyons  $e^* = -e/3$ , their statistical charge  $\alpha = -1/3$  (for comparison,  $\alpha = 0$  for bosons, and  $\alpha = 1$  for fermions),<sup>9</sup> and the number of anyons  $N = 3$ . If  $h \gg l$ , the mean separation between anyons in an exciton is about  $h$ , which is larger than the anyon size,  $l$ . Therefore, anyons are well defined particles, anyon-anyon and anyon–hole interactions follow a Coulomb law, and the Coulomb energy is small compared with the IQL gap width. When  $h < l$ , only qualitative results may be expected from the AE model. Nevertheless, we show that they are quite encouraging. Recently, there has been a significant activity in the hierarchy theory of the FQHE<sup>13,14</sup> and some experimental data have been discussed in these terms.<sup>15</sup> While the hierar-

chies provide a level classification for free anyons, AEs may also give insight into the effect of an external (Coulomb) field and the treatment of optical data.

We consider a model of an AE consisting of a hole and two semions, anyons with  $e^* = -e/2$  and  $\alpha = -1/2$ . If the hard-core constraint<sup>12,13</sup> is imposed, we should choose  $\alpha = 3/2$ . For spin polarized IQLs  $e^* = -e/q$ , where  $q$  is odd. Our simple two-anyon model gives insight into the properties of the more realistic models with  $q \geq 3$ . We assume that the magnetic field is strong, i.e., the Coulomb energy is  $\epsilon_C = e^2/\epsilon l \ll \hbar\omega_c$  ( $\omega_c$  is the cyclotron frequency), and use dimensionless variables scaled in units of  $\epsilon_C$  and  $l$ . Such an AE, which consists of three particles, is described by three quantum numbers. Since it is a neutral entity, a two component momentum  $\mathbf{K}$  may be attributed to it. Therefore, the internal motion in it is characterized by a single quantum number, and the charge fractionalization (CF) should result in a single-parameter array of energy bands, instead of the single band of a usual magnetoexciton.

The preexponential factor in the wave function of a positively (negatively) charged particle is a polynomial in the complex coordinate  $z$  ( $\bar{z}$ ),  $z = x + iy$ . We introduce for anyons the Jacobi coordinates  $z = z_1 - z_2$  and  $z_0 = (z_1 + z_2)/2$ , and relate  $z_0$  to a hole coordinate,  $z_3$ , by the usual exciton transformation. The new coordinates are  $z$ ,  $\zeta = z_0 - z_3$ , and  $Z = (z_0 + z_3)/2$ , and the Halperin pseudo wave functions<sup>9</sup> of the three noninteracting particles with a momentum  $\mathbf{K}$  are

$$\Psi_{\mathbf{K}n}(z_1, z_2, z_3 | \alpha) = B_n(\alpha) \exp\{i\mathbf{K}\mathbf{R} + i(\rho_x Y - \rho_y X)/2\} \\ \times \exp\{-(1/4)(\rho - \kappa)^2\} |z|^\alpha \bar{z}^m \exp(-|z|^2/16).$$

The vectors  $\mathbf{R}$ ,  $\rho$ , and  $\mathbf{r}$  correspond to complex coordinates  $Z$ ,  $\zeta$ , and  $z$ ;  $\kappa = \hat{z} \times \mathbf{K}$ ,  $\hat{z}$  is a unit vector in the direction perpendicular to the confinement plane,  $-\kappa$  is the dipole moment of the exciton, and  $B_n(\alpha)$  is a normalization factor. The quantum number  $n \geq 0$  is the relative angular momentum of anyons. The Coulomb interaction may be written as  $V = V_{\alpha\alpha} + V_{ah}$ . The first term is diagonal in  $n$ :

$$\langle n_1 | V_{\alpha\alpha} | n_2 \rangle = \delta_{n_1 n_2} \frac{\Gamma(n_1 + \alpha + 1/2)}{8 \sqrt{2} \Gamma(n_1 + \alpha + 1)}. \quad (1)$$

