Phonon contribution to the noise of a one-channel Landauer resistor

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We devise a theory of shot noise in semiconductor nanowires under conditions of phonon-assisted quasiballistic transport. A general expression for the noise spectral density is derived and used to investigate particular cases of interest. For low temperatures, a remarkable threshold effect for the noise is predicted. © *1995 American Institute of Physics.*

Recently a general relation between the shot noise spectral density P and the transmission matrix of the mesoscopic conductor has been derived for elastic electron scattering.[1–3] For a one-channel case the shot noise is $P=2e|V|(e^2/h)T_0(1-T_0)$, where T_0 is the channel transmission. The shot noise is finite if and only if the transmission coefficient is neither 0 nor 1. The shot noise due to the inelastic electron–phonon scattering has been pointed out by Kulik *et al.*[4] for a classical 3D ballistic point contact. The purpose of the present paper is to study effects of electron–phonon scattering on the noise of a Landauer resistor, where the resistance is strongly quantized.

We consider a wire of length L along x-axis. In the spirit of the Landauer approach we assume the wire to be connected with reservoirs which we call "left" (L) and "right" (R), each of them being in equilibrium with itself. If the wire is long enough, the electron motion along x axis may be treated classically, and one can use a semiclassical kinetic theory to treat the electron transport in this direction (see also Ref. 5). The introduction of phonons into this picture can be done along the lines worked out in Refs. 6 and 7.

Using an approach described in Refs. 8 and 9, we will study the time evolution equation for $\langle \delta F \delta F \rangle$ along with the equation for $\bar{F}$, which both have the form of quasiclassical Boltzmann equations. We recall an equation for the average distribution function $\bar{F}(x,p,t)$:

$$\partial \bar{F}(x,p,t)/\partial t + v\,\partial \bar{F}(x,p,t)/\partial x = I\{\bar{F}(x,p,t)\}, \tag{1}$$

where p and v are the x components of the electron quasimomentum and velocity. $I\{\bar{F}(x,p,t)\}$ is the electron–phonon collision integral, which, as usual, is a difference of "in" and "out" terms:

$$I^{(\mathrm{out})}\{F(p,x)\} = F(p,x)\int_{-\infty}^{\infty}\frac{dp'}{2\pi\hbar}[1-F(p',x)]\int\frac{d^2\mathbf{q}\perp}{(2\pi)^d}|\langle 0|\exp(i\mathbf{q}\perp\cdot\mathbf{r}\perp)$$
$$\times|0\rangle|^2 W\mathbf{q}[N\mathbf{q}\,\delta(\epsilon'-\epsilon-\hbar\omega\mathbf{q})+(N\mathbf{q}+1)\,\delta(\epsilon'-\epsilon+\hbar\omega\mathbf{q})], \tag{2}$$

$$I^{(\mathrm{in})}\{F(p,x)\}=[1-F(p,x)]\int_{-\infty}^{\infty}\frac{dp'}{2\pi\hbar}F(p',x)\int\frac{d^2\mathbf{q}\perp}{(2\pi)^d}|\langle 0|\exp(i\mathbf{q}\perp\cdot\mathbf{r}\perp)$$

$$\times|0\rangle|^2 W\mathbf{q}[(N\mathbf{q}+1)\delta(\epsilon'-\epsilon-\hbar\omega\mathbf{q})+N\mathbf{q}\delta(\epsilon'-\epsilon+\hbar\omega\mathbf{q})], \qquad (3)$$

where $|0\rangle$ is the ground state wave function of the transverse quantization, $\epsilon=p^2/2m$, and $\epsilon'=p'^2/2m$. In the isotropic approximation for scattering on acoustical phonons $W\mathbf{q}=(\pi\Lambda^2 q^2/\rho\omega\mathbf{q})$, where Λ is the deformation potential constant for the longitudinal phonons, and ρ is the mass density. We integrate in Eqs. (2) and (3) over the three components of the phonon wave vector. $\mathbf{q}\perp$ denotes the two transverse wave vector components. The third component is given by $q_x=\pm(p-p')/\hbar$. Therefore, the third integration is equivalent to the integration over the electron quasimomentum p', because of the conservation of quasimomentum.

