Angular distribution of ultracold neutrons produced by scattering of cold neutrons in superfluid ^{4}He

S. K. Lamoreaux
University of Washington Physics Department, FM-15, Seattle, WA 98195, USA

R. Golub
Hahn-Meitner Institut, Glienicker Str. 100, D-14109 Berlin-Wannsee, Germany

(Submitted 19 October 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 10, 844–847 (25 November 1993)

The angular distribution of ultracold neutrons (UCN) produced by the inelastic scattering of cold (8.9 Å) neutrons in superfluid ^{4}He has been calculated. These calculations have shown that this distribution is isotropic at zero UCN energy, while at the limiting velocity of UCN (8m/s) there is a 6% asymmetry in the forward-versus-backward directed UCN production rate. Although this is a relatively weak effect, it should be taken into account in accurate Monte Carlo simulations of proposed superfluid ^{4}He UCN sources and in those that exist now.

The production of ultracold neutrons (UCN) by scattering 8.9-Å neutrons to zero energy by emission of a single excitation in superfluid ^{4}He, as proposed by Golub and Pendlebury,[1,2] has received some experimental attention. The primary features of the theoretical treatment of the system have been verified.[3,4]

A heretofore unrecognized feature of the system, as will be theoretically demonstrated below, is that the production rate of UCN is not isotropic. Although the effect is only on the order of 10% at the highest UCN energies, it is important to take it into account in the Monte Carlo studies of the existing[5] and proposed UCN sources based on the scattering in superfluid ^{4}He ("superthermal" sources).

We consider first the kinematics of the UCN production. Incident neutrons with a "critical momentum" (where the free neutron dispersion curve $E=\hbar^2k_i^2/2m_n$ crosses the Landau–Feynman dispersion curve for the elementary excitations in superfluid ^{4}He) can be inelastically scattered to near-rest by emission of a single phonon. The critical momentum q^* corresponds to a wavelength of ≈ 8.917 Å and an energy of 11 K. Both energy and momentum are conserved in this process:

$$\mathbf{q}=\mathbf{k}_i-\mathbf{k}_f, \tag{1}$$

$$\hbar^2k_i^2/2m=\hbar^2k_u^2/2m+E(q), \tag{2}$$

where k_i is the incident neutron wave number, k_u is the final (UCN) neutron wave number, and $E(q)$ is the elementary excitation dispersion relation. For a fixed initial momentum k_i and a given final UCN momentum k_u, the direction θ of the UCN momentum relative to the incident neutron momentum and q are uniquely determined.

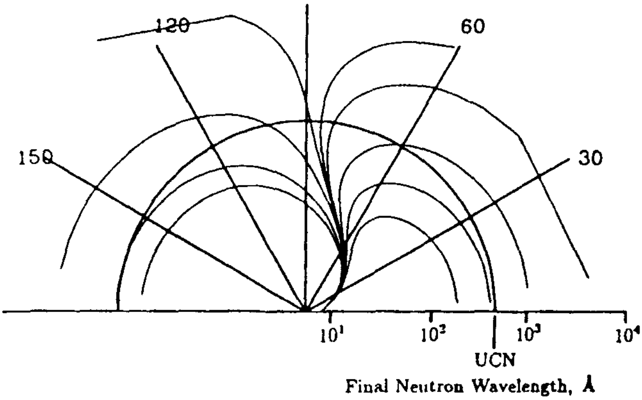

FIG. 1. Polar plot of kinematically allowed UCN wavelengths produced by down-scattering from a specified incident cold neutron wavelength. The incident wavelengths (from left to right) near the origin are as follows: 8.8, 8.85, 8.9, 8.916, 8.920, 8.925, 9.95, 9, and 9.1 Å. The semicircle at 500 Å indicates the threshold of UCN production.