When  $m = |n_1 - n_2| = \text{odd}$ , all  $\langle n_1 | V_{ah} | n_2 \rangle = 0$ , which ensures the correct interchange symmetry; only even  $n$  have a physical meaning. For even  $m$  we have

$$\langle n_1 | V_{ah} | n_2 \rangle \\ = - \left\{ \Gamma\left(\frac{n_1 + n_2 + m}{2} + \alpha + 1\right) / 2^{m/2} m! [\Gamma(n_1 + \alpha + 1) \Gamma(n_2 + \alpha + 1)]^{1/2} \right\} \\ \times \int_0^\infty q^m \exp(-q^2/2 - qh) J_m(Kq) \Phi\left(\frac{n_1 + n_2 + m}{2} + \alpha + 1, m + 1; -q^2/2\right) dq, \quad (2)$$

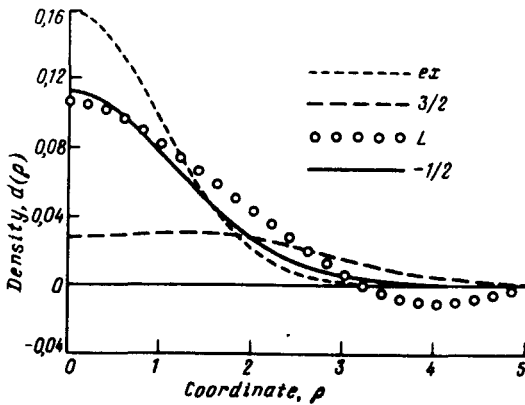


FIG. 1. Electron density,  $d(\rho)$ , for the ground state of a free magnetoexciton ( $ex$ ), an exciton in the presence of the IQL with  $\nu=1/3$  ( $L$ ), and for anyon excitons with statistical charges of anyons  $\alpha = -1/2$ , and  $3/2$ ;  $K=0$ ,  $h=0$ . The last curve also describes the first excited state for  $\alpha = -1/2$ .

where  $\Phi$  is a confluent hypergeometric function. For  $\alpha = -1/2$  the integral  $\langle n | V_{\alpha\alpha} | n \rangle$  diverges logarithmically at  $n=0$ . This is the price one pays for using the oversimplified model,  $N=2$ . We use a cutoff  $V_{\alpha\alpha} = 1/4(r^2 + \alpha^2)^{1/2}$ . For the hard-core model,  $\alpha=3/2$ , all integrals are regular.

For  $K=0$  only diagonal matrix elements survive, and Eqs. (1) and (2) give the exact solution of the three-particle problem.

Figure 1 shows the effect of statistics on the distribution of electron density,  $d(\rho)$ , near a hole in an AE for  $K=n=0$ . The functions  $d(\rho)$  for a free exciton,  $d_{ex}(\rho) = \exp(-r^2/2)/2\pi$ , and an exciton in the presence of an IQL ( $\nu=1/3$ ,  $h=0$ ) are also shown. In the last case the excess density,  $d_L(\rho)$ , is plotted.<sup>8</sup> The most striking property of  $d_L$  is a considerable increase in the spread of the density compared with  $d_{ex}$ , which is caused by the Pauli exclusion principle. This property is reproduced by the anyon model, primarily because of the increase in the magnetic length,  $l^* > l$ . The curve  $d_{-1/2}(\rho)$  is in reasonable quantitative agreement with  $d_L$ ; a realistic comparison may be done only for odd  $q$ . The first excited state of an  $\alpha = -1/2$  exciton coincides with the ground state of an  $\alpha = 3/2$  exciton. The latter curve, which has a flat minimum at  $\rho=0$ , closely resembles the distribution of the electronic density of an exciton against the background of an IQL,  $\nu=1/3$  (Fig. 2, curve 3 in Ref. 8). This similarity in  $d(\rho)$  for the two lowest states found by simulations and for AE strongly suggests that when  $h$  is small, the hard-core constraint for quasiparticles<sup>12,13</sup> is violated in an exciton by the attractive field of a hole.

