Percolation mechanism for vortex depinning in the resistive state of thin films of type-II superconductors

A. A. Snarskiĭ
Kiev Polytechnical Institute, 252056 Kiev, Ukraine

É. A. Pashitskiĭ
Institute of Physics, Ukrainian National Academy of Sciences, 252650 Kiev, Ukraine

A. M. Pal'ti and A. E. Morozovskiĭ
Institute of Metal Physics, Ukrainian National Academy of Sciences, 252680 Kiev, Ukraine

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A 2D percolation model successfully describes the depinning of vortices in thin films of type-II superconductors when there is a continuous distribution of pinning forces over a finite interval, and under certain other conditions. This model predicts a universal power-law current–voltage characteristic $E=(j-j_c)^\gamma$ ($\gamma\approx 1.3-1.6$) for the resistive state near the critical current j_c. This prediction agrees satisfactorily with experimental data on high-T_c superconductors. © *1995 American Institute of Physics.*

1. Some recent experimental studies of the resistive state of high-T_c superconductors,[1] including thin films,[1)] indicate a power-law nonlinearity of the initial regions of the current–voltage characteristic above the critical current:

$$U\propto(j-j_c)^\gamma. \tag{1}$$

This behavior stands in contrast with that of conventional (low-temperature) type-II superconductors, which typically have an exponential resistive current–voltage characteristic. In relatively weak magnetic fields $H\ll H_{c2}(T)$, where H_{c2} is the upper critical field, the numerical value of the exponent γ depends weakly on the type of high-T_c superconductor and on the quality of the sample, lying in the interval $\gamma\approx 1.2-1.5$ (Ref. 1). These results indicate that the behavior in (1) is universal, by analogy with critical phenomena near the points of second-order phase transitions.

In the present letter we show that current–voltage characteristics of this sort can arise in thin films of type-II superconductors if the film thickness is smaller than or on the order of the magnetic-field penetration depth. In this case the problem of the pinning (or depinning) of vortices is effectively a 2D problem, and the transition to the resistive state is associated with a viscous flow of vortices in a 2D percolation structure formed by a random distribution of the forces of the one-particle core pinning, F_p, over a finite interval.

A flow of vortices (depinning) becomes possible because of the formation of connected regions (a percolation cluster) with values of F_p below a certain critical F_{pc}. This approach makes it possible to express the exponent γ in terms of universal critical exponents of 2D percolation theory, and it leads to a satisfactory quantitative agreement with experiment.[1]

2. We consider a thin film of a type-II superconductor, whose thickness d is far smaller than the magnetic-field penetration depth $\lambda \approx \lambda_L(\xi/l_e)^{1/2}$, where λ_L is the London depth, ξ is the coherence length, and l_e is the electron mean free path. We assume that point centers of a vortex-core pinning, randomly distributed in the film, have a continuous distribution with respect to pinning force F_p over some finite interval $0 \leqslant F_p \leqslant F_{pm}$. This assumption means that for any fixed $F_p < F_{pm}$ the film can be partitioned into two phases: a "black" phase, for which the Lorentz force satisfies $F_L = j\Phi_0/c > F_p$, where Φ_0 is the flux quantum, j is the transport current density, and c is the velocity of light; and a "white" phase, in which the condition $F_L < F_p$ holds.

Since the values of F_p are different for different pinning centers, i.e., since the pinning force is nonuniform and varies in a random way over the plane of the film, there must exist a critical F_{pc} such that under the condition $F_p < F_{pc}$ the black phase forms a connected structure of channels which traverse the entire sample in the direction perpendicular to the transport current (Fig. 1a). In this case, under the equality[2]

$$F_p + F_\eta = F_L, \tag{2}$$

where $F_\eta = \eta v$ is the viscous-friction force for a vortex, there can be a steady-state flow of vortices at a velocity

$$v = (j\Phi_0/c - F_p)/\eta. \tag{3}$$

Figure 1b shows the directions of the force $\mathbf{F}_L$, of the vortex velocity $\mathbf{v}$, and of the transport current $\mathbf{j}$. Here it is being assumed that the force F_η is strong enough that we can ignore the contribution from regions in which vortices are accelerated and decelerated as they interact with pinning centers. Furthermore, we are assuming magnetic fields which are weak enough ($H \ll H_{c2}$) that we can ignore the forces of the interaction between vortices (this is the regime of "one-particle" pinning).

