One-dimensional Fermi liquid and symmetry breaking in the vortex core

Yu. G. Makhlin and G. E. Volovik

Low Temperature Laboratory, Helsinki University of Technology, 02150 Espoo, Finland; L. D. Landau Institute for Theoretical Physics, 117940 Moscow, Russia

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Fermions localized within vortex cores can form one-dimensional Fermi liquids. The nonzero density of states in these Fermi liquids can lead to instability of the symmetric structure of the vortex core. We consider a symmetry breaking which is obtained due to spontaneous admixture of the spin-triplet p-wave component of the order parameter in a conventional s-wave vortex in the presence of a magnetic field. This occurs at a rather low temperature and leads to an asymmetric shape of the vortex core. A similar phenomenon of symmetry breaking induced by core fermions occurs in electroweak Z strings.[8] © *1995 American Institute of Physics.*

INTRODUCTION

Recently new techniques have been developed that in principle allow one to probe the core structure of an individual Abrikosov vortex in a superconductor. These include scanning tunneling microscopy,[1] electron holography, Lorentz microscopy, etc. (see Ref. 2 and the references therein). It is therefore timely to carry out a theoretical investigation of the symmetry of the core and of the energy spectrum of the fermions localized in the core.

Three possible types of energy spectrum $E(Q,k_z)$ for fermions localized within the vortex core were discussed in Ref. 3. (i) The spectrum $E(Q,k_z)$, where Q is the generalized angular momentum, has a gap of the order of $\Delta^2/E_F{}^4$ (here Δ is the superconducting gap and E_F is the Fermi energy in the normal Fermi liquid). (ii) One or more branches of the spectrum with particular Q's cross the zero level as functions of the linear momentum k_z at some points $k_z = k_{F,Q}$. In this case the fermions occupying negative energy levels form one-dimensional (1D) Fermi liquids with Fermi points at $k_{F,Q}$ (see also an example of the continuous vortex in ^{3}He–A (Ref. 5). (iii) There is a flat band where all fermions have exactly zero energy in a finite region of the momenta k_z; this is called the Fermi condensate.[6]

It was emphasized that both the 1D Fermi liquid and 1D Fermi condensate can be unstable against a further breaking of the vortex symmetry at low temperatures; in particular, the vortex can become nonaxisymmetric or the vortex line can transform into a spiral[3] (see also Ref. 7).

Recently this kind of instability of the vortex caused by the fermion zero modes in the core has been discussed for the Z string solution in the Weinberg–Salam model of

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electroweak interactions.[8] In this case the situation corresponds to the case of a 1D Fermi liquid. Although it was stated in Ref. 8 that this can lead to absolute instability of the Z string, the situation is actually not critical for the existence of a locally stable Z string and is similar to that which was first discussed by Peierls.[9] In the Peierls model the 1D electron liquid coupled to the lattice is unstable at low temperature against the dimerization of the lattice. In the Z string the 1D massless relativistic fermions coupled to the bosonic field of the order parameter (Higgs field) lead, below some critical temperature, to spontaneous symmetry breaking, which comes about, e.g., through growth of a small upper component of the Higgs field or of another mode of instability. The transition temperature $T_{c\ \mathrm{core}}$ of such an instability is usually exponentially small.

We discuss here the analogous instability for condensed-matter vortices. We have found that in most cases the transition temperature $T_{c\ \mathrm{core}}$ is extremely small, of the order of $\Delta\exp(-E_F/\Delta)$, but in some cases $T_{c\ \mathrm{core}}$ can be reasonably large, of the order of Δ^2/E_F.

s-wave superfluids. Let us consider as an example the situation in superconductors with s-pairing. The spatial dependence of the order parameter around the axis of a one-quantum vortex is given by $\Delta(r)\cdot\exp(i\phi)$ in cylindrical coordinates. The magnitude of the gap $\Delta(r)$ equals zero at the axis $r=0$ and tends to the bulk value Δ for large distances $r\gg\xi$, where $\xi=v_F/\Delta$ is the coherence length.