The boundary conditions for Eq. (1) are:

$$\bar{F}(p>0,x=-L/2)=f_L(p)\equiv\frac{1}{\exp[(\varepsilon_p-\mu_L)/k_BT]+1}, \qquad (4)$$

while for $p<0$ we have the same condition with the replacements $(-L/2)\rightarrow(L/2)$, $f_L\rightarrow f_R$, and $\mu_L\rightarrow\mu_R$. Here T is the temperature, μ_L and μ_R are the chemical potentials of the reservoirs, and $eV=\mu_L-\mu_R$ is the voltage bias across the conductor.

In the absence of collisions and under stationary conditions the solution of Eq. (1) is $\bar{F}^{(0)}(x,p)=\theta(p)f_L(p)+\theta(-p)f_R(p)$, where $\theta(p)$ is the step function. This solution corresponds to a purely ballistic motion. Adding a weak electron–phonon interaction, we have $\bar{F}=\bar{F}^{(0)}+\overline{\Delta F}$, with $\overline{\Delta F}$ satisfying the first iteration of the transport equation $v\partial\overline{\Delta F}/\partial x = I\{\bar{F}^{(0)}\}$. Taking the boundary conditions and Eq. (4) into account and placing the origin of the coordinate system at the midpoint of the wire, we arrive at the solution of this equation in the form[6]:

$$\overline{\Delta F(x,p)}=(1/|v|)[x+(L/2)\mathrm{sign}\ p]I\{\bar{F}^{(0)}(p)\}. \qquad (5)$$

We assume that the phonons are in equilibrium and so $N_{\mathbf{q}}$ is the Bose function. Detailed balance guarantees a vanishing collision term for the equilibrium distribution function and constant temperature and chemical potential. This means that the distribution function $F^{(0)}$ gives a finite contribution to the collision term if and only if p and p' are of opposite sign, so that their chemical potentials are different. In other words, only those phonons contribute that can backscatter the electrons—see Ref. 6.

This gives the averaged electron distribution function along the wire in the presence of weak inelastic scattering. As to the fluctuating part of the distribution function, δF, according to Ref. 8 the correlation function $\langle\delta F(x',p',t')\delta F(x,p,t)\rangle$ for $t'>t$ satisfies the Boltzmann equation (1) in the first set of variables with the initial condition

$$\langle\delta F(x',p',t)\delta F(x,p,t)\rangle=h\delta(x-x')\delta(p-p')\bar{F}(x,p,t)[1-\bar{F}(x,p,t)]. \qquad (6)$$

The current through a cross section of the wire, which we choose to be near the right-hand reservoir, is:

$$J(t)=\frac{2e}{h}\int_{-\infty}^{\infty}dpvF(x=L/2,p,t). \tag{7}$$

The current has an average value $\bar{J}$ and a fluctuating part $\delta J(t)=J(t)-\bar{J}$. The current noise spectral density in the limit of zero frequency is given by

$$P=4\int_0^{\infty}dt\langle\delta J(t)\delta J(0)\rangle=4\left(\frac{e}{h}\right)^2\int_0^{\infty}dt\int_{-\infty}^{\infty}dp\int_{-\infty}^{\infty}dp'vv'\chi, \tag{8}$$

where $\chi=\langle\delta F(L/2,p',t)\delta F(L/2,p,0)\rangle$. Here we have made use of conservation of the current, which permits one to choose any cross section to calculate the current fluctuations for $\omega\to 0$.

Electrons with $p>0$ reach the right-hand reservoir without further scattering. Therefore, χ contains only terms proportional to $\delta(t)$:

$$\chi(p>0)=\frac{h}{|v'|}\delta(t)\delta(p'-p)\bar{F}(L/2,p)[1-\bar{F}(L/2,p)], \tag{9}$$

where $\bar{F}(L/2,p)$ is given by a sum of $F^{(0)}$ and Eq. (5), taken at $x=L/2$ and $p>0$:

$$\bar{F}(L/2,p>0)=f_L(p)+\frac{L}{|v|}I^{(\text{in})}\{f_L(p)\}-\frac{L}{|v|}I^{(\text{out})}\{f_L(p)\}. \tag{10}$$