The critical current j_c is determined by the critical pinning force F_{pc}:

$$j_c = cF_{pc}/\Phi_0. \tag{4}$$

Below we consider the behavior of the vortices near the critical current, under the condition

$$\tau_j = (j - j_c)/j_c \ll 1. \tag{5}$$

According to general ideas from percolation theory, the basic resistance of the two-phase black–white medium above the percolation threshold in samples with dimensions on the order of the correlation length L is due to long bridges of the black phase (Fig. 1a). Here L is to be understood as the distance over which there is a self-averaging of physical quantities, so that we have[3] $L \sim \tau^{-\nu}$, where ν is the critical exponent for the correlation length ($\nu = 4/3$ in 2D systems), and the parameter τ is a measure of the proximity to the percolation threshold:

$$\tau = (p - p_c)/p_c \ll 1. \tag{6}$$

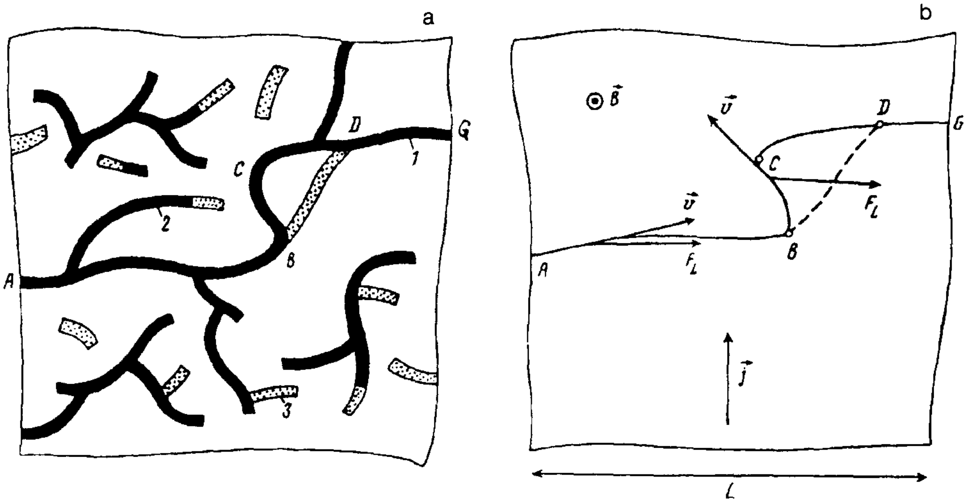

FIG. 1. a: Two-dimensional percolation structure in a system of centers which pin Abrikosov vortices in a thin film ($d<\lambda_L$). A steady-state flow of vortices can occur in the black regions (this is the "black phase"). 1—Infinite cluster *ABCDG*; 2—"dead ends"; 3—additional regions (shaded regions) of the black phase which arise upon an increase in the Lorentz force (the transport current); *ABDG*—the part of the infinite cluster in which there can be a steady-state flow of vortices, in the case of a fixed direction of the Lorentz force. b: Schematic diagram of the percolation structure. *BC*—A part of the infinite cluster in which vortices cannot move. As the Lorentz force (the transport current) increases, a new region of the infinite cluster, *BD*, which is compatible with the motion of vortices, arises. Here L is the correlation length.

Here p is the concentration of the black phase, and p_c is the critical concentration, at which an infinite black percolation cluster forms.

In this model, the quantity p is given by

$$p=\int_0^{F_p} D(F_p)dF_p, \tag{7}$$

where $D(F_p)$ is the distribution of pinning centers with respect to F_p. In particular, for a uniform distribution $D(F_p)=1/F_{pm}=\text{const}$, the parameters τ_j and τ are the same. For any smooth distribution they remain proportional to each other near the percolation threshold, where we have $(j-j_c)\ll j_c$.

