Raman scattering of light by electrons in superconductors with a small correlation length

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The reflection coefficient of pure anisotropic superconductors with a correlation length $\hbar v/\Delta$ much smaller than the penetration depth of light δ has been determined. The effect of anisotropy is discussed.

Abrikosov and Fal'kovskiĭ[1] and Abrikosov and Genkin[2] studied Raman-scattering spectra of pure superconductors with a large correlation length

$$\xi \sim \hbar v/\Delta \gg \delta , \qquad (1)$$

where v is the velocity at the Fermi surface, Δ is the superconducting gap, and δ is the penetration depth of light. Dierker *et al.*[3] reported the observation of Raman scattering in a superconducting Nb_3Sn. Since condition (1) does not hold for this material, and since the inverse inequality probably applies, Dierker *et al.*[3] have made relevant theoretical predictions. They failed, however, to take into account a series of diagrams (which were singled out in Ref. 1) which in the limiting case

$$\xi \ll \delta \qquad (2)$$

are of crucial importance. At the same time, the study of Raman electron scattering in superconductors in the case where condition (2) holds is of great current interest in view of the discovery of high-temperature superconductors with an extremely small correlation length. Experimental determination of Raman scattering of electrons[4] makes it possible to measure the energy gap of a contactless method.

We will derive here a theory in which all necessary diagrams are taken into account. Assuming that the temperature is zero, we will start with an isotropic model.

As was pointed out in Ref. 1, in addition to the zeroth-order diagram for electron interaction (Fig. 2a in Ref. 1), it is necessary to take the sum over the entire series of chain diagrams (Fig. 3a in Ref. 1). While, at $\xi \gg \Delta$ these diagrams determine the reflection coefficient only near the threshold, $\omega_0 = \omega - \omega' = 2\Delta$ (ω and ω' are the frequencies of the incident and scattered light), for $\xi \ll \delta$ they are important in a broad region, $\omega_0 - 2\Delta \sim \Delta$; we are using here a system of units with $\hbar = c = 1$.

The addition of new diagrams when the sueprconducting pairing is taken into account results in a replacement in the equation for the matrix element S_{j0} (see Ref. 1) of the expression $\psi_\alpha^+(x)\psi_\alpha(x)$ by the combination

$$R = A_1R_1 + A_2R_2 + A_3R_3 , \tag{3}$$

where $R_1 = \psi_\alpha^+\psi_\alpha$, $R_2 = I_{\alpha\beta}\psi_\beta\psi_\alpha$, and $R_3 = I_{\alpha\beta}\psi_\alpha^+\psi_\beta^+$; here $I_{\alpha\beta}$ is a 2×2 antisymmetric matrix, $I_{\alpha\beta} = -I_{\beta\alpha}$, and $I^2 = -1$.

The equations for the coefficient A_i can be obtained as follows. By adding one link to the sum of an infinite series of diagrams in Fig. 3a of Ref. 1 this series transforms into itself. We thus obtain (in zeroth approximation)

$$A_i = \delta_{i_1} + a_{ki}A_k , \tag{4}$$

where the coefficients a_{ik} correspond to different types of links. The coefficients which are important below, for example, are described by the diagrams in Fig. 1. Analyzing the properties of the coefficients a_{ik} and Eq. (4), we find that $A_3 = -A_2$ and $A_1 \approx 1$ within the electron coupling constant g, which is assumed to be small. We then find

$$A_2 = a_{12}/(1 - a_{22} + a_{32}). \tag{5}$$

We see from Fig. 1 that the diagram for a_{22} has a large logarithm. If the frequency transfer ω_0 and momentum transfer $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ are moderately large ($\omega_0 \lesssim \Delta, vq \lesssim \Delta$), the coefficient a_{22} will contain the term $q\ln(2\omega_D/\Delta)$ which cancels out with 1 by virtue of the equation for the gap; the parameter ω_D limits the frequencies at which there is an attraction between electrons: this is the Debye frequency for the phonon mechanism. As a result, the terms of order g remain both in the numerator and denominator of A_2, i.e., $A_2 \sim 1$. In the case of a large energy transfer or large momentum transfer the interaction is unimportant, $A_2 \ll 1$, and only the first term is retained in (3).

Substituting into the matrix element S_{j0} the combination R instead of $\psi_\alpha^+\psi_\alpha$ (see Ref. 1) and performing the same transformations as in Ref. 1, we find (we assume, for

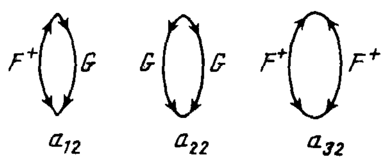

FIG. 1.

simplicity, that the incidence and reflection are normal)

$$d\sigma = \frac{2^9 e^4 \cos^2\theta\, d\omega' d\Omega'}{\pi m^2 \delta^2 [(n+1)^2 + \kappa^2][(n-1)^2 + \kappa^2]} \int dq F(q)/(q^2 + 4\delta^{-2})^2 \,, \qquad (6)$$

$$F(q) = \int |uv' + vu' - 2A_2(uu' + vv')|^2 \delta(\omega_0 - \epsilon - \epsilon') d^3 p / (2\pi)^3 \,, \qquad (7)$$

where n is the refractive index at the frequency ω; κ is the absorption coefficient which determines the field penetration depth, $\delta = c/\omega\kappa$; u and v are the Bogolyubov conversion factors; $u^2, v^2 = (1 \pm \xi/\epsilon)/2$; u' and v' corresond to the momentum $\mathbf{p} + \mathbf{q}$; the vector $\mathbf{q}$ is directed normal to the surface, with an absolute q over which the intergration is carried out; and θ is the angle between the polarization of incident light and the polarization of scattered light.