Electrons with $p<0$ can be backscattered within the wire and recross the $x=L/2$ cross section. In order to take into account all backscattering trajectories we first divide the wire into small pieces $[x_1,x_2]$, $[x_2,x_3]$,... $[x_i,x_{i+1}]$,... . Then we introduce the probability per unit time for the electron from the right-hand reservoir to be backscattered, $\mathcal{U}(p<0\to p_k>0)$, and sum over the contributions from all the pieces. As a result, we obtain:

$$\chi(p<0)=\frac{h}{|v'|}\left[\delta(t)\delta(p'-p)+\sum_{ik}h\frac{\Delta x_i}{|v|}\mathcal{U}(p<0\to p_k>0)\delta(t-t_i)\delta(p'-p_k)\right]$$
$$\times\bar{F}(L/2,p)[1-\bar{F}(L/2,p)]. \tag{11}$$

Here p_k is the final electron state after backscattering, t_i is the time spent in the wire, and $\Delta x_i\equiv x_{i+1}-x_i$. The distribution function $\bar{F}(L/2,p)$ for electrons with $p<0$ is exactly $f_R(p)$, as is seen from Eq. (5).

Substituting Eqs. (9) and (11) into the expression for the shot noise spectral density, Eq. (8), and integrating over t and p', we finally obtain the shot noise power:

$$P=4\left(\frac{e}{h}\right)^2\int_{-\infty}^{\infty}dp|v|\left\{\theta(p)\frac{1}{2}\left(f_L(p)+\frac{L}{|v|}I^{(\text{in})}\{f_L(p)\}-\frac{L}{|v|}I^{(\text{out})}\{f_L(p)\}\right)\right.$$
$$\times\left(1-f_L(p)-\frac{L}{|v|}I^{(\text{in})}\{f_L(p)\}+\frac{L}{|v|}I^{(\text{out})}\{f_L(p)\}\right)+\theta(-p)$$
$$\left.\times\left[\frac{1}{2}-\frac{L}{|v|}\frac{\delta I^{(\text{out})}\{f_R(p)\}}{\delta f_R(p)}\right]f_R(p)[1-f_R(p)]\right\}. \tag{12}$$

Here $\delta I^{(\mathrm{out})}\{f_R(p)\}/\delta f_R(p)=\Sigma_{p'}\mathscr{W}(p\langle 0\rightarrow p'\rangle 0)$ is the variational derivative of the collision integral. Equation (12) should be understood in the following way. It contains terms of the zeroth, first, and second order in the small parameter $(L/|v|)I^{(\mathrm{in,out})}\{f_{L,R}(p)\}$. In general, it is not always permissible to retain the terms of second order. However, we have not discarded them because they are meaningful in some cases (see below).

This formula allows us to study various cases of interest. In particular, we will be interested in 2 regimes: $eV\gg k_BT$, and $eV\ll k_BT$.

Let us first assume that the voltage across the conductor, $eV=\mu_L-\mu_R$, is finite and that $T=0$. Then $N_q=0$, and only phonon emission processes can occur. The distribution functions for electrons in the leads are step functions: $f_{L,R}(E)=\theta(\mu_{L,R}-E)$. This leads to a substantial simplification of the problem. In particular, $I^{(\mathrm{in})}\{f_L(p)\}$ and $\delta I^{(\mathrm{out})}\{f_R(p)\}/\delta f_R(p)$ become zero, while $(L/|v|)I^{(\mathrm{out})}\{f_L(p>0)\}=f_L(p)\mathscr{R}(\epsilon_p)$, where:

$$\mathscr{R}(\varepsilon_p)=\frac{L}{|v|}\int_{-\infty}^{\infty}\frac{dp'}{2\pi\hbar}\int\frac{d^2\mathbf{q}\perp}{(2\pi)^d}\theta([\varepsilon_p-\mu_R]-\hbar\omega_q)|\langle 0|$$

$$\times\exp(i\mathbf{q}\perp\cdot\mathbf{r}\perp)|0\rangle|^2W_q\delta(\varepsilon_p-\hbar\omega_q-\varepsilon_{p'}). \tag{13}$$

Here we have introduced the effective "inelastic reflection" coefficient $\mathscr{R}(\epsilon_p)$, which describes the efficiency of the electron backscattering in the course of phonon emission.