There are several distinctive features of AEs caused by CF. If  $K=0$ , and  $h$  increases,  $\Psi_{Kn}$  remain exact eigenfunctions, but the level arrangement is changed; the larger is  $h$ , the higher is the value of  $n$  for the ground level, and the wider is the density distribution,  $d(\rho)$ , for it. The  $n=0$  and  $n=2$  levels interchange at  $h_{cr} \approx 1.66$  (for  $a=1$ ). At this point  $d(\rho)$  for the ground state changes abruptly from  $\alpha = -1/2$  for  $\alpha=3/2$  curve in Fig. 1. Since the  $K$ -dependence of the diagonal matrix elements,

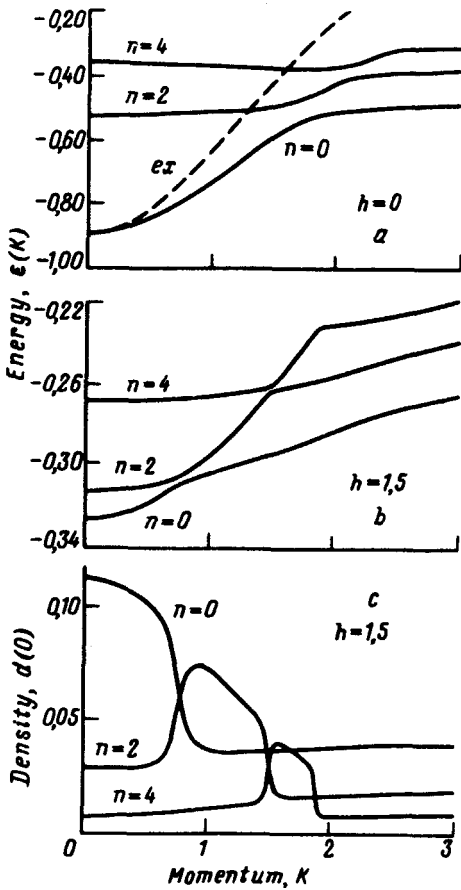


FIG. 2. Dispersion law,  $\epsilon(K)$ , for the three lowest spectrum branches;  $\alpha = -1/2$ . When  $h$  increases, anticrossings become narrower, and  $\epsilon(K)$  for the ground state flatter. The curve for a free exciton (ex) is shifted to facilitate a comparison with  $n=0$  curve.

$\langle n | V | n \rangle$ , increases with decreasing  $n$ , the energy levels converge at different values of  $K$ . At  $K=0$ , nondiagonal elements of  $V$  vanish and the level rearrangement shows the patterns of the level crossing, while at  $K \neq 0$  it shows the patterns of the level anticrossing. This behavior is seen in Fig. 2, where dispersion laws,  $\epsilon(K)$ , and densities,  $d(0)$ , at  $\rho=0$  are shown. While  $\epsilon(K)$  are always monotonic for the ground state, and change smoothly for excited states,  $d(0)$  show dramatic changes near level anticrossings. The humps seen in Fig. 2c reflect a transfer of  $n=0$  component between wave functions of adjacent states. Dispersion is strongly suppressed as compared with a usual exciton (Fig. 2a), since in the  $K \rightarrow \infty$  limit only one anyon moves away from the hole, while another remains in a bound state and makes an ion with the hole (the existence of anyon ions was established in Ref. 5c, which is supported by more recent calculations.<sup>16</sup> If  $h > h_{cr}$ , the dispersion in the ground state is even more suppressed. The abrupt change in the ground state with increasing  $h$  and the suppression of the dispersion are in agreement with the patterns found by simulations.<sup>5,8</sup>