The coefficients appearing in A_2 are

$$a_{ik} = \frac{\lambda}{16} \int_{-\infty}^{+\infty} d\xi \int_{-1}^{1} d\mu [(\epsilon + \epsilon' + \omega_0 - i0)^{-1} + (\epsilon + \epsilon' - \omega_0 - i0)^{-1}] \rho_{ik} \,,$$

where the weight function ρ_{12} for a_{12} is $-\Delta\omega_0/2\epsilon\epsilon'$, and for the difference $a_{22} - a_{32}$ we have $\rho = 1 + (\xi\xi' + \Delta^2)/\epsilon\epsilon'$, $\lambda = g p_0^2/\pi^2 v$ is a dimensionless coupling constant, and $\mu = \cos \mathbf{vq}$.

Since we are considering the case (2), and since the important terms in the integration over $\mathbf{q}$, according to (6), are the terms $q \sim 1/\delta$, we must set $q = 0$ in the zeroth approximation. However, the expression inside the modulus in (7), with allowance for the δ function, will then vanish. This occurs precisely because of the incorporation of the chain diagrams. If these diagrams were ignored, as was the case in Ref. 3, then the first term, which is retained inside the modulus in (7), will, at $q = 0$, lead to a divergence near the threshold, $d\sigma \sim (\omega_0^2 - 4\Delta^2)^{-1/2}$.

Let us analyze the function $F(q)$ in the region $q \lesssim \Delta/v$. If we are near the threshold, i.e., if the condition $\omega_0 - 2\Delta \ll \Delta$ holds, then the value $q_m = (\omega_0^2 - 4\Delta^2)^{1/2}/v$, falls within this region, beginning with which the function $F(q)$ reaches its asymptotic behavior $F(q) \sim (vq/\Delta)^3 \ln^{-2}(q/q_m)$. At $q \ll q_m$ the function $F(q) \sim (vq/\Delta)^3 q/q_m$. Clearly the principal contribution to the integral over q comes from $q \lesssim \max\{q_m, \delta^{-1}\}$. We thus find the following expression (in standard units), depending on the relationship between q_m and $1/\delta$:

$$d\sigma = \frac{2^5}{\pi^4} \left(\frac{e^2 v}{\hbar c^2}\right)^2 \left(\frac{\hbar v}{\Delta\delta}\right)^2 \frac{\cos^2\theta\, f\, d\omega' d\Omega'}{\Delta[(n+1)^2 + \kappa^2][(n-1)^2 + \kappa^2]} \,, \qquad (8)$$

where

$$f \approx \begin{cases} 1 & \text{for} \quad q_m \gg 1/\delta & (8a) \\ \dfrac{\pi^2}{8} \ln(\delta/\xi)/\ln(1/q_m\delta)\ln(1/q_m\xi) & \text{for} \quad q_m \ll 1/\delta. & (8b) \end{cases}$$

Equation (8) can be used when $\omega_0 - 2\Delta \lesssim \Delta$. Far from the threshold, i.e., with the condition $\omega_0 - 2\Delta \gg \Delta$, satisfied, the result should correspond to the normal metal. A simple calculation leads to an expression which can be derived from (8) through the substitution

$$f = 4\pi^2(\Delta/\omega_0)^3 t^4 [\ln(1+t^{-2}) - (1+t^2)^{-1}], \qquad t = \omega_0 \delta/2v . \tag{8c}$$

This result describes Raman scattering by a normal metal for any relationship between the penetration depth and the frequency transfer. For case (2) which we are considering, Eqs. (8a) and (8c) are matched at $\omega_0 \approx 2.7\Delta$.

Equation (8) implies that near the threshold $d\sigma$ increases rapidly from zero to a value determined by (8) and (8a). A numerical estimate yields the value

$$d\sigma \approx 10^{-11}(\xi/\delta)^2(\hbar d\omega'/\Delta)d\Omega' . \tag{9}$$

Let us now consider qualitatively the consequences of anisotropy. The principal change is that the function $F(q)$ in (7) does not vanish at $q = 0$. Allowance for the anisotropy of the mass tensor ($m^{-1}A^2 \rightarrow m_{ik}^{-1}A_iA_k$, where $\mathbf{n} = \mathbf{p}/p$) shows that the result is proportional to $(\overline{m_{ik} - \bar{m}_{ik})^2}$, where an average is weighted in a way that depends on the nature of electron interaction and on $\Delta(\mathbf{n})$. The final expression

$$d\sigma_0 \sim \pi^{-2}(e^2/\hbar c)^2(v/c)^2\delta d\omega' d\Omega'/v[(1+n)^2+\kappa^2][(1-n)^2+\kappa^2] \tag{10}$$

does not depend on ξ far from the threshold. Near the threshold it has the form

$$d\sigma \sim d\sigma_0 [(\omega_0 - 2\Delta_{min})/\Delta_{min}]^\alpha . \tag{11}$$

If the minimum of $\Delta(\mathbf{n})$ corresponds to a single point of the Fermi surface, we would have $\alpha = 1/2$. If the minimum falls on the line (in the strictly two-dimensional case, for example), we would have $\alpha = 0$.

Our calculations show that integration involves the entire Fermi surface and that the result remains the same, regardless of whether we are dealing with a single crystal or a polycrystal comprised of crystallites with identical parameters but different orientations.

Substituting the numerical values ($\Delta \sim 10^2$ K) in (9), we find

$$d\sigma_0 \sim 10^{-11}\hbar d\Omega' d\omega'/\Delta .$$

This result corresponds to the result obtained experimentally in Ref. 4. Note that our result was obtained under the assumption that $\delta \gg \xi$, whereas the experiment of Ref. 4 apparently was carried out in accordance with the condition $\delta \sim \xi$.

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