Limiting rate of inductionless current drive in a tokamak

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The rate of current drive by lower hybrid waves in a tokamak is shown to be limited by the heating of the plasma. The corresponding maximum temporal rate of increase of the injected power is found. A deviation from this maximum rate lowers the efficiency at which the energy of rf waves is converted into energy of the magnetic field.

Recent experiments on inductionless current drive in tokamaks and also on the magnetization reversal of an inductor by means of lower hybrid waves have achieved efficiencies up to 25% in the conversion of rf energy into inductive energy of the magnetic field.[1,2] The results of these experiments can be described well by a kinetic theory with fixed values of the plasma parameters.[3] Attempts to increase the efficiency of the energy conversion by varying the plasma parameters in accordance with the existing theory have not been successful, however. In the present letter we show that in addition to the kinetic effects[3] one must also consider the heating of the plasma associated with the injection of additional power. The heating increases the conductivity of the plasma and also the time scale of changes in the plasma current. Under certain conditions an increase in the "external" current source—accompanied by an increase

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in the injected power—leads to a decrease in the current drive rate and thereby a decrease in the efficiency at which rf energy is converted into inductive energy of the plasma current. It turns out that the heating of the plasma accounts for the existence of a limiting time dependence of the current, $I(t)$, for each specific device. The only way to exceed this limiting time dependence is to raise the efficiency of the current drive or to reduce the energy lifetime of the plasma.

To incorporate the heating of the plasma in a description of current drive by lower hybrid waves, we work from the system of equations

$$\frac{dW}{dt} = \frac{I_{rf}}{\eta} + \frac{U^2}{R} - \frac{W}{\tau_E}, \tag{1}$$

$$Q_{rf} = \frac{I_{rf}}{\eta} - I_{rf}U, \tag{2}$$

$$u = (I - I_{rf})R, \tag{3}$$

$$\frac{d}{dt}\frac{LI^2}{2} + IU = 0. \tag{4}$$

Here W, I, U, R, L, and τ_E are respectively the energy, current, bypass voltage, resistance, external inductance, and energy lifetime of the plasma column; Q_{rf} is the rf power absorbed; I_{rf} is the current of fast, weakly collisional electrons; and η is the efficiency of the rf current drive in the absence of an electric field (i.e., with $U = 0$). The first two equations describe the energy balance in the thermal component and in the beam of fast electrons in the quasisteady stage of the discharge. The third equation is Ohm's law. The last equation of this system (the equation of the external circuit of the tokamak) describes the energy flux out of the plasma column into the poloidal magnetic field.

It follows from Eqs. (1)–(3) that the power which must be injected at a given instant in order to sustain the instantaneous value of the plasma current (with $U = 0$) is $Q_{rf} = I/\eta$. By increasing the power one can drive a plasma current, i.e., arrange $dI/dt > 0$ ($U < 0$). In addition, an increase in Q_{rf} leads to an increase in the temperature and thus a decrease in the plasma resistance R; such a decrease corresponds to a tendency toward a decrease in dI/dt. It would evidently be possible to arrange a situation in which the heating effect turns out to be stronger, and beginning at a certain $Q_{rf} = Q_{rf}^{opt}$ a further increase in the injected power causes a decrease in the rate at which inductive energy is built up.

Figure 1 shows the efficiency at which rf energy is converted into energy of the poloidal magnetic field as a function of the amount of power absorbed. The η_{el} (Q_{rf}) curve is seen to go through a maximum at $Q_{rf} = Q_{rf}^{opt}$. The condition for this maximum can be written $\partial\eta_{el}/\partial Q_{rf} = 0$ or

$$\partial U/\partial Q_{rf} = U/Q_{rf}. \tag{5}$$

The derivative here is calculated at a constant current, and system of equations

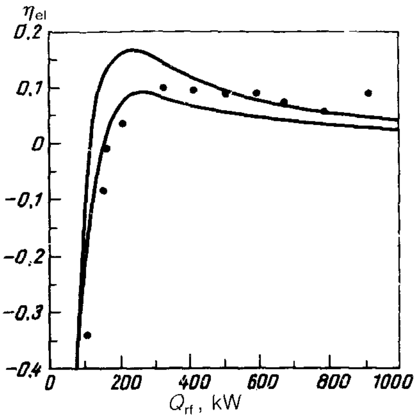

FIG. 1. The efficiency of the energy conversion $\eta_{el} = (dLI^2/2/dt)/Q_{rf}$ versus the injected power. The plasma was assumed to have a Spitzer conductivity in these calculations; $I = 90$ kA, $\eta = 0.6$ A/W; the energy lifetime $\tau_E = 3$ ms was chosen on the basis of the experimental data of Ref. 4. 1—$\langle n_e \rangle = 2.5 \times 10^{13}$ cm^{-3}; 2—$\langle n_e \rangle = 2 \times 10^{13}$ cm^{-3}; points—experimental results from the ALCATOR C tokamak.[2]

(1)–(3), (5) can be used to find Q_{rf}^{opt} as a function of the current I, the plasma density n, the energy lifetime τ_E, and the current drive efficiency η. Figure 2 shows the results of system (1)–(3), (5). The good agreement between the calculations and the experimental results demonstrates that the simple model outlined above correctly reflects the