At  $K=0$ , the level intersections have important implications for optical transitions. The matrix elements for them are

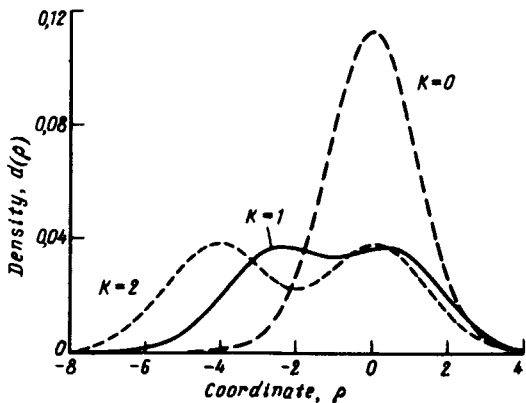


FIG. 3. Distribution of the electron density in the ground state of an anyon exciton along the direction  $\rho \parallel \mathbf{K}$ ;  $h = 1.5$ .

$$M_n(\alpha) \propto \int \Psi_{0n}(z_1, z_2, z_3 | \alpha) \delta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) d\mathbf{r}_1 d\mathbf{r}_2.$$

Here  $\delta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$  is a  $\delta$ -shape function of  $r_{13}$  and  $r_{23}$  with a width of about 1, in units of  $l$ . After the angular integration over  $\mathbf{r}$ , only the  $M_n(\alpha)$  with  $n=0$  survive. Therefore, exciton transitions are allowed in the emission at  $T=0$ , i.e., from the ground state, only if  $h < h_{cr}$ . This result is in agreement with the numerical data<sup>5,8</sup> which show that only weak transitions assisted by magnetorotons<sup>17</sup> (MR) are allowed at  $h > h_{cr}$ .

The effect of the CF becomes even more spectacular when the ground state density,  $d(\rho)$ , is plotted for  $K \neq 0$ . In Fig. 3,  $d(\rho)$  is shown along the symmetry line,  $\rho \parallel \mathbf{K}$ ; it is symmetric with respect to  $\rho = -K$  for all  $K$ . When  $K$  increases, a single-humped distribution changes into a camel-back type. The right-hand part, centered near  $\rho = 0$ , corresponds to the ground state of the ion, and the left-hand part corresponds to a free anyon. When  $K \rightarrow \infty$ , the separation between maxima approaches  $K/|e^*| = 2K$ , and the electron density distribution in each wing approaches  $d(\rho) = \exp(-\rho^2/4)/8\pi$ , which describes the shape of both a free anyon and an ion. In the same limit the energy is  $\varepsilon(K) \approx \sqrt{\pi}/4 - 1/8K$ ; the second term is the same as for MR's.

We have concentrated on the region of small  $h$ , the least favorable for the AE model, since the comparison with numerical data is available only for it.

The success of the model is nonetheless impressive. However, there are two peculiarities of the exciton ground state found by simulations<sup>5,8</sup> which the simple AE model does not describe: (i) at  $h=0$  the function  $d(\rho)$  is of a single-hump type and depends only slightly on  $K$ , and (ii) at  $h=h_{cr}$  the minimum of  $\varepsilon(K)$  shifts from  $K=0$  to  $K_{min} \neq 0$ , where  $K_{min}$  is close to the roton minimum.<sup>17</sup> These facts unambiguously signal that the AE-MR coupling should be invoked. The importance of this coupling is implied by Fig. 3. The separation between an anyon and ion increases with  $K$ , and an AE produces a strong Coulomb field which acts on the IQL and which is known to be unstable when an external charge about  $e^*$  approaches it.<sup>18</sup>

