Quasiperiodic and chaotic Langmuir cavitons

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The evolution of 3D packets of Langmuir waves is analyzed. The packets are described by the Zakharov equation with several additional terms to simulate Landau excitation and damping. The damping stabilizes the packet at only a small amplitude and at length scales much larger than the Debye length. A caviton with a quasiperiodic or stochastic electric field confined in it is formed. The transition to the stochastic regime with increasing excitation intensity occurs via period-doubling bifurcations.

The collapse of Langmuir waves[1] is a fundamental concept in the theory of strong plasma turbulence. Despite numerous studies along this line, the final stage of the collapse remains unresolved. Several hypotheses involve the formation of localized, long-lived zones in the plasma in which energy is dissipated (a "funnel" effect[2] or "dissipative" solitons"[3]). In a recent experiment[4] long-lived dips have been observed in the plasma density ("cavitons"), in which there has been an absorption of the energy of the plasma waves generated continuously by an electron beam. A model of a Langmuir caviton based on the Zakharov equation[1] with several additional terms to describe the generation at large length scales and a Landau damping was proposed in Ref. 3. Steady-state cavitons with monochromatic modulated waves were found. These formations were called "dissipative" solitons" there, since they were analogs of dissipative structures in chemistry and biology.[5] This article,[3] however, did not attract much attention, since just how stable these formations were was not clear. Moreover, if one assumes that cavitons play a definite role in strong plasma turbulence, then the turbulence spectrum must be a line spectrum. However, experiments on artificial heating of the ionosphere have revealed not only a line spectrum[6] but also a broad-band spectrum[7] of plasma waves. In addition, some numerical simulations[8] revealed the appearance of several well-defined peaks in the spectrum. In the present letter we resolve several contradictions of the "caviton" theory of Langmuir turbulence: We determine the range of stability of steady-state cavitons, and we find cavitons of a new type, whose modulated waves are quasiperiodic or even stochastic and broad-band.

Assuming spherical symmetry, we write the Zakharov equation[1] as follows:

$$i\partial E/\partial t+RE+|E|^2E=-i\{\alpha RE+R^2E\},$$

$$R=(\partial/\partial r)r^{-2}(\partial/\partial r)r^2, \quad (1)$$

where (following Ref. 3) we have introduced on the right side several additional terms to simulate the excitation and absorption of waves. The coefficient α characterizes the

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excitation intensity. Equation (1) is written in dimensionless form, with the time t being normalized to the reciprocal of the plasma frequency ω_p, the radius r to a quantity on the order of the Debye length r_D, and the field E to the quantity $\sqrt{16\pi p}$, where p is the plasma pressure.

Expanding the solution of (1) in a series in eigenfunctions of the operator

$$\varphi_k(r)=r^{-1/2}J_{3/2}(kr), \tag{2}$$

we find that in the linear approximation the additional terms on the right side of (1) correspond to the presence of a growth rate

$$\Gamma_k=\alpha k^2-k^4. \tag{3}$$

The plasma waves in the long-wave region $0<k^2\leqslant\alpha$ are thus unstable and bring about an influx of energy into the system. In the short-wave region, $k^2>\alpha$, on the other hand, plasma waves are rapidly absorbed. These effects cancel out each other and account for the existence of a soliton.

Equation (1) was solved numerically in the region $0\leqslant r\leqslant L$ under the boundary conditions $E(0,t)=E(L,t)=0$. Solutions of (1) were expanded in eigenfunctions (2), and then the system of ordinary differential equations was solved for the coefficients of the Fourier series, $E_n(t)$, by Runge–Kutta methods of fourth-order precision. The number of harmonics $E_n(t)$ was limited by $n\leqslant N$, where N was chosen large enough that this "truncation" procedure did not affect the results. In the calculations, N was varied over the range 10–20, and the size of the computation region was fixed at $L=25$. Small-amplitude perturbations were adopted as initial conditions. The results were asymptotically $(t\to\infty)$ independent of the initial conditions and were a self-consistent solution of (1) determined exclusively by the parameter α.

For $\alpha<\alpha_0=0.0316$, any initial perturbations in (1) die out as time elapses, and the solution asymptotically approaches a zero solution $E=0$.

At $\alpha>\alpha_0$, Eq. (1) has spatially localized solutions

$$E(r,t)=A(r)\exp\{i\lambda t\}, \tag{4}$$

whose amplitude and frequency depend on the parameter α. These solutions correspond to cavitons with monochromatic modulated waves: steady-state density cavities $\sim|E|^2$ in which plasma waves of frequency $\omega_p(1-\lambda)$ are trapped. Solutions of the type in (4) were studied in detail in Ref. 3. In the range $\alpha_0<\alpha<\alpha_1=0.049$ any solution of (1) asymptotically in time approaches a steady-state caviton of the type in (4).

