Slightly nonideal 2D Fermi gas in a magnetic field

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(Submitted 19 July 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 5, 349–352 (10 September 1993)

The properties of a slightly nonideal 2D Fermi gas in a magnetic field are analyzed by a Feynman-diagram technique. Expressions for the eigenenergy part of the one-particle Green's function are derived in up to second-order perturbation theory. Because the spectrum of the 2D particles is discrete in a magnetic field, there is no attenuation of the quasiparticles, and there is a splitting of seed Landau levels. The eigenenergy part has a pole at zero frequency. The maximum filling factor for each sublevel is $\nu_{\max}=1/2$.

Research on the thermodynamic properties of a system in a magnetic field, in particular, on oscillations of the magnetic susceptibility,[1] is based on certain ideas regarding the electron energy spectrum in a magnetic field H. The effect of the interparticle interaction on the de Haas–van Alphen effect was studied in the model of an isotropic Fermi liquid in Refs. 2 and 3. The results found there show that the quasiparticle spectrum in a weak magnetic field is the same as that of free electrons, whose mass was renormalized as a result of the interaction.

The properties of a system of 2D electrons in a magnetic field have attracted much interest, and one is led to ask whether the results of Refs. 2 and 3 apply to such a system. To answer this question we need to determine the nature of the spectrum of interacting 2D electrons in a weak magnetic field.

A slightly nonideal 3D Fermi gas is a system whose properties correspond completely to Landau's theory of a Fermi liquid.[4,5] In particular, the class of diagrams describing the properties of a Fermi liquid or a slightly nonideal gas was established in Ref. 5. In Ref. 6, on the other hand, the properties of a 3D system in a weak magnetic field were studied in the model of a slightly nonideal Fermi gas. It was shown there that the results are in complete agreement with the representation of the system as a Fermi liquid.

In the present letter we use an approximation corresponding to the approach taken in Ref. 5 to study the spectrum of a 2D electron gas in a magnetic field in the semiclassical case. We accordingly assume that the chemical potential of the system corresponds to a high-index Landau level and that the condition for a slight deviation from an ideal situation holds:

$$p_F|f_0|^2/\hbar\ll 1, \tag{1}$$

where p_F is the Fermi momentum, and f_0 is the amplitude for the scattering of "slow" particles by each other. In the Born approximation we have

$$|f_0| = \frac{mU_0}{4\hbar^2}\left(\frac{2\hbar}{\pi p_F}\right)^{1/2}, \quad U_0 = \int U(r)d^2x, \tag{2}$$

where m is the unrenormalized particle mass, and $U(r)$ is the short-range interaction potential. We ignore Zeeman splitting. There is a circumstance which needs to be emphasized here: Because of the degeneracy of Landau levels, it is necessary to use a temperature diagram technique. The transition to zero temperature is made (in order to determine the quasiparticle spectrum) by analytic continuation.[7] We will make use of some results from the theory of a slightly nonideal Fermi gas in the form presented in Ref. 7.

The correction of first-order perturbation theory to the eigenenergy part of Σ is

$$\Sigma^{(1)} = \frac{1}{2} n^{(0)}(\mu) U_0, \tag{3}$$

where $n^{(0)}(\mu)$ is the density of the 2D gas in the magnetic field as a function of the chemical potential μ. The simplest way to derive the correction of second order, Σ, in a weak magnetic field is to take the following approach: In the absence of a magnetic field, the corresponding expression is

$$\Sigma^{(2)}(i\omega_l, p) = U_0^2 \int \frac{d^2p_1\, d^2p_2\, d^2p_3}{(2\pi\hbar)^4} \delta(p+p_1-p_2-p_3) \times \left\{ \frac{n(p_1)[1-n(p_2)-n(p_3)]+n(p_2)n(p_3)}{i\omega_l+\mu+\epsilon(p_1)-\epsilon(p_2)-\epsilon(p_3)} - \mathrm{P}\frac{n(p_1)}{\epsilon(p)+\epsilon(p_1)-\epsilon(p_2)-\epsilon(p_3)} \right\}. \tag{4}$$

Here $n(p)$ is a Fermi distribution function, the symbol P means that we are to discard terms in which the denominator is zero, and we have $\omega_l = \pi T(2l+1)$, where T is the temperature. The further calculations reduce to the following. The δ-function in (4) should be expressed as a standard integral of an exponential function over the variable r, and then the integration should be carried out over all angles. As a result, we find the multiple integral

$$\int r dr\, J_0\left(\frac{pr}{\hbar}\right) J_0\left(\frac{p_1 r}{\hbar}\right) J_0\left(\frac{p_2 r}{\hbar}\right) J_0\left(\frac{p_3 r}{\hbar}\right) d\left(\frac{p_1^2}{2}\right) d\left(\frac{p_2^2}{2}\right) d\left(\frac{p_3^2}{2}\right). \tag{5}$$

The next step is to replace the integration over the variable $p_i^2/2$ by a summation over Landau levels by means of the substitution $p_i^2 = 2m\hbar\omega_c(N_i + 1/2)$, where N_i is the level index, and ω_c the cyclotron frequency.