Figure 1 is the result of a numerical calculation of $\lambda_u = 2\pi/|\mathbf{k}_u|$ and θ, with $k_i \approx q^*$, and using $E(q)$ as parameterized by Maris.[6] This plot should be compared with that in Fig. 2 of Ref. 7, where k_u is plotted as a function of k_i and θ. We are interested in the curves for the incident wave number $k_i \approx 0.68$ Å^{-1}, for which $k_u \approx 0$ at $\theta = 90°$. In our case, however, we have plotted λ_u which shows more readily the angular dependence of the kinematically allowed scattering to near-zero final energy. Figure 1 indicates that UCN can be produced at any θ, while the complexity of the kinematics for $k_u \to 0$ simply is not evident on the scale at which Fig. 2 of Ref. 7 is drawn.

We next consider the dynamics of UCN production. Following Ref. 7, the cross section for the production of neutrons in a given final state is given by the Born approximation:

$$\sigma = \frac{2a^2}{k_i} \int S(\mathbf{q}) \delta\left(k_u^2 - k_i^2 + \frac{2m}{\hbar^2} E(q)\right) d^3k_u, \tag{3}$$

where $a \approx 1$ barn is the ^{4}He-n coherent scattering length, $S(q)[=S(q)$ for a liquid] is the structure function, and $d^3k_u = k_u^2 dk_u d\Omega$. We are interested in the production rate to a given angle for a fixed incident neutron momentum. Following Cohen and Feynman,[7] Eq. (3) can be integrated over k_u first to give

$$\frac{d\sigma}{d\Omega} = \frac{2a^2}{k_i} S(q) \frac{k_u^2}{|f'(k_u)|}, \tag{4}$$

where

$$f(k_u) = k_u^2 - k_i^2 + \frac{2m}{\hbar^2} E(q), \tag{5}$$

$$f'(k_u)=2\left(k_u+\frac{m}{\hbar^2}E'(q)\frac{\partial q}{\partial k_u}\right), \tag{6}$$

and m is the neutron mass. From Eq. (1) we have

$$q^2=k_u^2+k_i^2-2k_uk_i\cos\theta, \tag{7}$$

which can be differentiated to yield

$$qdq=(k_u-k_i\cos\theta)dk_u \tag{8}$$

or

$$\frac{\partial q}{\partial k_u}=\frac{k_u}{q}\left(1-\frac{k_i}{k_u}\cos\theta\right). \tag{9}$$

We thus obtain[7]

$$\frac{d\sigma}{d\Omega}=\frac{a^2k_uS(q)}{k_i\left|\left(1+\frac{mE'(q)}{hq}\left(1-\frac{k_i}{k_u}\cos\theta\right)\right)\right|}, \tag{10}$$

where k_i is determined by $f(k_u)=0$.

Since the incident neutron beam is an incoherent mixture of the plane wave states [specified by a spectral density $d\Phi(k_i)/dk_i$ in units of neutrons/$\text{Å}^{-1}\,\text{s}\cdot\text{cm}^2$], to calculate the differential UCN production rate at a specified k_u in the range dk_u, we must multiply the incident spectral density, using Eqs. (1) and (2), by the kinematic factor which is

$$\left|\frac{\partial k_i}{\partial k_f}\right|_\theta=\frac{\left|k_u+\frac{mE'(q)}{\hbar^2 q}(k_u-k_i\cos\theta)\right|}{\left|-k_i+\frac{mE'(q)}{\hbar^2 q}(k_i-k_u\cos\theta)\right|}. \tag{11}$$

The differential production rate is thus

$$\begin{aligned}\frac{d^2P}{d\Omega dk_u}&=Na^2S(q)\frac{d\Phi(k_i)}{dk_i}\left|\frac{\partial k_i}{\partial k_u}\right|_\theta\frac{d\sigma}{d\Omega}\\&=Na^2\frac{d\Phi(k_i)}{dk_i}\frac{k_u^2}{k_i^2}\frac{1}{\left|-1+\frac{mE'(q)}{\hbar^2 q}\left(1-\frac{k_u}{k_i}\cos\theta\right)\right|},\end{aligned} \tag{12}$$

where N is the ^{4}He density. The same result can be obtained by integrating first Eq. (3) over k_i.