With increasing value $\alpha>\alpha_1$, the steady-state caviton in (4) becomes unstable. The resulting solution is quasiperiodic with two independent frequencies, f_0 and f_1. The frequency f_0 is close to the corresponding frequency λ of the steady-state solution, while the amplitude fluctuates around $A(r)$. As can be seen from Fig. 1, the spectrum of the integral

$$I(t)=\int_0^L |E|^2 r^2 dr, \tag{5}$$

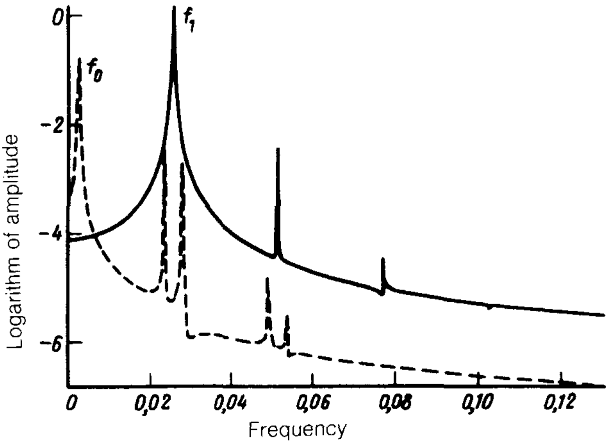

FIG. 1. Fourier spectra of a caviton with quasiperiodic modulated waves for $\alpha = 0.052$. Solid line—The $I(t)$ spectrum; dashed—$E_1(t)$.

which is proportional to the total energy of a caviton with quasiperiodic modulated waves, consists of frequencies which are multiples of f_1. The spectrum of the field E itself is more complicated. The spectrum of the first harmonic, $E_1(t)$, for example, consists of f_0 and linear combinations $mf_1 \pm f_0$ (Fig. 1).

In terms of the theory of dynamic systems,[9] a limiting cycle arises in the phase space of the system at the value $\alpha = \alpha_0$. This cycle remains stable over the interval $\alpha_0 < \alpha < \alpha_1$. At $\alpha > \alpha_1$, the cycle is destroyed, and an attractive 2D torus is created. As α increases further, the structure of the 2-torus type remains unchanged until stochastic states arise.

As α is increased over the interval $\alpha_1 < \alpha < \alpha_\infty \approx 0.05523$, we observe a sequence of period-doubling bifurcations of the oscillations at the second frequency, f_1: $f_1 \to f_1/2 \to f_1/4$ ... . The overall structure of the $I(t)$ and $E_1(t)$ spectra remains the same as described above. The first four points of a period-doubling bifurcation are $\alpha_2 \approx 0.0531$, $\alpha_3 \approx 0.0546$, $\alpha_4 \approx 0.05505$, and $\alpha_5 \approx 0.05516$.

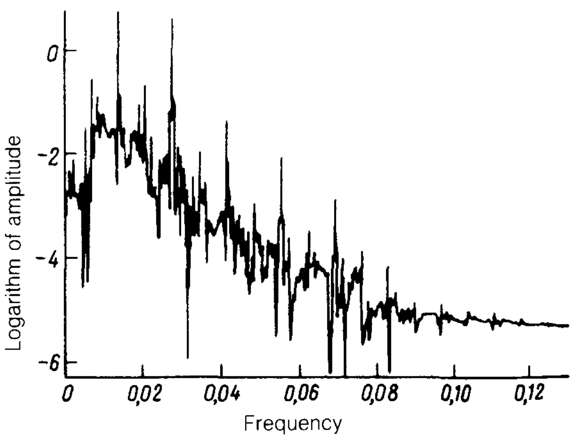

FIG. 2. Fourier spectrum of the integral $I(t)$ for a caviton with chaotic modulated waves, for $\alpha = 0.05525$.

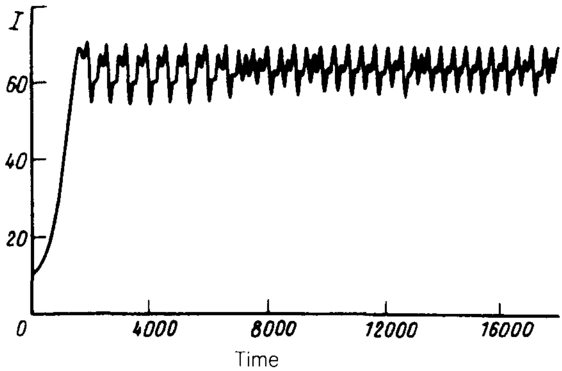

FIG. 3. Evolution of the integral $I(t)$ for a caviton with a chaotic filling, for $\alpha=0.05525$.

In the narrow interval $\alpha_\infty < \alpha < \alpha_1' \approx 0.05528$ we observe random oscillations of the field with a wide-band spectrum (Fig. 2). Figure 3 shows the time evolution of the integral $I(t)$ in the stochastic region. Fluctuations of $I(t)$ occur around the value $I=I_0 \approx 68.52$, which corresponds to the steady-state solution of the type in (4) for the corresponding value of α.

As $\alpha > \alpha_1'$ is increased further, we again observe a growth of quasiperiodic oscillations with two independent frequencies. Later, at $\alpha=\alpha_2' \approx 0.0555$, there is a period-doubling bifurcation at the second frequency, f_1, then, at $\alpha=\alpha_\infty' \approx 0.0557636$, we observe an abrupt transition to a chaotic state, which persists at all $\alpha > \alpha_\infty'$. In contrast with the first stochastic band (Fig. 2), the oscillation spectrum in the region $\alpha > \alpha_\infty'$ is essentially continuous, without any clearly expressed resonant peaks.

Qualitatively, the overall picture of the onset of quasiperiodic states and the transition to chaos in Eq. (1) generally reproduces the corresponding processes in the Ginzburg–Landau equation.[10] The only distinction is the second stochastic region, $\alpha > \alpha_\infty'$, which turns out to be unbounded in terms of the bifurcation parameter α. The apparent reason for this result is a breakdown in Eq. (1), which describes non-one-dimensional processes.

We note in conclusion that the length scales of the cavitons which arise are much larger than r_D, and the frequency scales of both the quasiperiodic and chaotic regimes are close to the plasma frequency (most of the spectrum is at frequencies well below one; Figs. 1 and 2). In addition, the cavitons which arise are small-amplitude entities: $|E|^2 \ll 1$.

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