A simple idea that the AE creates a virtual MR and makes a bound state with it describes the behavior at  $h \approx 0$  very well.<sup>8</sup> When  $h$  increases, the exciton dispersion

curves flatten and converge (Fig. 2b). As a result, a pseudo-Jahn–Teller effect sets in, and a new ground state with a broken symmetry and the momentum  $K_{\min}$  close to the MR minimum appears. In this state the shape of  $d(\rho)$  found by simulations (Ref. 5) is reminiscent of the curve  $K=1$  (Fig. 3). The AE–MR coupling manifests itself also in the oscillatory behavior of  $d_L(\rho)$  (Fig. 1) (similarly to the screening of charged impurities).<sup>17,18</sup>

In conclusion, we have proposed a model of anyon excitons for the description of optical properties of IQL's. We showed that the model reflects distinctive features of excitons in the presence of an IQL, and establishes properties of excitons, for which their coupling to magnetorotons is of crucial importance.

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<sup>1</sup>E. I. Rashba and M. E. Portnoi, *Bull. Amer. Phys. Soc.* **38**, 136 (1993).

<sup>2</sup>I. V. Kukushkin *et al.*, *Surf. Sci.* **263**, 30 (1992); B. B. Goldberg *et al.*, *ibid.* **263**, 9 (1992); A. J. Turberfield *et al.*, **263**, 1 (1992); E. M. Goldys *et al.*, *Phys. Rev. B* **46**, 7957 (1992), and references cited there.

<sup>3</sup>R. B. Laughlin, *Phys. Rev. Lett.* **50**, 13 (1983).

<sup>4</sup>D. C. Tsui, H. L. Stormer, and A. C. Gossard, *Phys. Rev. Lett.* **48**, 1559 (1982).

<sup>5</sup>V. M. Apal'kov and E. I. Rashba, (a) *Pis'ma Zh. Eksp. Teor. Fiz.* **53**, 420 (1991), (b) *ibid.* **54**, 160 (1991), (c) *ibid.* **55**, 38 (1992).

<sup>6</sup>B.-S. Wang, J. L. Birman, and Z.-B. Su, *Phys. Rev. Lett.* **68**, 1605 (1992).

<sup>7</sup>A. H. MacDonald, E. H. Rezayi, and D. Keller, *Phys. Rev. Lett.* **68**, 1939 (1992).

<sup>8</sup>V. M. Apalkov and E. I. Rashba, *Phys. Rev. B.* **46**, 1628 (1992).

<sup>9</sup>B. I. Halperin, *Phys. Rev. Lett.* **52**, 1583 (1984).

<sup>10</sup>D. Arovas, J. R. Schrieffer, and F. Wilczek, *Phys. Rev. Lett.* **53**, 722 (1984).

<sup>11</sup>F. Wilczek, *Phys. Rev. Lett.* **49**, 957 (1982).

<sup>12</sup>Y.-S. Wu, *Phys. Rev. Lett.* **52**, 2103 (1984); **53**, 111 (1984).

<sup>13</sup>S. He, X.-C. Xie, and F.-C. Zhang, *Phys. Rev. Lett.* **68**, 3460 (1992).

<sup>14</sup>G. Dav and J. K. Jain, *Phys. Rev. Lett.* **69**, 2843 (1992).

<sup>15</sup>R. G. Clark *et al.*, *Phys. Rev. Lett.* **60**, 1747 (1988); S. Kivelson, *ibid.* **65**, 3369 (1990); S. W. Hwang *et al.*, *Phys. Rev. B.* **44**, 13497 (1991); F. G. Pinkus and A. L. Efros, *Phys. Rev. B*, in press.

<sup>16</sup>X. M. Chen and J. J. Quinn, *Phys. Rev. Lett.* **70**, 2130 (1993).

<sup>17</sup>S. M. Girvin, A. H. MacDonald, and P. M. Platzman, *Phys. Rev.* **33**, 2481 (1986).

<sup>18</sup>C. Zhang *et al.*, *Phys. Rev.* **32**, 6920 (1985); E. H. Rezayi and F. D. M. Haldane, *Phys. Rev.* **32**, 6924 (1985).

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