The system is invariant under translations along the z axis. If the spin–orbit interaction is neglected there is also the $SU(2)_S$ group of spin rotation, which is reduced to the $SO(2)_S$ subgroup if a magnetic field is applied. The "axial" symmetry of the order parameter in the vortex is characterized by another subgroup $SO(2)_Q$, with a generator that is a combination of the orbital momentum L_z and the particle number operator **I** (Ref. 10).

$$\mathbf{Q}=\mathbf{L}_z-\frac{m}{2}\mathbf{I}, \tag{1}$$

where $m=1$ is the winding number of the vortex. The fermion eigenstates are characterized by the quantum numbers k_z, Q, and the direction of the spin (↑ or ↓). The energy spectrum of low-lying excitations is given by[4]

$$E(Q,k_z)=Q\cdot\omega_0(k_z) \tag{2}$$

with $\omega_0\sim\Delta^2/E_F$ for $k_z\ll k_F$ and $\omega_0\to\infty$ for $k_z\to k_F$ (this approximation is true for $\omega_0\ll\Delta$).[4] For an $m=1$-quantum vortex, Q can assume only half-integer values (L_z is integer, and I is $+1$ and -1 for particles and holes, respectively).

Let us apply a magnetic field H parallel to the axis of the vortex. Then the energy spectrum is shifted down (up) for fermions with spin up (down):

$$E_{\uparrow,\downarrow}(Q,k_z)=Q\cdot\omega_0(k_z)\mp\mu H, \tag{3}$$

and we get a picture (Fig. 1) wherein the branches $E_{Q,\uparrow}(k_z)$ and $E_{Q,\downarrow}(k_z)$ intersect for $\mu H>Q\omega_0(0)/2$.

The fermions near the points $k_{F,Q}$ can be treated as a one-dimensional Fermi-liquid, and $k_{F,Q}$ plays the part of the Fermi momentum. The nonzero density of states can lead

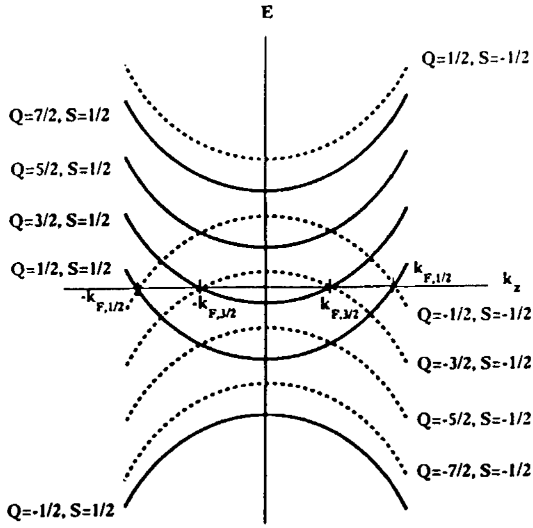

FIG. 1. Spectrum of fermions in the vortex in an s-wave superconductor. The levels are shifted by the magnetic field. Some branches cross the zero energy level, forming a 1D Fermi liquid. Fermions with opposite spins at the Fermi level are paired at low T, which leads to broken symmetry of the core.

to a new pairing of the one-dimensional fermions, the character of which depends on the sign and the magnitude of the interaction of 1D fermions in different channels. Let us show that both symmetries $SO(2)_S$ and $SO(2)_Q$ will be broken at low enough temperatures if the interaction in the spin-triplet p-wave channel is attractive, and the core of the vortex can become anisotropic.

p-wave component in the core. In our model an extra interaction of the fermions in a 1D Fermi liquid results from the interaction of the initial bare fermions in the p-channel, and we check for instability of the vortex core against nucleation of an admixture of the spin triplet p-wave component in the core, of the type

$$\delta\hat{\Delta}=g\Delta(\mathbf{d}\cdot\hat{\boldsymbol{\sigma}})i\hat{\sigma}_2(\mathbf{a}\cdot\mathbf{k})f(r)\exp(iN\phi). \tag{4}$$

As in Ref. 8 we assume that $f(r)$ is concentrated in the core, since it should influence only the core fermions. $f(r)$ is of the order of unity in the core of the vortex and decreases to zero outside the core (i.e., at $r\sim\xi$); in addition $f(0)=0$ if $N\neq 0$. The parameter Δ is the bulk value of the s-wave gap, and the dimensionless parameter g is thus the magnitude of the admixture of the p-wave order parameter in the core region as compared to the s-wave component. For simplicity we can assume a trial function of the form $f(r)=\exp(-r/\xi)$ for $N=0$. The spin vector $\mathbf{d}$ should be orthogonal to the magnetic field in order to break the spin rotation symmetry.