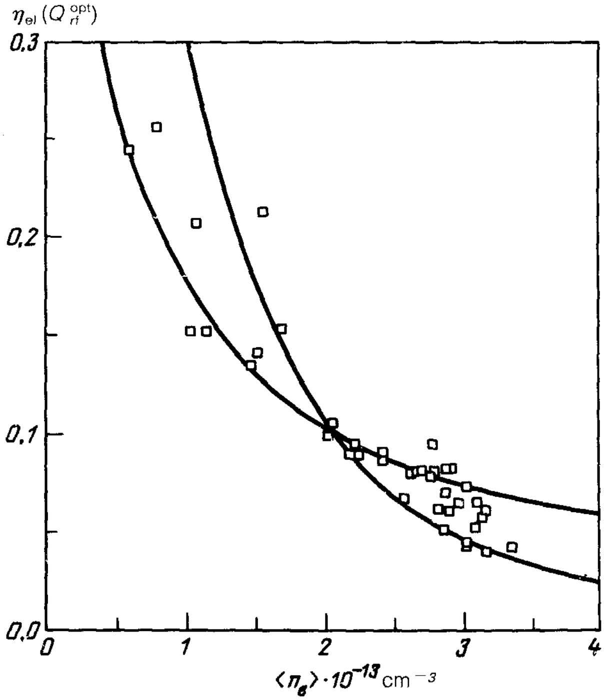

FIG. 2. Optimum efficiency $\eta_{el}(Q_{rf}^{opt})$ as a function of the plasma density (otherwise the same as in Fig. 1). Points—experimental data of Ref. 2; Line 1—$\tau_E = 3$ ms; 2—$\tau_E = 1.5 \times 10^{-13} \langle n_e \rangle$ (cm).

basic experimental behavior, so that further consequences of this model deserve consideration.

We first note that the condition for a maximum of the energy conversion efficiency which we discussed above is not the same as the condition for a maximum of the rate of current drive. It can be seen from Eq. (4) that the condition for a maximum of the derivative dI/dt at a given current I is

$$\partial U/\partial Q_{\mathrm{rf}} = 0. \tag{6}$$

Using system (1)–(3), we can easily put condition (6) in the form

$$\frac{\partial}{\partial T}\left[R\left(\frac{W}{\tau_E}-\frac{I}{\eta}\right)\right]=0. \tag{7}$$

Assuming, for simplicity, that the plasma has a Spitzer conductivity, we find from (7) that the current drive rate dI/dt goes through a maximum as a function of the injected power Q_{rf} if the energy lifetime of the plasma, τ_E, falls off no more rapidly than $T^{-1/2}$ with increasing temperature. In particular, for the ALCATOR scaling of the thermoconductivity, $\tau_E \sim n$, condition (6) is the same as $Q_{\mathrm{rf}} = 3I/\eta - IU$. Since the second term on the right side is usually less than 10% of the first, we can use the approximation

$$Q_{\mathrm{rf}} = 3I/\eta. \tag{8}$$

The maximum current rise rate or, equivalently, the maximum rate at which energy of the external rf source is converted in energy of the poloidal magnetic field is thus reached when the injected power is three times the power required to sustain a given current at a constant level. A comparison of conditions (5) and (6) shows that the maximum efficiency of the energy conversion is always reached at a lower level of the injected power, i.e., at $Q_{\mathrm{rf}}^{\mathrm{opt}} \lesssim 3I/\eta$. This conclusion agrees well with the condition $2 \leqslant \eta Q_{\mathrm{rf}}^{\mathrm{opt}}/I \leqslant 3$, which has been established experimentally over broad ranges of the parameters.[2]

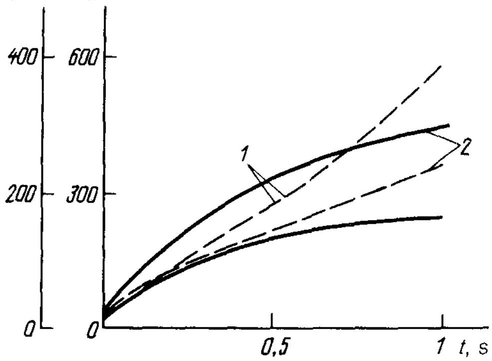

FIG. 3. Maximum possible current $I_{\lim}(t)$ (solid lines) and corresponding rf power $Q_{\mathrm{rf}}^{\mathrm{opt}}(t)$ (dashed lines) for the T-7 tokamak with $\tau_E = 10^{-12}\langle n_e\rangle$ (ms) and $\eta = 2\times10^{13}/\langle n_e\rangle$ (A/W). Lines 1—$\langle n_e\rangle = 10^{13}$ cm^{-3}; 2—$\langle n_e\rangle = 0.2\times10^{13}$ cm^{-3}.

We have established that there exists a limiting rate at which current can be driven in tokamaks by lower hybrid waves. This limiting rate is determined by the energy lifetime of the plasma, the efficiency of the current drive, and the instantaneous value of the current. If the plasma density is fixed, only the dependence on the magnitude of the current remains. This statement means that there is a certain limiting time dependence of the current, $I_{\lim}(t)$. This dependence is shown in Fig. 3 for various values of the density for the T-7 tokamak. The region in the (t,I) plane below the $I_{\lim}(t)$ curve is the "allowed" region for a tokamak. The working point may lie slightly above this curve only because our analysis has ignored the short time scales for changes in the structure of the energy in the plasma column.

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