The connecting bridges consist of N_1 links,[4] where $N_1\sim\tau^{-\alpha_1}$, with $\alpha_1=1$ in models of the Nodes–Links–Blobs type,[4–6] and with $\alpha_1=t$ in the 2D version of the Weak-Links model,[7] where t is the critical exponent of the conductivity ($t\approx1.33$).

3. Let us find the energy which is dissipated as the vortices traverse a bridge of length $l=a_0N_1$, where a_0 is a length scale of the variations in the field of pinning forces. In the case at hand, this length scale is on the same order of magnitude as the average distance between pinning centers. The energy dissipated in the traversal of one vortex over a time t_0 is

$$Q_1=\int_0^{t_0} F_\eta v\,dt=\int_0^{l} F_\eta dx=l\int_0^{F_{pc}}(F_L-F_p)D(F_p)dF_p, \tag{8}$$

while the time required to traverse the bridge, t_0, is given by

$$t_0=\int_0^{t_0} dt=\int_0^{l} dx/v=\eta l\int_0^{F_{pc}} D(F_p)dF_p/(F_L-F_p). \tag{9}$$

For a uniform distribution $D(F_p)=1/F_{pm}$ we then find

$$Q_1=lF_{pc}^2/2F_{pm}, \quad t_0=(\eta l/F_{pm})\ln(1/\tau). \tag{10}$$

The concentration of vortices is V/Φ_0, and the number of vortices in a bridge is

$$n=(B/\Phi_0)a_0 l\sim(a_0^2B/\Phi_0)\tau^{-\alpha_1},$$

where B is the magnetic induction. Consequently, (on the one hand) the rate at which the energy of all the vortices over an area L^2 is dissipated is

$$\dot{Q}=nQ_1/t_0=a_0BF_{pc}^2l/2\eta\Phi_0\ln(1/\tau). \tag{11}$$

On the other hand, we can express $\dot{Q}$ in terms of the electric field E and the current j (Ref. 2):

$$\dot{Q}=jEL^2. \tag{12}$$

Comparing (11) and (12), and using $j\approx j_c$, $l\sim\tau^{-\alpha_1}$, and $L\sim\tau^{-\nu}$, we find

$$E=\frac{a_0BF_{pc}^2l}{2\eta j_c\Phi_0L^2\ln(1/\tau)}\sim\left(\frac{j-j_c}{j_c}\right)^\gamma/\ln\left(\frac{j_c}{j-j_c}\right), \tag{13}$$

where

$$\gamma=2\nu-\alpha_1. \tag{14}$$

In Nodes–Links–Blobs models with $\nu=4/3$, we thus find $\gamma=2\nu-1\approx1.66$, while for the Weak-Links model we have $\gamma=2(4/3)-1.33\approx1.33$.

However, we should note that the Lorentz force which "pushes" the vortices is always in the same direction, so vortices cannot move in regions of bridges which are directed opposite the force F_L (see region *BC* in Fig. 1b). We should accordingly replace the standard percolation model by the "diode" percolation model.[8,9] The latter incorporates only those links in which the motion is directed along the Lorentz force. In this case the structure of the infinite cluster changes: The new parts of the cluster required here arise as the Lorentz force increases (see *ABDG* in Fig. 1a). In the Weak-Links model we then have, in place of (14),

$$\gamma=2\nu_\perp-t_+, \tag{15}$$

in which we have $\nu_\perp\approx1.10$ and $t_+\approx0.63$ according to the numerical calculations of Refs. 8–11. We thus have $\gamma\approx1.57$ (in place of the value $\gamma\approx1.33$ for the isotropic Weak-Links model).

4. In thin films of a type-II superconductor with a thickness smaller than the magnetic-field penetration depth, power-law current–voltage characteristics can be observed in the resistive state in weak fields $H \ll H_{c2}$. As was mentioned above, current–voltage characteristics of this type, with an exponent $\gamma = 1.2-1.5$, have been observed both in thin films of high-T_c superconductors and in high-T_c single crystals.[1] This result may be a consequence of the layered structure of the latter samples. However, whether the 2D percolation model is valid for describing the depinning of vortices in layered superconductors requires a special analysis.

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[1)]Private communication from V. M. Pan.

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