The suggested change of the order parameter breaks the Q symmetry as well. To show this, note that $\delta\hat{\Delta}$ is the sum of three orbital harmonics: $\mathbf{a}\cdot\mathbf{k}=a_+k_-+a_-k_++a_0k_z$, where $k_\pm=k_x\pm ik_y$. We suppose that only one a is nonzero, namely a_l with some value $l=0,\pm 1$ of the z-projection of the Cooper pair orbital momentum. It is easy to check that new order parameter is not Q-invariant for $N+l\neq 1$, and this signifies a breaking of the axial symmetry of the whole core structure. g is a dimensionless parameter that shows the strength of the deformation. This symmetry breaking leads to mixing of $Q,\uparrow$ and $-Q,\downarrow$ fermionic states in the same way as u and d quarks are mixed in the Z string in the Naculich scenario[8] of the instability of the Z string. Due to the noncrossing theorem for levels of the same symmetry, the two branches which crossed each other at $k_{F,Q}$ repel each other, and a gap between the states appears which is proportional to g.

Deformation of the fermionic spectrum. Without the admixture, the Bogolyubov–Nambu Hamiltonian is given by the 4×4 matrix

$$\mathcal{H}_0=\begin{pmatrix} \xi & 0 & 0 & \Delta \\ 0 & \xi & -\Delta & 0 \\ 0 & -\Delta^* & -\xi & 0 \\ \Delta^* & 0 & 0 & -\xi \end{pmatrix}. \tag{5}$$

Here $\xi=(1/2m^*)(-d^2/d\mathbf{r}^2-k_F^2)$. The eigenstates of $\mathcal{H}_0$ with energies (2) for spin up and distances $r\gg k_F^{-1}$ are given by[4]

$$\chi_{k_z,Q,\uparrow}=\begin{pmatrix} \chi_1 \\ 0 \\ 0 \\ \chi_2 \end{pmatrix}=\text{const}\cdot e^{ik_z z}e^{iQ\phi}H_\nu(qr)e^{-K(r)}\cdot\begin{pmatrix} \exp\left(\frac{i}{2}(\phi+\psi(r))\right) \\ 0 \\ 0 \\ -i\exp\left(-\frac{i}{2}(\phi+\psi(r))\right) \end{pmatrix}, \tag{6}$$

where H_ν is the Hankel function with index $\nu=\sqrt{Q^2+1/4}$; $q^2=k_F^2-k_z^2$,

$$K(r)=\frac{m^*}{q}\int_0^r\Delta(r')dr',\quad \psi(r)=-\int_r^\infty \exp(2(K(r)-K(r')))\left(\frac{2Em^*}{q}+\frac{Q}{qr'^2}\right)dr'.$$

For spin down the eigenstates are

$$\chi_{k_z,Q,\downarrow}=\begin{pmatrix} 0 \\ \chi_1 \\ -\chi_2 \\ 0 \end{pmatrix}. \tag{7}$$

The change $\delta\hat{\Delta}$ leads to perturbation of the mean-field Bogolyubov–Nambu Hamiltonian. The angular integral in the matrix element of this perturbation between the states $\chi_{k_z,Q,\uparrow}$ and $\chi_{k_z-Q,\downarrow}$ is nonvanishing only for

$$N+l+1\pm 2Q=0, \tag{8}$$

(i.e., $N+l$ should be an even integer). In this case the matrix element between the spin-up and spin-down states is of the order of $g\Delta$. For other Q the integral over $d\phi$ in the matrix element vanishes. The 2×2 Hamiltonian for two mixing states $(k_z,Q,\uparrow)$ and $(k_z,Q,\downarrow)$ for k_z near $k_{F,Q}$ is then given by:

$$\begin{pmatrix} E_{\text{old}} & g\Delta \\ g\Delta & -E_{\text{old}} \end{pmatrix}. \tag{9}$$

Here $E_{\text{old}}=v_{F,Q}(k_z-k_{F,Q})$, the derivative $v_{F,Q}=Q\omega_0'$ of the energy over momentum k_z represents the "Fermi" velocity of the 1D Fermi liquid. The modified energy spectrum takes the form

$$E_{\text{new}}=\pm\sqrt{v_{F,Q}^2(k_z-k_{F,Q})^2+g^2\Delta^2}. \tag{10}$$

At zero temperature, fermions occupying the states with negative energy near $k_{F,Q}$ gain in energy after the change in the order parameter. Let us now compare the energy gain and energy loss due to the perturbation g.