We finally find the following expression for the second-order correction to the eigenenergy part in a weak magnetic field:

$$\Sigma^{(2)}(i\omega_l,N)=\frac{m^3U_0^2(\hbar\omega_c)^3}{(2\pi)^2\hbar^6}\sum_{N_1,N_2,N_3} I(N,N_1,N_2,N_3)$$

$$\times\left\{\frac{n(N_1)[1-n(N_2)][1-n(N_3)]+[1-n(N_1)]n(N_2)n(N_3)}{i\omega+\mu+\epsilon(N_1)-\epsilon(N_2)-\epsilon(N_3)}\right.$$

$$\left.-\mathrm{P}\frac{n(N_1)}{\epsilon(N)+\epsilon(N_1)-\epsilon(N_2)-\epsilon(N_3)}\right\}. \qquad (6)$$

The quantity $I(N, N_1, N_2, N_3)$ in (6) is an integral of Bessel functions:

$$I(N,N_1,N_2,N_3)=\int rdrJ_0(\alpha r)J_0(\alpha_1 r)J_0(\alpha_2 r)J_0(\alpha_3 r), \qquad (7)$$

where $\alpha_i=[2c\hbar/eH(N_i+1/2)]^{1/2}$. This integral has been evaluated[8]; the result is

$$I=\frac{1}{\pi^2}\begin{Bmatrix}\frac{1}{b}K\left(\frac{a}{b}\right), & b>a, \\ \frac{1}{a}K\left(\frac{b}{a}\right), & a>b\end{Bmatrix}, \qquad (8)$$

where

$$a^2=\alpha\alpha_1\alpha_2\alpha_3,\quad 16b^2=[(\alpha+\alpha_1)^2-(\alpha_2-\alpha_3)^2][(\alpha_2+\alpha_3)^2-(\alpha-\alpha_1)^2],$$

and $K(k)$ is the complete elliptic integral of the first kind.

When we take the limit $T=0$, the first term in curly brackets in (4) (in the absence of a magnetic field) describes, in particular, an attenuation of quasiparticles. The situation changes fundamentally for a 2D system in a magnetic field, because the spectrum is discrete. A summation carried out over the numbers N_i in the first term in curly brackets in (6), gives rise to terms for which the condition $N_2+N_3=N+N_1$ $[\epsilon(N)=\hbar\omega_c(N+1/2)]$ holds. In other words, the following expression arises in $\Sigma^{(2)}$:

$$\frac{A_N}{i\omega_l+\mu-\epsilon(N)}. \qquad (9)$$

In an analytic continuation in the $T=0$ case, the imaginary frequencies $i\omega_l$ should be replaced by the real frequency ω. Now the presence of a term of type (9) in $\Sigma^{(2)}$ corresponds to a resonance in a system with a discrete spectrum. Since the resonant terms are the leading terms in the case of a weak interaction, we find that the one-particle Green's function takes the following form at $T=0$:

$$G(\omega,N)=\frac{\omega+\mu-\epsilon(N)}{[\omega+\mu-\epsilon(N)]^2-A_N}. \qquad (10)$$

It follows that the discrete spectrum of quasiparticles in this approximation is

$$E(N)=\epsilon(N)\pm A_N^{1/2}. \qquad (11)$$

In other words, the Landau levels are split because of resonance effects.

In our approximation of a slightly nonideal system, each sublevel corresponds to a maximum filling factor $\nu_{\max}=1/2$; i.e., the total number of states in the given Landau level is divided equally between the two sublevels, which are separated by an energy gap.

Some fairly simple calculations of the quantity I [which make use of expression (8)] lead to the following results. For $N=N_0$ ($N_0=\mu/\hbar\omega_c$), we find

$$A_{N_0}=\frac{\alpha}{\pi^4}\nu(1-\nu)\left(\frac{mU_0}{\hbar^2}\right)^2\frac{(\hbar\omega_c)^3}{\mu}\ln N_0, \tag{12}$$

where ν is the filling factor of the Landau level corresponding to the chemical potential, and α is a numerical coefficient on the order of one. In the region $1\ll|N-N_0|\ll N_0$ we find

$$A_N=\frac{1}{8\pi^4}[\epsilon(N)-\epsilon(N_0)]^2\left(\frac{mU_0}{\hbar^2}\right)\frac{\hbar\omega_c}{\mu}\ln\left(\frac{4N_0}{|N-N_0|}\right). \tag{13}$$

From these results we can draw the following conclusions regarding the properties of a slightly nonideal 2D system of electrons in a weak magnetic field. First, resonances between different states of the system arise because of the discrete nature of the spectrum. For a particle whose energy is above the chemical potential, a resonant state corresponds to five particles and one hole. As a result, the seed Landau levels split, but the spectrum remains discrete. This circumstance apparently confirms the heuristic arguments, advanced in Ref. 9, regarding the nature of the spectrum of this system.

Second, the radical change in the systematics of the quasiparticle levels, which results from the splitting of the Landau levels, casts doubt on the idea that the numbers of particles and quasiparticle are equal—an assertion which is of fundamental importance in the derivation of the theory of a Fermi liquid.

Third, the pole in the eigenenergy part $\Sigma^{(2)}$ at the frequency $\omega=0$ for the level corresponding to the chemical potential implies that the properties of the system are similar to those of a "marginal" Fermi liquid.[10]

There is an extremely important circumstance to be noted here: A state which resonates with a particle above the Fermi level consists of two particles and a hole. To what extent can such a state be regarded as a particle and a magnetic exciton which are interacting with each other? The exciton is apparently playing the role in this system which is played by zero sound in the theory of a classical Fermi liquid. This very important point, however, lies outside the scope of the present letter.

Our last comment is that a change in the systematics of the quasiparticle spectrum in a slightly nonideal 2D system in a magnetic field due to a splitting of Landau levels contradicts an assumption which was made in Ref. 11 in an analysis of the de Haas–van Alphen effect.

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Translated by D. Parsons