For the shot noise spectral density we now have, making use of Eq. (12):

$$P=\frac{2e^2}{h}\int_{\mu_R}^{\mu_R+e|V|}dE\,\mathscr{R}(E)[1-\mathscr{R}(E)]. \tag{14}$$

Therefore, the shot noise power is determined by the energy dependence of "inelastic reflection" coefficient. We wish to emphasize that here we retain the terms quadratic in the reflection coefficient (along with terms linear in $\mathscr{R}$). At the same time we do not take into account the quadratic terms while calculating the corrections due to the electron scattering to the distribution function itself [see Eq. (5)]. This is permissible, because in the case under discussion ($eV\gg k_BT$) the latter are of third order in $\mathscr{R}$. Indeed, the second-order corrections to ΔF could appear after taking into account the second scattering event for an electron which has already experienced a scattering. However, the probability of this event is proportional to the number of empty states $[1-F(p)]$, which in its turn is proportional to $\mathscr{R}$.

We consider electron backscattering. Therefore, there should be some minimum wave vector for an acoustical phonon to be emitted. Namely, $q_{\mathrm{min}}=2p_F/\hbar$, where p_F is the Fermi momentum. This leads to a threshold in the energy dependence of the "inelastic reflection" coefficient and, hence, in $P(V)$.

Indeed, $\mathscr{R}(E)=0$ for $E<2p_Fs\equiv E_{\mathrm{th}}$, where we have introduced the notation $E=\epsilon_p-\mu_R$. One can show that for energies near the threshold, when $(E-E_{\mathrm{th}})/E_{\mathrm{th}}\ll 1$, in the first approximation in this small parameter the coefficient of inelastic reflection is:

$$\mathscr{R}(E)=\frac{E-E_{\mathrm{th}}}{E_{\mathrm{th}}}R_0, \tag{15}$$

where

$$R_0=\frac{L}{|v_F|}\frac{mW_{2p_{F/\hbar}}p_F}{\hbar^4\pi^2}. \tag{16}$$

Here we have taken into account that in real structures $E_{\rm th}=2p_F s$ is of the same order as $\hbar s/d$. The shot noise power in this limit is $P(V)=(e^2/h)R_0[(e|V|-E_{\rm th})^2/E_{\rm th}]$.

Well above the threshold, when $E/E_{\rm th}\gg 1$, the matrix element $\langle 0|\exp(i\mathbf{q}\perp\mathbf{r}\perp)|0\rangle\propto(q\perp d)^{-2}$, where d is the thickness of the nanowire. Then $R(E)=R_0$, while the shot noise power becomes proportional to the applied voltage:

$$P(V)=\frac{2e^2}{h}e|V|R_0. \tag{17}$$

One sees that the voltage dependence of the nonequilibrium noise differs from a simple linear law on account of the inelastic electron backscattering within the wire.

It is worthwhile to mention the case of inelastic optical phonon emission. An optical phonon is a simplest example of a phonon mode with a non-vanishing minimum frequency. We denote the optical phonon energy by $\hbar\omega_0$ and assume that dispersion is absent completely. Under this condition the contribution of optical phonon emission processes to $\mathscr{R}$ vanishes for electron energies smaller than $\hbar\omega_0$, while in the vicinity of $\hbar\omega_0$ the reflection coefficient exhibits a jump. Therefore, the shot noise, $P(V)$, has a bend when V crosses $\hbar\omega_0/e$, which can be observed in experiment.

For small applied voltage $eV=\mu_L-\mu_R\ll k_BT$, expansion of Eq. (12) in powers of eV gives a voltage-dependent correction, $P_1(V)$, to the thermal noise:

$$P_1(V)=\frac{1}{3}\frac{(eV)^2}{k_BT}\frac{e^2}{h}R_0, \tag{18}$$

see Eq. (16).

To summarize, we have developed a theory of the shot noise in quantum quasiballistic channels under conditions of phonon-assisted transport. We have derived a formula for the shot noise caused by weak electron–phonon scattering in otherwise purely ballistic quantum channels and have used it to work out expressions for some particular cases of interest. We have studied the cases of low ($k_BT\ll eV$) and high ($k_BT\gg eV$) temperatures. For low temperatures a remarkable threshold effect for the shot noise is predicted.

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