Energy balance and zero-temperature deformation of the vortex core. The main contribution to the difference between the energy of the fermionic vacuum in symmetric and asymmetric states comes from the logarithmic term, which arises due to appearance of the energy gap in 1D Fermi-liquid:

$$E_{\text{gain}}=\int\frac{dk_z}{2\pi}\left(E_{\text{new}}-E_{\text{old}}\cong-\frac{g^2\Delta^2}{v_{F,Q}}\int_{k_1}^{k_2}\frac{d(k-k_{F,Q})}{|k-k_{F,Q}|}\right.. \tag{11}$$

The lower cutoff is determined by the gap itself:

$$k_1\cong\frac{g\Delta}{v_{F,Q}}. \tag{12}$$

The most interesting situation takes place for $k_{F,Q}$ not too close to k_F, in which case the Fermi velocity $v_{F,Q}$ is small and the energy gain increases. This occurs in fields H near the threshold values $H_c(Q)$ at which the branches $Q,\uparrow$ and $-Q,\downarrow$ start to cross each other and the "Fermi momentum" $k_{F,Q}$ appears for the first time. The "Fermi momentum" $k_{F,Q}$ is given by the condition $Q\omega_0(k_{F,Q})=\mu H$, that is we are interested in the fields $\mu H\sim Q\Delta^2/E_F$. In this region the upper cutoff in the logarithmically divergent integral in (11) is $k_2\sim k_{F,Q}\sim k_F\sqrt{\mu H/Q\omega_0(0)-1}$. This leads to the result

$$E_{\text{gain}}\cong E_F k_F g^2\frac{k_F}{|Q|k_{F,Q}}\ln\left(\frac{|Q|\Delta k_{F,Q}^2}{gE_Fk_F^2}\right). \tag{13}$$

On the other hand the energy loss due to the perturbation of the superconducting order parameter in the core of the vortex is of the order of $E_{\text{loss}}\cong E_Fk_Fg^2/\gamma_1$. Here γ_1 is the relative magnitude of the attractive interaction of bare fermions in the p channel compared to that in the s channel. For $N\neq 0$ the contribution to the gradient energy from the spatial inhomogeneity of $\delta\Delta$ increases by an amount $\sim E_Fk_FN^2g^2$. This analysis is valid only if the latter term is smaller than the former one.

Comparison of the energies shows an instability, the relative magnitude g of the new p-wave order parameter at zero temperature being of the order of

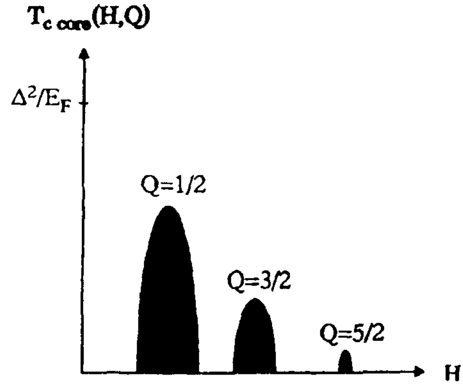

FIG. 2. Schematic dependence of the temperature of the core transition on the magnetic field. Within the shaded area the symmetry of the vortex core is broken.

$$g \cong \frac{\Delta}{E_F}|Q|\left(\frac{k_{F,Q}}{k_F}\right)^2 \exp\left(-|Q|\frac{k_{F,Q}}{k_F}\left(\frac{1}{\gamma_1}+N^2\right)\right). \tag{14}$$

Here we omitted the coefficients of the order of unity. $N(Q)$ is given by (8). At small Q the exponent $\exp(-O(1))$ is not very small for $k_{F,Q} \sim k_F$, and the transition temperature is reasonable. On the other hand, the numerical factor in the exponent $O(1)$ can be large. In this case (even for large Q) we can take $k_{F,Q}/k_F$ to be small, which leads to an exponential increase in the magnitude of g followed by just a power-law decrease in the pre-exponential factor.

Symmetry-breaking scheme. The above result means that there should be a second-order phase transition at $T_{c\text{ core}}$ into the broken-symmetry state. This is the state with the additional superfluid order parameter (4) with p-pairing. The $\mathbf{k}$ dependence of this order parameter is described by the quantum number $l=0,\pm 1$, and the spatial ϕ dependence by another integer $N=\pm 2Q-l-1$. Thus there are six competing structures (for three values of l) with comparable transition temperatures. They correspond to different symmetry groups of the new vortex core structure below $T_{c\text{ core}}$.