Since only those incident neutrons in a very narrow range of momenta near q^* are scattered to UCN, we have $k_i, q\approx q^*$. Defining the group velocity and the critical velocity, respectively, as

$$v_g=E'(q^*)/\hbar, \tag{13}$$

$$v_n^* = \hbar q^*/m, \tag{14}$$

and integrating over the azimuthal angle, we find the differential production rate, in the limit $k_u \ll k_i, q^*$, keeping all terms to first order in k_u/q^*,

$$\frac{d^2P}{d\cos\theta\, dk_u} \approx 2\pi N a^2 S(q^*) \frac{d\Phi(q^*)}{dk_i} \left(\frac{k_u}{q^*}\right)^2 \frac{v_n^*}{v_n^* - v_g} \left(1 + \left(\frac{k_u}{q^*}\right) \frac{v_n^*}{v_n^* - v_g} \cos\theta\right). \tag{15}$$

Here we have assumed the phonon dispersion to be linear, and we also assumed that $S(q)$ and $d\Phi(k_i)/dk_i$ are approximately constant with $k_i, q = q^*$. From the parametrization of the dispersion curve given in Ref. 6 we find

$$\alpha = \frac{v_n^*}{v_n^* - v_g} = 1.437. \tag{16}$$

The integration of Eq. (15) over $\theta = 0$ to π yields the same total UCN production rate as was obtained previously.[1,2]

If we consider Eq. (12) directly without assuming a linear phonon dispersion, we find (numerically)

$$\frac{d^2P}{d\cos\theta dk_u} \approx 2\pi N a^2 S(q^*) \frac{d\Phi(q^*)}{dk_i} (1.438) \left(\frac{k_u}{q^*}\right)^2 \left(1 + (1.56)\left(\frac{k_u}{q^*}\right)\cos\theta\right). \tag{17}$$

In this expression the extra angular dependence reflects the nonlinearity of the phonon dispersion relation.

For a UCN of wavelength 500 Å the fractional difference in the UCN production rate between $\theta = 0°$ and $\theta = 180°$ is approximately 6%. This asymmetry falls off as $1/\lambda_u$, the UCN wavelength.

In conclusion, we have shown that the angular distribution of UCN produced by inelastic scattering in superfluid ^{4}He is not isotropic. Although the effect is relatively small, taking it into account in the Monte Carlo modeling of existing and proposed sources is very important, particularly when a high-accuracy comparison between experimental results and theory is desirable.[8,9]

SKL was supported by NSF grant 9100719.

[1] R. Golub and J. M. Pendlebury, Phys. Lett. **53A**, 133 (1975).
[2] R. Golub and J. M. Pendlebury, Phys. Lett. **62A**, 337 (1977).
[3] R. Golub, C. Jewell, P. Ageron *et al.*, Z. Phys. **B51**, 187 (1983). A. I. Kilvington, R. Golub, W. Mampe, and P. Ageron, Phys. Lett. **125A**, 416 (1987).
[4] Ultracold Neutrons, R. Golub, D. J. Richardson, and S. K. Lamoreaux (Adam-Hilger, 1991).
[5] H. Yoshiki, K. Sakai, M. Ogura *et al.*, Phys. Rev. Lett. **68**, 1323 (1992).
[6] H. Maris, Rev. Mod. Phys. **49**, 341 (1977).
[7] M. Cohen and R. P. Feynman, Phys. Rev. **107**, 13 (1957).
[8] R. Golub and S. K. Lamoreaux, Phys. Rev. Lett. **70**, 517 (1993).
[9] H. Yoshiki *et al.*, Phys. Rev. Lett. **70**, 518 (1993).

Published in English in the original Russian journal. Reproduced here with the stylistic changes by the Translations Editor.