The spin structure of the new order parameter is described by the vector $\mathbf{d}$ in the xy plane. For $\mathbf{d}=\hat{x}\pm i\hat{y}$ the symmetry $SO(2)_Q \times SO(2)_S$ is reduced to the combined symmetry group $SO(2)_{Q\pm 2|Q\pm 1|S}$. For other $\mathbf{d}$ in the plane the symmetry is reduced to the discrete group $Z_{4|Q\pm 1|}$, the elements of which are $\mathbf{Q}$-rotations accompanied by the possible inversion of $\mathbf{d}$. In the former case, say, the density is still axially symmetric, and in the latter case it is not.

Transition temperatures. The temperature $T_{c\text{ core}}$ at which the transition to the asymmetric core state occurs is

$$T_{c\text{ core}} \sim g\Delta \tag{15}$$

(see the phase diagram, Fig. 2). The maximum transition temperature in the Qth region

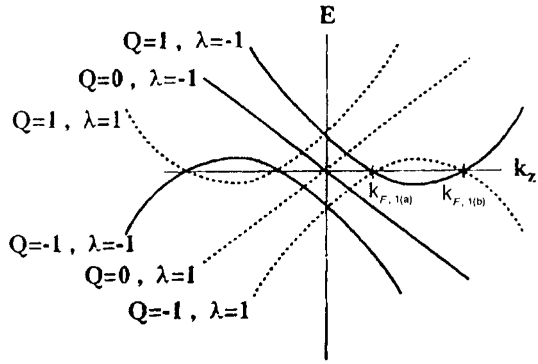

FIG. 3. Spectrum of fermions in the most symmetric vortex in ^{3}He–B (Ref. 3). $\lambda=\pm1$ is the helicity.

and the range of the fields in which the bare structure is unstable (the height and width of the region in $H-T$ plane) decrease with Q:

$$T_{c,\max}(Q)\sim\Delta\mu H(Q)\sim\frac{\Delta^2}{E_F}\frac{1}{Q^5}. \tag{16}$$

The minimum field is of order $(\Delta^2/E_F)/\mu$. This field can be less than the upper critical field H_{c2} if the effective fermion mass m^* is larger than the bare electron mass m.

Vortex in ^{3}He–B. In the spin-triplet superfluid ^{3}He–B the situation is almost equivalent to that in s-superfluids. In the most symmetric B-phase vortex the branches of the energy spectrum are described by quantum numbers k_z, Q, and helicity $\lambda=\pm1$ (Ref. 3). The branches $E_{Q=0,\lambda=1}(k_z)$ and $E_{Q=0,\lambda=-1}(k_z)$ intersect at the point $k_z=0$, $E=0$ even in zero external magnetic field. The branches $E_{Q,\lambda=1}$ and $E_{-Q,\lambda=-1}$ begin to intersect at $k_z\sim k_F$ in fields $\mu H_c(Q)\sim Q\omega_0\sim Q\Delta^2/E_F$ (Fig. 3).

In case there is no conservation of longitudinal spin component S_z, and the perturbing order parameter may have spin anisotropy vector $\mathbf{d}$ parallel to the axis z. The deformation of the vortex can occur within the triplet pairing state; singlet pairing (e.g., s-pairing) is allowable as well.

The transition temperatures, critical fields, and the whole phase diagram are more or less the same as for s-superfluids. For the two branches mentioned above which intersect in zero field the transition temperature is of the order of $T_c\sim\Delta^2/E_F$. It was found experimentally and theoretically that the most symmetric vortex in ^{3}He–B is unstable against symmetry breaking at least at high temperatures of the order of T_c (Refs. 10–12). Thus if the symmetry is restored at lower temperature it is broken again at very low T.

States with definite λ are mixtures of states with different particle number and different $S_z=\pm1/2$. The analysis shows that in the Qth region the angle dependence of the perturbation of the order parameter can be given by an integer

$$N=\pm2Q-l-m+\Sigma$$

where $\Sigma=0,\pm2$. Therefore, for each l we get three different values of N and thus several competing types of symmetry breaking.

CONCLUSION

A vortex structure which contains a one-dimensional Fermi liquid of core fermions is unstable against symmetry breaking. In the case of an Abrikosov vortex in s-wave superconductors this leads to the breaking of the axial symmetry of the vortex core in some regions in the $H-T$ plane.

Similar symmetry breaking occurs in the electroweak Z string, where the mixing of the d and u quarks opens up a gap in the fermionic spectrum.[8] In the Abrikosov and the ^{3}He–B vortices this corresponds to hybridization of the $Q,\uparrow$ and $-Q,\downarrow$ states of fermions. For an Abrikosov vortex this gives rise to a p-wave pairing component in the core.

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